Branching Immediate Observation Petri Nets

A strong class with simple reachability

Chana Weil-Kennedy

joint work with Javier Esparza and Mikhail Raskin
Branching Immediate Observation Petri Nets

A strong class with simple reachability

non-semilinear

PSPACE

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Petri nets
Petri nets

S \xrightarrow{t_1} C \xrightarrow{t_3} R \xrightarrow{t_4}

S \xrightarrow{t_1} W \xrightarrow{t_2} R \xrightarrow{t_4}

C
Petri nets
Petri nets
Petri nets
Petri nets

S → W → t2 → R → t4
S → t1 → C → t3 → R

C. Weil-Kennedy, TUM
Petri nets

[C. Weil-Kennedy, TUM]
Petri nets
Petri nets
Petri nets
Petri nets
Petri nets

\begin{align*}
S & \xrightarrow{t_1} C & S & \xrightarrow{t_3} R \\
W & \xrightarrow{t_2} R & (1,3,0,0) & \xrightarrow{t_4} (1,0,0,0)
\end{align*}
Petri nets

\[
\begin{align*}
S & 
\rightarrow 
\quad W & 
\rightarrow 
\quad t_2 & 
\rightarrow 
\quad R & 
\rightarrow 
\quad t_4 \\
\rightarrow 
\quad t_1 & 
\rightarrow 
\quad C & 
\rightarrow 
\quad t_3 \\
\end{align*}
\]

\[
S \ C \ W \ R \quad \rightarrow^* \quad S \ C \ W \ R
\]

\[
(1,3,0,0) \quad \rightarrow^* \quad (1,0,0,0)
\]
Reachability problem: Given a Petri net $\mathcal{N}$, and markings $M_0$ and $M$ can marking $M_0$ reach marking $M$ in $\mathcal{N}$?
Reachability Problem

Reachability problem: Given a Petri net \( \mathcal{N} \), and markings \( M_0 \) and \( M \) can marking \( M_0 \) reach marking \( M \) in \( \mathcal{N} \) ?

- verification of systems modelled by Petri nets
- many problems are interreducible with reachability in Petri nets in:
  - formal languages (e.g. shuffle closure of regular language)
  - logic (e.g. logics on data words)
  - process calculi (e.g. fragment of \( \pi \)-calculus)  

[survey by S. Schmitz, ’16]
Reachability Problem

**Reachability problem:** Given a Petri net $\mathcal{N}$, and markings $M_0$ and $M$ can marking $M_0$ reach marking $M$ in $\mathcal{N}$?

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  - process calculi (e.g. fragment of $\pi$-calculus) [survey by S. Schmitz, '16]

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non-elementary complexity [Czerwinski, Lasota, Lazic, Leroux, Mazowiecki, '19]
Reachability Problem

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  - process calculi (e.g. fragment of $\pi$-calculus) [survey by S. Schmitz, ’16]

Study subclasses of Petri nets

non-elementary complexity
[Czerwinzki, Lasota, Lazic, Leroux, Mazowiecki, ’19]
Branching immediate observation nets

- Token creation and destruction
- Communication-free

[Christensen et al., '93]
[Yen, '97]  [Lasota, '09]
[Mayr, Weihmann, '15]
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Branching Parallel Processes (BPP)

[Christensen et al., '93]
[Yen, '97] [Lasota, '09]
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Branching immediate observation nets

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References:

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Branching Parallel Processes (BPP)

- Conservative
- Communication

Immediate Observation nets (IO)

References:

- Christensen et al., ’93
- Yen, ’97
- Lasota, ’09
- Mayr, Weihmann, ’15
- Esparza, Raskin, W.-K., ’19
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[Christensen et al., ’93]
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Branching Parallel Processes (BPP)

- Token creation and destruction
- Communication-free

Immediate Observation nets (IO)

- Conservative
- Communication

[Esparza, Raskin, W.-K., ’19]
Definition

Branching Immediate Observation nets (BIO)

- Token creation and destruction
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Definition

Branching Immediate Observation nets (BIO)

- Token creation and destruction
- Communication

\[ \text{Card}(\bullet t - t^*) \leq 1 \]
Branching Immediate Observation nets
Branching Immediate Observation nets
A strong class with simple reachability

- General Petri nets
- Branching Immediate Observation (BIO)
- Branching Parallel Processes (BPP)
- Immediate Observation (IO)
- Conservative

- non-elementary
  - [Czerwinzki, Lasota, Lazic, Leroux, Mazowiecki, ’19]
- PSPACE-complete
  - [Esparza, Raskin, W.-K., ’19]
- NP-complete
  - [Esparza, ’97]
A strong class with simple reachability

General Petri nets

Branching Immediate Observation (BIO)

Branching Parallel Processes (BPP)

Immediate Observation (IO)

Conservative

Non-elementary

PSPACE-complete

[Czerwinski, Lasota, Lazic, Leroux, Mazowiecki, ’19]

PSPACE-complete

[Esparza, Raskin, W.-K., ’19]

NP-complete

[Esparza, ’97]
A strong class with simple reachability

Unbounded Petri net classes with provably simpler reachability then the general case have **semilinear** reachability sets (e.g. BPP nets, reversible Petri nets…)
A strong class with simple reachability

Unbounded Petri net classes with provably simpler reachability than the general case have **semilinear** reachability sets (e.g. BPP nets, reversible Petri nets...)

BIO nets may have **non-semilinear** reachability set

[Hopcroft, Pansiot, ’79] example of a 3-dimensional VASS

[VASS to Petri net classic translation]
A strong class with simple reachability

**Unbounded** Petri net classes with provably simpler reachability than the general case have *semilinear* reachability sets (e.g. BPP nets, reversible Petri nets…)

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[Hopcroft, Pansiot, ’79] example of a 3-dimensional VASS

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[Diagram]: A Petri net with places labeled $c_1$, $c_2$, and $c_3$, and transitions labeled $t_1$, $t_2$, $t_3$, and $t_4$. The places are connected by transitions and the net is colored to represent the different types of tokens.
A strong class with simple reachability

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BIO nets may have **non-semilinear** reachability set

[Hopcroft, Pansiot, ’79] example of a 3-dimensional VASS

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**Diagram:**

- VASS to Petri net classic translation
- BIO net
- c
- p
- c
- q
- t
- t
- t
- t
- t
- c

A strong class with simple reachability

**Unbounded** Petri net classes with provably simpler reachability than the general case have *semilinear* reachability sets (e.g. BPP nets, reversible Petri nets…)

BIO nets may have *non-semilinear* reachability set

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BIO nets may have **non-semilinear** reachability set

\[ c_2 + c_3 \leq 2^{c_1} \]

[Hopcroft, Pansiot, ’79] example of a 3-dimensional VASS
PSPACE reachability

BIO nets reachability is a **PSPACE-complete** problem

- **PSPACE-hard** by weakly simulating bounded tape Turing machines
PSPACE reachability

BIO nets reachability is a **PSPACE-complete** problem

- **PSPACE-hard** by weakly simulating bounded tape Turing machines
- Solvable in **PSPACE** via a **main theorem** which provides firing sequences of **bounded length** and **bounded token count**.
PSPACE reachability
Main Theorem

In a BIO net with n places, and transitions producing \( \leq \gamma \) tokens

If \( M_0 \xrightarrow{*} M \)

then \( \exists \) markings \( M_1, M_2, \ldots, M_l \)

\( \exists \) transitions \( t_1, t_2, \ldots, t_l \)

\( \exists \) constants \( k_1, k_2, \ldots k_l \geq 0 \)

\( M_0 \xrightarrow{t_{k_1}^1} M_1 \xrightarrow{t_{k_2}^2} M_2 \xrightarrow{\ldots} M_l = M \)
PSPACE reachability

Main Theorem

In a BIO net with n places, and transitions producing $\leq \gamma$ tokens

If $M_0 \rightarrow^* M$

then $\exists$ markings $M_1, M_2, \ldots, M_l$

$\exists$ transitions $t_1, t_2, \ldots, t_l$

$\exists$ constants $k_1, k_2, \ldots k_l \geq 0$

such that $l \in O(\frac{|M|}{\gamma} n)^n$  \hspace{1cm} \text{bound on (accelerated) length}
**Main Theorem**

*In a BIO net with $n$ places, and transitions producing $\leq \gamma$ tokens*

If $M_0 \rightarrow^* M$

then there exist markings $M_1, M_2, \ldots, M_l$,

transitions $t_1, t_2, \ldots, t_l$

and constants $k_1, k_2, \ldots, k_l \geq 0$

such that $l \in O(|M| n)^n$ bound on (accelerated) length

and $\forall i, M_i \in O(|M_0| |M| n\gamma)^n$ bound on token count

$M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \rightarrow \ldots \rightarrow M_l = M$

$l$
PSPACE reachability

Main Theorem

In a BIO net with $n$ places, and transitions producing $\leq \gamma$ tokens

If $M_0 \rightarrow^* M$

then $\exists$ markings $M_1, M_2, \ldots, M_l$

$\exists$ transitions $t_1, t_2, \ldots, t_l$

$\exists$ constants $k_1, k_2, \ldots, k_l \geq 0$

such that $l \in O(|M| n)^n$ bound on (accelerated) length

and $\forall i, M_i \in O(|M_0| |M| n\gamma)^n$ bound on token count

\[ M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \rightarrow \ldots \rightarrow M_l = M \]

accelerated length

token count

$|M_0|$

$|M|$

C. Weil-Kennedy, TUM
Main Theorem

In a BIO net with $n$ places, and transitions producing $\leq \gamma$ tokens

If $M_0 \rightarrow^* M$

then $\exists$ markings $M_1, M_2, \ldots, M_l$

$\exists$ transitions $t_1, t_2, \ldots, t_l$

$\exists$ constants $k_1, k_2, \ldots, k_l \geq 0$

such that $l \in O(|M| n^n)$ bound on (accelerated) length

and $\forall i, M_i \in O(|M_0| |M| n\gamma^n)$ bound on token count

accelerated length

| $M_0$ | $|M|$ |

token count

PSPACE reachability

Main Theorem

In a BIO net with $n$ places, and transitions producing $\leq \gamma$ tokens

If $M_0 \overset{*}{\rightarrow} M$

then $\exists$ markings $M_1, M_2, \ldots, M_l$ $\exists$ transitions $t_1, t_2, \ldots, t_l$ $\exists$ constants $k_1, k_2, \ldots, k_l \geq 0$

$M_0 \overset{t_1^{k_1}}{\rightarrow} M_1 \overset{t_2^{k_2}}{\rightarrow} M_2 \rightarrow \ldots \overset{t_l^{k_l}}{\rightarrow} M_l = M$

such that $l \in O(|M|n)^n$ and $\forall i, M_i \in O(|M_o| |M| \gamma^n)^n$

NPSPACE algorithm for reachability

$M_0 \overset{*}{\rightarrow} M$?
PSPACE reachability

Main Theorem

In a BIO net with n places, and transitions producing $\leq \gamma$ tokens

If $M_0 \ast \rightarrow M$

then $\exists$ markings $M_1, M_2, \ldots, M_l$ $\exists$ transitions $t_1, t_2, \ldots, t_l$ $\exists$ constants $k_1, k_2, \ldots, k_l \geq 0$

$M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \rightarrow \ldots \xrightarrow{t_l^{k_l}} M_l = M$

such that $l \in O(|M|n)^n$ and $\forall i, M_i \in O(|M_o| |M| n \gamma)^n$

NPSPACE algorithm for reachability

$M_0 \ast \rightarrow M$ ?

Guess the first marking $M_1$ Check that there $\exists t, k$ such that $M_0 \xrightarrow{t^k} M_1$ Guess the next marking $M_2$ ...

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PSPACE reachability
PSPACE reachability

\[ M_0 \]

\[ \begin{align*}
& p \\
& q \\
& q
\end{align*} \]
PSPACE reachability

$M_0$ $M_1$
PSPACE reachability

\[ M_0 \quad M_1 \]

\[ p \quad q \quad r \]

\[ 2 \]

C. Weil-Kennedy, TUM
PSPACE reachability

\[ p \rightarrow r \rightarrow r \rightarrow r \rightarrow q \rightarrow q \rightarrow q \rightarrow q \rightarrow M_0 \rightarrow M_1 \rightarrow M_2 \]

\[ p \rightarrow 2 \rightarrow q \rightarrow r \]

C. Weil-Kennedy, TUM
PSPACE reachability

\[ q \quad q \quad q \quad M_0 \quad M_1 \quad M_2 \quad p \]

\[ r \quad r \quad r \quad p \quad q \quad r \quad r \]

\[ q \quad q \quad q \quad r \quad q \quad q \quad p \]

\[ q \quad q \quad q \quad q \quad q \quad q \quad q \]
PSPACE reachability
PSPACE reachability
PSPACE reachability
PSPACE reachability
PSPACE reachability

\[ M_0 \quad M_1 \quad M_2 \quad M_3 \quad M \]

final tokens

\[ p \quad q \quad q \quad q \quad q \]

\[ p \quad r \quad r \quad r \quad r \]

\[ q \quad q \quad q \quad q \quad q \]

\[ p \quad r \quad r \quad r \quad r \]

\[ q \quad q \quad q \quad q \quad q \]

\[ M \]
PSPACE reachability

\[ p \rightarrow r \rightarrow r \rightarrow r \rightarrow r \rightarrow r \rightarrow r \]

\[ q \rightarrow q \rightarrow q \rightarrow q \rightarrow q \rightarrow q \]

\[ M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow M \]

**final tokens**

**helper tokens**

\[ p \rightarrow 3 \rightarrow r \]

\[ q \rightarrow q \]

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PSPACE reachability

**Diagram:**

- **Nodes:**
  - $p$
  - $q$
  - $r$
  - $M_0$, $M_1$, $M_2$, $M_3$, $M$

- **Edges:**
  - Green edges: $r$
  - Blue edges: $q$
  - Black edges: $2$

- **Tokens:**
  - Green tokens: final tokens
  - Blue tokens: helper tokens

**Legend:**
- $p$
- $q$
- $r$
- $M_0$, $M_1$, $M_2$, $M_3$, $M$
PSPACE reachability
PSPACE reachability

\[ M_0 \quad M_1 \quad M_2 \quad M_3 \quad M \]

final tokens

helper tokens

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PSPACE reachability
PSPACE reachability

1. Keep the **final** tokens
PSPACE reachability

1. Keep the final tokens
2. Reduce the number of helper tokens
PSPACE reachability

1. Keep the final tokens

2. Reduce the number of helper tokens
PSPACE reachability

1. Keep the final tokens
2. Reduce the number of helper tokens
PSPACE reachability

1. Keep the final tokens

2. Reduce the number of helper tokens
PSPACE reachability

1. Keep the final tokens
2. Reduce the number of helper tokens
3. Cut out repetitions of final + helper
PSPACE reachability

1. Keep the final tokens

2. Reduce the number of helper tokens

3. Cut out repetitions of final + helper
PSPACE reachability

1. Keep the final tokens
2. Reduce the number of helper tokens
3. Cut out repetitions of final + helper
PSPACE reachability

1. Keep the **final** tokens \[\leq |M|\] per intermediate marking

2. Reduce the number of **helper** tokens

3. Cut out repetitions of **final** + **helper**
PSPACE reachability

1. Keep the final tokens $\leq |M|$ per intermediate marking

2. Reduce the number of helper tokens $\leq n^2$ per intermediate marking

3. Cut out repetitions of final + helper
PSPACE reachability

1. Keep the \textit{final} tokens $\leq |M|$ per intermediate marking
2. Reduce the number of \textit{helper} tokens $\leq n^2$ per intermediate marking
3. Cut out repetitions of \textit{final} + \textit{helper} accelerated length $\leq O(\# \text{ such configurations})$
Flatness

[Leroux, Sutre, ’05]

flat \( \exists \) sequence \( t_1^* t_2^* \ldots t_l^* \) such that

\[
M_0 \rightarrow^* M \text{ iff } M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l}} M
\]
Flatness

[Leroux, Sutre, ’05]

\textbf{flat} \ \exists \ \text{sequence} \ t_1^* t_2^* \ldots t_l^* \ \text{such that}

\[
M_0^* \to M \quad \text{iff} \quad M_0 \xrightarrow{t_1^k_1 t_2^k_2 \ldots t_l^k_l} M
\]

BPP, IO nets
Flatness

[Le Roux, Sutre, '05]

**flat** \( \exists \) sequence \( t_1^* t_2^* \ldots t_l^* \) such that 
\[
M_0^* \rightarrow M \text{ iff } \begin{array}{c}
M_0 \\
\downarrow
\end{array} \rightarrow \begin{array}{c}
M \\
\end{array}
\]

BPP, IO nets

**pre*-flat** \( \forall M, \exists \) sequence \( t_1^* t_2^* \ldots t_l^* \) such that 
\[
M_0^* \rightarrow M \text{ iff } \begin{array}{c}
M_0 \rightarrow \begin{array}{c}
t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l}
\end{array}
\end{array} \rightarrow \begin{array}{c}
M \\
\end{array}
\]
Flatness

[Leroux, Sutre, '05]

\textbf{Flat} \quad \exists \text{ sequence } t^*_1t^*_2 \ldots t^*_l \text{ such that } \quad \textbf{pre}^*\text{-flat} \quad \forall M, \exists \text{ sequence } t^*_1t^*_2 \ldots t^*_l \text{ such that }

\[
M_0 \xrightarrow{*} M \iff M_0 \xrightarrow{t^*_1t^*_2 \ldots t^*_l} M
\]

BPP, IO nets

Main Theorem

In a BIO net with \( n \) places, and transitions producing \( \leq \gamma \) tokens

\[
\text{If } M_0 \xrightarrow{*} M
\]

then \( \exists \) markings \( M_1, M_2, \ldots, M_l \) \( \exists \) transitions \( t_1, t_2, \ldots, t_l \) \( \exists \) constants \( k_1, k_2, \ldots k_l \geq 0 \)

\[
M_0 \xrightarrow{t^*_1} M_1 \xrightarrow{t^*_2} M_2 \rightarrow \ldots \rightarrow M_l = M
\]

such that \( l \in O(|M|n)^n \) and \( \forall i, M_i \in O(|M_o| |M|n\gamma)^n \)
Flatness

[LEROUX, SUTRE, ’05]

**Flat** \( \exists \) sequence \( t_1^* t_2^* \ldots t_l^* \) such that

\[
\begin{align*}
M_0 \rightarrow^* M & \iff M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l}} M \\
\downarrow & \\
\text{BPP, IO nets}
\end{align*}
\]

**Pre-flat** \( \forall M, \exists \) sequence \( t_1^* t_2^* \ldots t_l^* \) such that

\[
\begin{align*}
M_0 \rightarrow^* M & \iff M_0 \xrightarrow{t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l}} M \\
\downarrow & \\
\text{BIO nets}
\end{align*}
\]

**Main Theorem**

*In a BIO net with \( n \) places, and transitions producing \( \leq \gamma \) tokens*

If \( M_0 \rightarrow^* M \)

then \( \exists \) markings \( M_1, M_2, \ldots, M_l \) \( \exists \) transitions \( t_1, t_2, \ldots, t_l \) \( \exists \) constants \( k_1, k_2, \ldots k_l \geq 0 \)

\[
M_0 \xrightarrow{t_1^{k_1}} M_1 \xrightarrow{t_2^{k_2}} M_2 \rightarrow \ldots \rightarrow M_l = M
\]

such that \( l \in O(|M| n)^n \) and \( \forall i, M_i \in O(|M_o| |M| n \gamma)^n \)
Flatness

[Le roux, Sutre, '05]

**flat** \( \exists \) sequence \( t^*_1t^*_2\ldots t^*_l \) such that

\[
M_0 \xrightarrow{t^*_1} M \iff M_0 \xrightarrow{\neg t^*_1t^*_2\ldots t^*_l} M
\]

\downarrow

BPP, IO nets

**pre*-flat** \( \forall M, \exists \) sequence \( t^*_1t^*_2\ldots t^*_l \) such that

\[
M_0 \xrightarrow{t^*_1} M \iff M_0 \xrightarrow{t^*_1t^*_2\ldots t^*_l} M
\]

\downarrow

BIO nets

Main Theorem

*In a BIO net with \( n \) places, and transitions producing \( \leq \gamma \) tokens*

*If* \( M_0 \xrightarrow{*} M \)

*then* \( \exists \) markings \( M_1, M_2, \ldots, M_l \) \( \exists \) transitions \( t_1, t_2, \ldots, t_l \) \( \exists \) constants \( k_1, k_2, \ldots k_l \geq 0 \)

\[
M_0 \xrightarrow{t^*_1} M_1 \xrightarrow{t^*_2} M_2 \xrightarrow{t^*_l} M_l = M
\]

*such that* \( l \in O(|M|n)^n \) and \( \forall i, M_i \in O(|M_o| |M| n\gamma)^n \)

**model checking tools with acceleration techniques**

*e.g. FAST* [Bardin, Finkel, Leroux, Petrucci, '03]
Conclusion

- Non-elementary
  [Czerwinksi, Lasota, Lazic, Leroux, Mazowiecki, ’19]

- \textbf{PSPACE-complete}
  [Esparza, Raskin, W.-K., ’20]

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- \textbf{NP-complete}
  [Esparza, ’97]
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Other results:

• reachability between possibly infinite sets of markings (cubes) is also PSPACE-complete
• this also holds for coverability, liveness, and more
Conclusion

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• reachability between possibly infinite sets of markings (*cubes*) is also PSPACE-complete
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Future: Investigate consequences in chemical reaction networks, formal languages, etc.