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Introduction

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- These can be defined by using different frameworks.
- A unified presentation of both, the observational and the equational semantics, has been discussed here previously.
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Introduction

The classical ltbt-spectrum
Next we present a unified view of the logical semantics.

Each language $\mathcal{L} \subseteq \text{HML}$ defines a preorder $<_{\mathcal{L}}$, given by

$$(p <_{\mathcal{L}} q \leftrightarrow (p \models \varphi \Rightarrow q \models \varphi)).$$

(A bit surprisingly!) We look for sets of formulas, characterizing each semantics, as large as possible.
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Introduction

(A part of) the enlarged spectrum
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Some useful definitions

All the semantics that we consider can be defined over arbitrary (possibly infinite) processes whose operational semantics is defined by a lts

\[ \mathcal{P} = (\text{Proc}, \text{Act}, \rightarrow) \]

We will use \( p \xrightarrow{a} p' \) to represent the transitions between processes.

**Definition (BCCSP)**

Given a set of actions Act, the set BCCSP(Act) of processes is defined by the BNF-grammar:

\[
p ::= 0 \mid a \ p \mid p + q
\]
Some useful definitions

**Definition (N-Constrained simulation)**

Given a relation $N$ over BCCSP processes, an $N$-constrained simulation is a relation $S_N$ such that $S_N \subseteq N$, and whenever $p S_N q$ and $p \xrightarrow{a} p'$, there exists $q'$ with $q \xrightarrow{a} q'$ and $p' S_N q'$. We say that $p$ is $N$-simulated by $q$, or that $q$ $N$-simulates $p$, written $p \sqsubseteq_{NS} q$, when there exists an $N$-constrained simulation $S_N$ such that $p S_N q$.

**Definition (Hennessy-Milner logic, HML)**

The set $\mathcal{L}_{HM}$ of Hennessy-Milner logical formulas is defined by:

- If $\varphi_i \in \mathcal{L}_{HM}$ and $i \in I$ then $\bigwedge_{i \in I} \varphi_i \in \mathcal{L}_{HM}$.
- If $a \in \text{Act}$ and $\varphi \in \mathcal{L}_{HM}$ then $a \varphi \in \mathcal{L}_{HM}$.
- If $\varphi \in \mathcal{L}_{HM}$ then $\neg \varphi \in \mathcal{L}_{HM}$.
Some useful definitions

**Definition (Satisfaction relation)**

For each lts $P$, the satisfaction relation $\models \subseteq P \times \mathcal{L}_{HM}$ is defined by:

- $p \models a\varphi$ if there exists $q \in P : p \xrightarrow{a} q$ and $q \models \varphi$;
- $p \models \bigwedge_{i \in I} \varphi_i$ if for all $i \in I : p \models \varphi_i$.
- $p \models \lnot \varphi$ if $p \not\models \varphi$.

**Definition**

Any subset $\mathcal{L}$ of $\mathcal{L}_{HM}$ induces a logical semantics for processes, given by the preorder $\sqsubseteq_{\mathcal{L}}$: We have $p \sqsubseteq_{\mathcal{L}} q$ if, and only if, for all $\varphi \in \mathcal{L}$ ($p \models \varphi \Rightarrow q \models \varphi$). We say that $\mathcal{L}$ and $\mathcal{L}'$ are equivalent, and we write $\mathcal{L} \sim \mathcal{L}'$, if they induce the same semantics, that is $\sqsubseteq_{\mathcal{L}} = \sqsubseteq_{\mathcal{L}'}$. 
## Van Glabbeek’s logical characterizations

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<thead>
<tr>
<th>Formulas</th>
<th>Semantics ($\mathcal{Z}$)</th>
<th>T</th>
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Van Glabbeek’s logical characterizations

Some syntactic sugar for Van Glabbeek’s logics...

\[ \tilde{X} := \bigwedge_{a \in X} \neg a \top \quad \tilde{X} \varphi' := \tilde{X} \land \varphi' \quad 0 := \tilde{\text{Act}} \]

\[ \varphi_1 \land \varphi_2 := \bigwedge_{i \in \{1,2\}} \varphi_i \quad X := \bigwedge_{a \in X} a \top \land \bigwedge_{a \notin X} \neg a \top \]

\[ X \varphi' := X \land \varphi' \quad \tilde{a} := \neg a \top \]
Van Glabbeek’s logical characterizations

**THEOREM (Disjunction theorem)**

If we define $\mathcal{L}_Z^\vee$ with $Z \in \{\text{T, CT, F, FT, R, RT, PF, S, CS, RS, 2S, PW, B}\}$, by adding to the definition of $\mathcal{L}_Z$ the clause

$$\sigma_i \in \mathcal{L}_Z^\vee \ \forall i \in I \Rightarrow \bigvee_{i \in I} \sigma_i \in \mathcal{L}_Z^\vee$$

and replacing $\mathcal{L}_Z$ by $\mathcal{L}_Z^\vee$ in each of the other clauses, and we take

$$p \models \bigvee \sigma_i ::= \exists i \in I: p \models \sigma_i$$

then we have $\mathcal{L}_Z^\vee \sim \mathcal{L}_Z$ for any such $Z$. 
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Main ideas

- In order to obtain a uniform presentation of the logics characterizing each of the semantics we look for (simple) sets of formulas, but as large as possible.
- As for other previously studied frameworks, we divide the semantics in two classes: the branching and the linear ones.
- The key point to get the different logics is to use in the proper way in each case, negations and conjunctions.
- Whenever a semantics is finer than other, the logic characterizing the first will contain that for the latter, thus making trivial that relationship.
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- Whenever a semantics is finer than other, the logic characterizing the first will contain that for the latter, thus making trivial that relationship.
The layer of RS provides some illustrative examples

- The most important layer in the extended spectrum is that corresponding to ready simulation.

- The logical language characterizing the set of initial actions of a process: \( L_I = \{ aT \mid a \in \text{Act} \} \).
The layer of RS provides some illustrative examples

- Simulation Game.
- Set of Initial actions

\[ P_1 \sim_{\text{RS}} P_2 \]
The layer of RS provides some illustrative examples

\[ P_2 \not\models_{RS} P_3 \]

\[ P_2 \models (bc \land bd), \text{ but } P_3 \text{ does not.} \]

- Branching semantics.
- Unrestricted use of conjunctions.
The layer of RS provides some illustrative examples

- Linear semantics (prefix operator).
- Positive information about initial actions.

\[ P_4 \not
\subseteq_{\{RT,R\}} P_3 \]
\[ P_4 \models ab(c \land d), \text{ but } P_3 \not\models ab(c \land d) \]
The layer of RS provides some illustrative examples

- Linear semantics (prefix operator).
- Negative information about initial actions.
The layer of RS provides some illustrative examples

- Linear semantics (prefix operator).
- Positive (resp. negative) information about initial actions for the case or R (resp. FT).

\[ P_7 \not\equiv_{R,FT} P_6 \]
\[ P_7 \models a(\neg e \land c) \]
The layer of RS provides some illustrative examples

- Linear semantics (prefix operator).
- Negative information about initial actions.
Uniform logical characterizations of the semantics at the layer of RS

**Definition (Negative closure $\mathcal{L}_N^-$)**

Given a set of formulas $\mathcal{L}_N'$, we define $\mathcal{L}_N^-$ by:

- $\sigma \in \mathcal{L}_N' \Rightarrow \neg \sigma \in \mathcal{L}_N^-$
- $\sigma_i \in \mathcal{L}_N^- \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in \mathcal{L}_N^-$

**Definition (Failure semantics)**

Inspired by the order $\leq_{I}^{lf}$, we define the set of formulas $\mathcal{L}_F'$ by:

- $\top \in \mathcal{L}_F'$
- $\sigma \in \mathcal{L}_I^- \Rightarrow \sigma \in \mathcal{L}_F'$
- $\varphi \in \mathcal{L}_F', \ a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}_F'$
Uniform logical characterizations of the semantics at the layer of RS

**Definition (Negative closure \( \mathcal{L}_N^- \))**

Given a set of formulas \( \mathcal{L}_N' \), we define \( \mathcal{L}_N^- \) by:

- \( \sigma \in \mathcal{L}_N' \Rightarrow -\sigma \in \mathcal{L}_N^- \)
- \( \sigma_i \in \mathcal{L}_N^- \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in \mathcal{L}_N^- \)

**Definition (Failure trace semantics)**

Inspired by the order \( \leq_{I}^{12} \), we define the set of formulas \( \mathcal{L}_{FT}' \) by:

- \( \top \in \mathcal{L}_{FT}' \)
- \( \varphi \in \mathcal{L}_{FT}', \sigma \in \mathcal{L}_I^- \Rightarrow \sigma \land \varphi \in \mathcal{L}_{FT}' \)
- \( \varphi \in \mathcal{L}_{FT}', a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}_{FT}' \)
Uniform logical characterizations of the semantics at the layer of RS

**Definition (Symmetric closure $\mathcal{L}_N^\equiv$)**

Given a set of formulas $\mathcal{L}_N'$, we define $\mathcal{L}_N^\equiv$ by:

- $\sigma \in \mathcal{L}_N' \Rightarrow \sigma \in \mathcal{L}_N^\equiv$
- $\sigma \in \mathcal{L}_N' \Rightarrow \neg \sigma \in \mathcal{L}_N^\equiv$
- $\sigma_i \in \mathcal{L}_N^\equiv \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in \mathcal{L}_N^\equiv$

**Definition (Readiness semantics)**

Inspired by the order $\leq_I^{lf}$, we define the set of formulas $\mathcal{L}_R'$ by:

- $\top \in \mathcal{L}_R'$
- $\sigma \in \mathcal{L}_I^\equiv \Rightarrow \sigma \in \mathcal{L}_R'$
- $\varphi \in \mathcal{L}_R'$, $a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}_R'$
Uniform logical characterizations of the semantics at the layer of RS

**Definition (Symmetric closure \( \mathcal{L}_{N}^{=} \))**

Given a set of formulas \( \mathcal{L}_{N}' \), we define \( \mathcal{L}_{N}^{=} \) by:

- \( \sigma \in \mathcal{L}_{N}' \Rightarrow \sigma \in \mathcal{L}_{N}^{=} \)
- \( \sigma \in \mathcal{L}_{N}' \Rightarrow \neg \sigma \in \mathcal{L}_{N}^{=} \)
- \( \sigma_{i} \in \mathcal{L}_{N}^{=} \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_{i} \in \mathcal{L}_{N}^{=} \)

**Definition (Ready trace semantics)**

Inspired by the order \( \leq_{I}^{1} \), we define the set of formulas \( \mathcal{L}_{RT}' \) by:

- \( \top \in \mathcal{L}_{RT}' \)
- \( \varphi \in \mathcal{L}_{RT}', \sigma \in \mathcal{L}_{I}^{=} \Rightarrow \sigma \land \varphi \in \mathcal{L}_{RT}' \)
- \( \varphi \in \mathcal{L}_{RT}', a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}_{RT}' \)
Uniform logical characterizations of the semantics at the layer of RS

Definition (Ready simulation semantics)

We define the set of formulas $\mathcal{L}_{IS}'$, which is also denoted by $\mathcal{L}_{RS}'$, that defines the I-constrained simulation semantics by:

- $\sigma \in \mathcal{L}'_I \Rightarrow \sigma \in \mathcal{L}'_{RS}$
- $\sigma \in \mathcal{L}'_I \Rightarrow \neg \sigma \in \mathcal{L}'_{RS}$
- For any set $I$, $\varphi_i \in \mathcal{L}'_{RS} \forall i \in I \Rightarrow \bigwedge_{i \in I} \varphi_i \in \mathcal{L}'_{RS}$
- $\phi \in \mathcal{L}'_{RS}, \ a \in \text{Act} \Rightarrow a\phi \in \mathcal{L}'_{RS}$

Theorem

We have (1) $\mathcal{L}_{RS} \sim \mathcal{L}'_{RS}$; (2) $\mathcal{L}_{RT} \sim \mathcal{L}'_{RT}$; (3) $\mathcal{L}_{FT} \sim \mathcal{L}'_{FT}$; (4) $\mathcal{L}_R \sim \mathcal{L}'_R$ and (5) $\mathcal{L}_F \sim \mathcal{L}'_F$. 
Uniform logical characterizations of the semantics at the layer of RS

**Definition (Ready simulation semantics)**

We define the set of formulas $\mathcal{L}'_{IS}$, which is also denoted by $\mathcal{L}'_{RS}$, that defines the I-constrained simulation semantics by:

- $\sigma \in \mathcal{L}'_I \Rightarrow \sigma \in \mathcal{L}'_{RS}$
- $\sigma \in \mathcal{L}'_I \Rightarrow \neg \sigma \in \mathcal{L}'_{RS}$
- For any set $I$, $\varphi_i \in \mathcal{L}'_{RS} \forall i \in I \Rightarrow \bigwedge_{i \in I} \varphi_i \in \mathcal{L}'_{RS}$
- $\varphi \in \mathcal{L}'_{RS}, a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}'_{RS}$

**Theorem**

We have (1) $\mathcal{L}_{RS} \sim \mathcal{L}'_{RS}$; (2) $\mathcal{L}_{RT} \sim \mathcal{L}'_{RT}$; (3) $\mathcal{L}_{FT} \sim \mathcal{L}'_{FT}$; (4) $\mathcal{L}_{R} \sim \mathcal{L}'_{R}$ and (5) $\mathcal{L}_{F} \sim \mathcal{L}'_{F}$.
Uniform logical characterizations of the semantics at the layer of RS

**Definition (Possible Worlds semantics)**

We define the formulas of $\mathcal{L}'_{D_1}$, which is also denoted $b\mathcal{L}'_{PW}$, by:

- $\top \in \mathcal{L}'_{PW}$
- $\phi \in \mathcal{L}'_{PW}, \; \sigma \in \mathcal{L}^{=I} \Rightarrow \sigma \land \phi \in \mathcal{L}'_{PW}$
- $X \subseteq \text{Act}, \; \phi_a \in \mathcal{L}'_{PW} \; \forall a \in X \Rightarrow \land_{a \in X} a\phi_a \in \mathcal{L}'_{PW}$

In this case $\mathcal{L}_{PW}$ and $\mathcal{L}'_{PW}$ are not equivalent, but this is caused by the fact that the original logical characterization $\mathcal{L}_{PW}$ was wrong!
Our new logical characterizations of the semantics

Uniform logical characterizations of the semantics at the layer of RS

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In this case \( \mathcal{L}_{PW} \) and \( \mathcal{L}'_{PW} \) are not equivalent, but this is caused by the fact that the original logical characterization \( \mathcal{L}_{PW} \) was wrong!
Uniform logical characterizations of the semantics at the layer of RS

\[ \varphi \equiv a(\neg d \land bc) \in \mathcal{L}_{PW}' \]

\[ P \models \varphi \quad \text{Q} \not\models \varphi \]

- \( T \in \mathcal{L}_{PW}' \)
- \( \varphi \in \mathcal{L}_{PW}' \), \( \sigma \in \mathcal{L}_{I}^{\equiv} \Rightarrow \sigma \land \varphi \in \mathcal{L}_{PW}' \)
- \( X \subseteq \text{Act}, \ \varphi_a \in \mathcal{L}_{PW}' \ \forall a \in X \Rightarrow \bigwedge_{a \in X} a\varphi_a \in \mathcal{L}_{PW}' \)
Uniform logical characterizations of the semantics at the layer of RS

\[ \varphi \equiv a(\neg d \land bc) \notin \mathcal{L}_{PW} \]

- \( X \subseteq \text{Act} \Rightarrow X \in \mathcal{L}_{PW} \)
- \( X \subseteq \text{Act}, \ \varphi_a \in \mathcal{L}_{PW} \ \forall a \in X \Rightarrow \bigwedge_{a \in X} a \varphi_a \in \mathcal{L}_{PW} \)
Uniform logical characterizations of all the semantics

Logical characterizations of the semantics used as constraints in the N-constrained semantics

<table>
<thead>
<tr>
<th>Constraints ((\mathcal{N}))</th>
<th>U</th>
<th>C</th>
<th>I</th>
<th>T</th>
<th>S</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\top \in \mathcal{L}'_N)</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>ν</td>
<td>ν</td>
</tr>
<tr>
<td>(\neg \top = \bot \in \mathcal{L}'_N)</td>
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<td>ν</td>
<td>ν</td>
<td>ν</td>
<td>ν</td>
<td>ν</td>
</tr>
<tr>
<td>(\neg 0 \in \mathcal{L}'_N)</td>
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<td>●</td>
<td>ν</td>
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<td>ν</td>
</tr>
<tr>
<td>(a \in \text{Act} \Rightarrow a\top \in \mathcal{L}'_N)</td>
<td>●</td>
<td>ν</td>
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</tr>
<tr>
<td>(\varphi \in \mathcal{L}'_N, a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}'_N)</td>
<td>●</td>
<td>●</td>
<td>●</td>
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<tr>
<td>(\varphi_i \in \mathcal{L}'<em>N \forall i \in I \Rightarrow \bigwedge</em>{i \in I} \varphi_i \in \mathcal{L}'_N)</td>
<td>●</td>
<td>●</td>
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<tr>
<td>(\varphi \in \mathcal{L}'_N \Rightarrow \neg \varphi \in \mathcal{L}'_N)</td>
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<td>●</td>
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Uniform logical characterizations of all the semantics

<table>
<thead>
<tr>
<th>Formulas</th>
<th>$\leq_{F}^{1}$</th>
<th>$\leq_{R}^{1}$</th>
<th>$\leq_{T}^{1}$</th>
<th>$\leq_{N}^{1}$</th>
<th>$D_{N}$</th>
<th>$NS$</th>
<th>$N \in {U,C,I,T,S}$</th>
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</thead>
<tbody>
<tr>
<td>$\top \in L_{y_{N}}'$</td>
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<td>$\bullet$</td>
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<td>$\bullet$</td>
<td>$\nu$</td>
<td>when $N = I$</td>
</tr>
<tr>
<td>$\phi \in L_{y_{N}}'$, $a \in Act \Rightarrow$</td>
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<td>$\nu$</td>
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<tr>
<td>$a \phi \in L_{y_{N}}'$</td>
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<td>$\phi \in L_{N} \Rightarrow$</td>
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<td>$\varphi \in L_{y_{N}}'$</td>
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<tr>
<td>$\phi \in L_{y_{N}}'$, $\sigma \in L_{N} \Rightarrow$</td>
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<td>$\sigma \land \phi \in L_{y_{N}}'$</td>
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<td>$\phi \in L_{y_{N}}'$, $\sigma \in L_{y_{N}} \Rightarrow$</td>
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<td>$\sigma \land \phi \in L_{y_{N}}'$</td>
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<tr>
<td>$X \subseteq Act$, $\phi_{a} \in L_{y_{N}}'$, $\forall a \in X \Rightarrow$</td>
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<td>$\land_{a \in X} a \phi_{a} \in L_{y_{N}}'$</td>
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<td>$\phi_{i} \in L_{y_{N}}'$, $\forall i \in I \Rightarrow$</td>
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<tr>
<td>$\neg \phi \in L_{y_{N}}'$</td>
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   - Uniform logical characterizations of all the semantics

4 The complete structure of the extended spectrum
   - Partial offers trace and partial offers semantics
   - Meet and join semantics

5 Conclusions and future work
Partial offers trace and partial offers semantics

- Using a new closure we can define two more semantics at each layer of the spectrum.

- These semantics are defined by observing partial offers along a computation or just at its end.

- Duality between failures and partial offers, causes the picture of the complete layer of linear semantics for each N to become two diamonds, sharing the side corresponding to the readies-based semantics.
**Definition (Positive closure \( L_N^\vee \))**

Given a logical set \( L'_N \) with \( N \in \{U, C, I, T, S\} \), we define:

1. \( \sigma \in L'_N \Rightarrow \sigma \in L_N^\vee \)
2. \( \sigma_i \in L_N^\vee \forall i \in I \Rightarrow \bigwedge_{i \in I} \sigma_i \in L_N^\vee \)

**Definition (Partial offers trace semantics)**

For the constraint \( N \), is that defined by the logic \( L'_\text{POT} \), with

1. \( \top \in L'_\text{POT} \)
2. \( \varphi \in L'_\text{POT}, \sigma \in L_N^\vee \Rightarrow \sigma \land \varphi \in L'_\text{POT} \)
3. \( \varphi \in L'_\text{POT}, a \in \text{Act} \Rightarrow a\varphi \in L'_\text{POT} \)
Partial offer traces and partial offers semantics

**Definition (Positive closure $L'_N^\vee$)**

Given a logical set $L'_N$ with $N \in \{U, C, I, T, S\}$, we define:

- $\sigma \in L'_N \implies \sigma \in L'_N^\vee$
- $\sigma_i \in L'_N^\vee \forall i \in I \implies \bigwedge_{i \in I} \sigma_i \in L'_N^\vee$

**Definition (Partial offer semantics)**

For the constraint $N$ is that defined by the logic $L'_PO$ with

- $\top \in L'_PO$
- $\sigma \in L'_N \implies \sigma \in L'_PO$
- $\varphi \in L'_PO, \ a \in \text{Act} \implies a\varphi \in L'_PO$
Meet and join semantics

- There are another two semantics in each layer of the extended spectrum.

- For the particular case $N = I$, the meet semantics $R \lor FT$ has been previously studied by Roscoe.

The double diamond below ready simulation

- These semantics are in the linear side of the spectrum, therefore they have a similar structure to those linear semantics studied before.
Meet and join semantics

**Definition (Join semantics)**

We define the set of formulas $\mathcal{L}'_{\leq I}^{1_{\exists f}}$, by

- $\top \in \mathcal{L}'_{\leq I}^{1_{\exists f}}$; $\sigma \in \mathcal{L}_{\leq I}^{=}$ $\Rightarrow \sigma \in \mathcal{L}'_{\leq I}^{1_{\exists f}}$
- $\phi \in \mathcal{L}'_{\leq I}^{1_{\exists f}}$, $\sigma \in \mathcal{L}_{\leq I}^{-}$ $\Rightarrow \sigma \land \phi \in \mathcal{L}'_{\leq I}^{1_{\exists f}}$
- $\phi \in \mathcal{L}'_{\leq I}^{1_{\exists f}}$, $a \in \text{Act}$ $\Rightarrow a\phi \in \mathcal{L}'_{\leq I}^{1_{\exists f}}$

**Definition (Meet semantics)**

We define the set of formulas $\mathcal{L}'_{\leq I}^{1_{\exists !}}$ by

- $\top \in \mathcal{L}'_{\leq I}^{1_{\exists !}}$
- $\sigma, \sigma_j \in \mathcal{L}'_{I}$ $\forall j \in J$ $\Rightarrow (\sigma \land \bigwedge_{j \in J} \neg \sigma_j \top) \in \mathcal{L}_{\leq I}^{1_{\exists !}}$
- $\phi \in \mathcal{L}'_{\leq I}^{1_{\exists !}}$, $a \in \text{Act}$ $\Rightarrow a\phi \in \mathcal{L}'_{\leq I}^{1_{\exists !}}$
### Meet and join semantics

**Definition (Join semantics)**

We define the set of formulas $\mathcal{L}' \leq_{N}^{12\text{af}}$, by

- $\top \in \mathcal{L}' \leq_{N}^{12\text{af}}$; $\sigma \in \mathcal{L}^{=}_{N} \Rightarrow \sigma \in \mathcal{L}' \leq_{N}^{12\text{af}}$
- $\varphi \in \mathcal{L}' \leq_{N}^{12\text{af}}$, $\sigma \in \mathcal{L}^{\leq}_{N} \Rightarrow \sigma \wedge \varphi \in \mathcal{L}' \leq_{N}^{12\text{af}}$
- $\varphi \in \mathcal{L}' \leq_{N}^{12\text{af}}$, $a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}' \leq_{N}^{12\text{af}}$

**Definition (Meet semantics)**

We define the set of formulas $\mathcal{L}' \leq_{N}^{12\text{vf}}$ by

- $\top \in \mathcal{L}' \leq_{N}^{12\text{vf}}$
- $\sigma, \sigma_j \in \mathcal{L}'_{N}$ $\forall j \in J \Rightarrow (\sigma \wedge \bigwedge_{j \in J} \neg \sigma_j \top) \in \mathcal{L} \leq_{N}^{12\text{vf}}$
- $\varphi \in \mathcal{L}' \leq_{N}^{12\text{vf}}$, $a \in \text{Act} \Rightarrow a\varphi \in \mathcal{L}' \leq_{N}^{12\text{vf}}$
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- With this presentation we conclude the work on unification of all the concrete process semantics.

- Our main objective here was to develop a unified approach to the characterisation of the underlying semantics using sublogics of HML with a simple structure.

- The key point was to properly identify the rules for managing the use of negation and conjunction. As a result, we clarify the difference between branching-time and linear-time semantics.

- We have found out two more linear semantics in each layer and we have discovered that the classic logical characterizations of PW was wrong.
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Future work

- We got some partial results in the unified logical characterization of the weak semantics.

- We obtain some interesting results on the characterization of modal semantics.

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THANKS!