Branching Immediate Observation
Petri Nets

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joint work with Javier Esparza and Mikhail Raskin
Reachability Problem

Reachability problem: Given a Petri net $\mathcal{N}$, and markings $M_0$ and $M$, can marking $M_0$ reach marking $M$ in $\mathcal{N}$?
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Study subclasses of Petri nets
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Study subclasses of Petri nets

Study Branching Immediate Observation (BIO) nets!
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\( \equiv \) at most one “pure input” place
Branching Immediate Observation (BIO) nets

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\(\equiv\) at most one "pure input" place

Extend
- BPP nets:
- IO nets:
Flatness

[Leroux, Sutre, ’05]

\[ \text{flat} \ \exists \ \text{sequence} \ \ t_1^* t_2^* \ldots t_l^* \ \text{such that} \]

\[ M_0 \to M \iff M_0 t_1^{k_1} t_2^{k_2} \ldots t_l^{k_l} \to M \]
Flatness

[Leroux, Sutre, ’05]

\[ \exists \text{ sequence } t_1^* t_2^* \ldots t_l^* \text{ such that } \]

\[ M_0 \xrightarrow{k_1 t_1 k_2 t_2 \ldots k_l t_l} M \]

BPP, IO nets
Flatness

[Leroux, Sutre, ’05]

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M_0 \overset{*}{\rightarrow} M \text{ iff } M_0 \xrightarrow{t_1^* t_2^* \ldots t_l^*} M
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BPP, IO nets

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BIO nets
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BIO nets

check **reachability properties** with model checking **tools** that use acceleration techniques

**e.g. FAST** [Bardin, Finkel, Leroux, Petrucci, '03]
A strong class with simple reachability

- General Petri nets
  - Branching Immediate Observation (BIO)
    - Branching Parallel Processes (BPP)
    - Immediate Observation (IO)
      - Conservative
        - Non-elementary
          - PSPACE-complete
            - PSPACE-complete
              - NP-complete

[Czerwinski, Lasota, Lazic, Leroux, Mazowiecki, ’19]
[Esparza, Raskin, W.-K., ’19]
[Esparza, ’97]
A strong class with simple reachability

BIO nets can have **non-semilinear** reachability set

[Hopcroft, Pansiot, ’79] example of a 3-dimensional VASS

\[ c_2 + c_3 \leq 2^{c_1} \]
Branching Immediate Observation (BIO)

- unbounded (token creation and destruction)
- \(\textit{pre}^*\)-flat reachability relation \(\rightarrow\) use of model-checking tools like FAST
- \textbf{PSPACE-complete} reachability problem
- \textbf{non-semilinear} reachability
Branching Immediate Observation (BIO)

- unbounded (token creation and destruction)
- $pre^\ast$-flat reachability relation $\rightarrow$ use of model-checking tools like FAST
- **PSPACE-complete** reachability problem
- non-semilinear reachability

**Open Problems:**
- Applications for BIO nets (e.g. chemical reaction networks)
- Consequences of this result in other domains (data nets, process calculi, formal languages…)

**Article:** https://drops.dagstuhl.de/opus/volltexte/2020/12857/

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