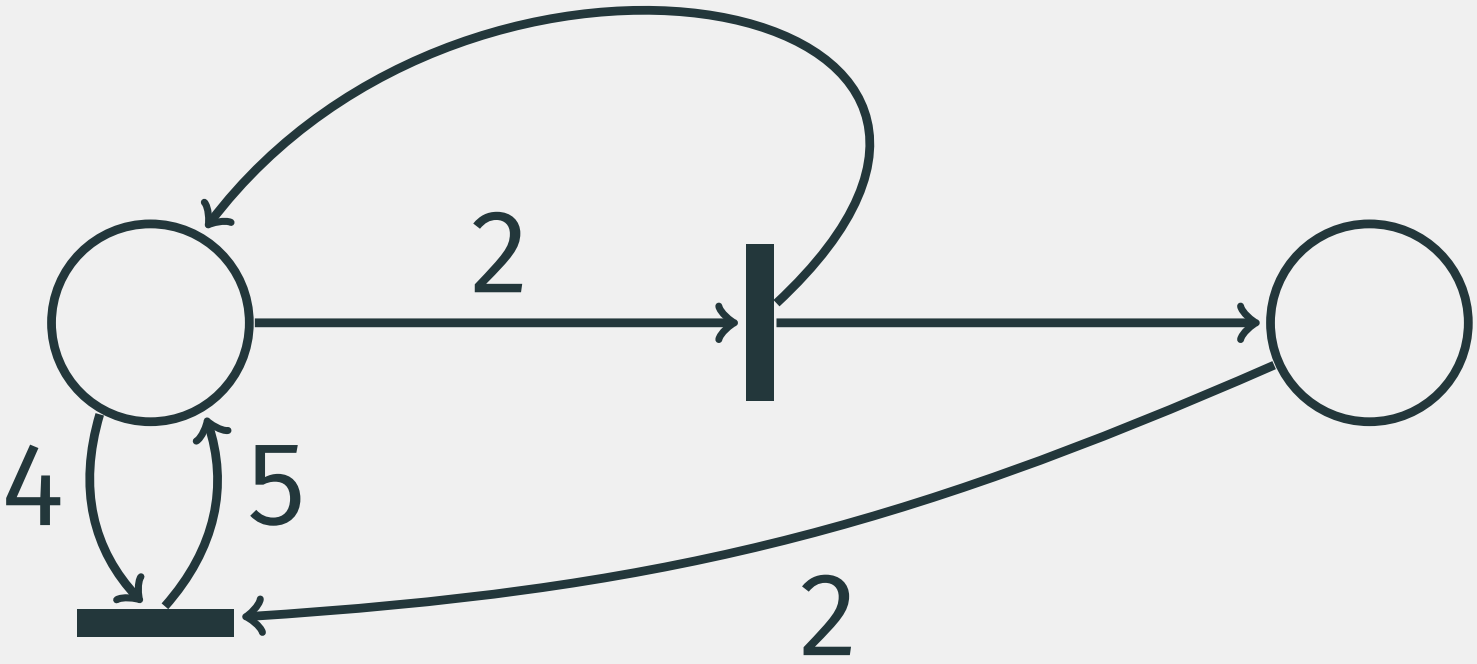
Approaching the Coverability Problem Continuously

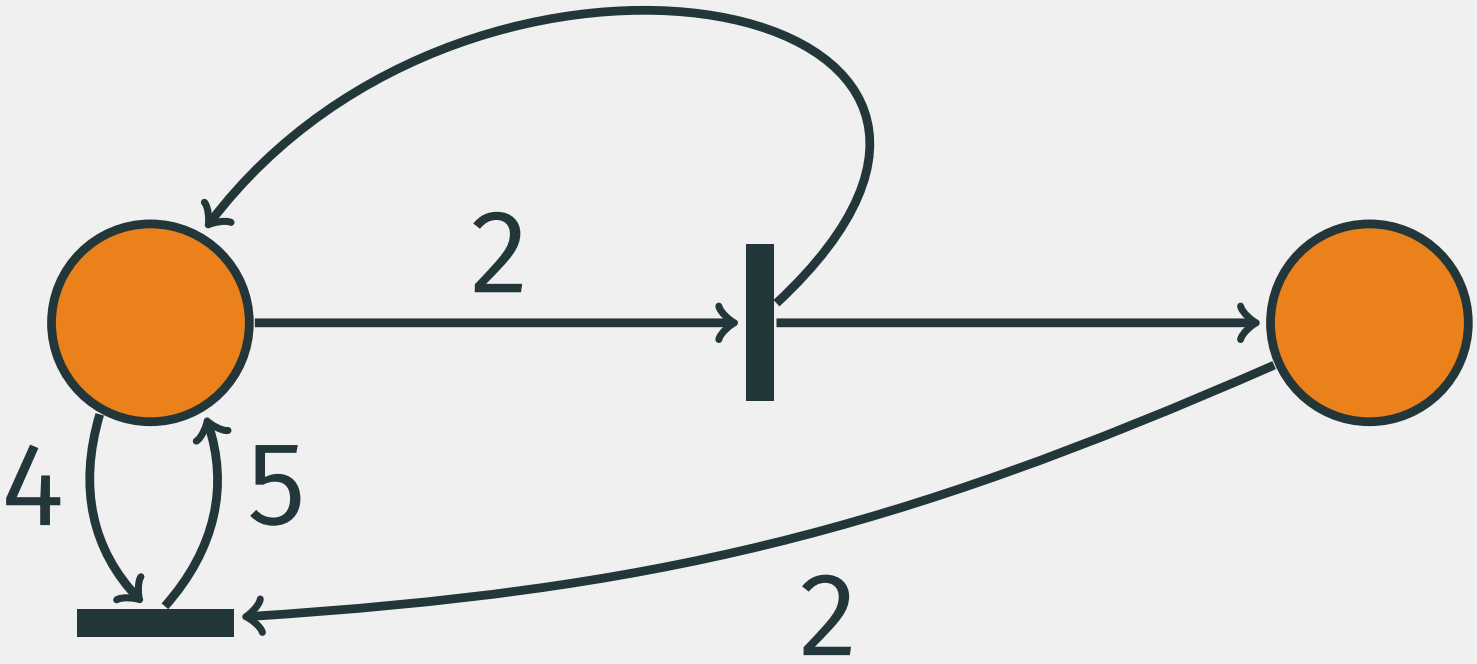
Michael Blondin

Joint work with Alain Finkel, Christoph Haase, Serge Haddad

(Discrete) Petri nets

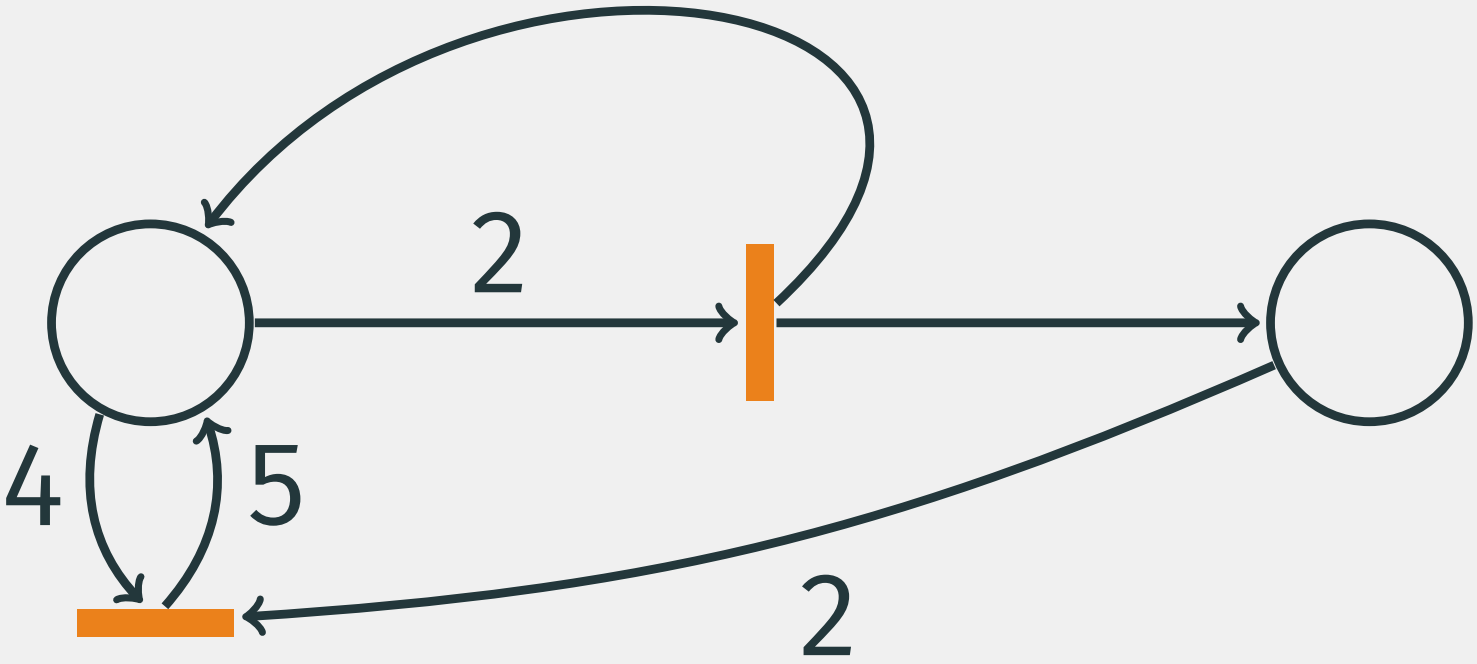


(Discrete) Petri nets



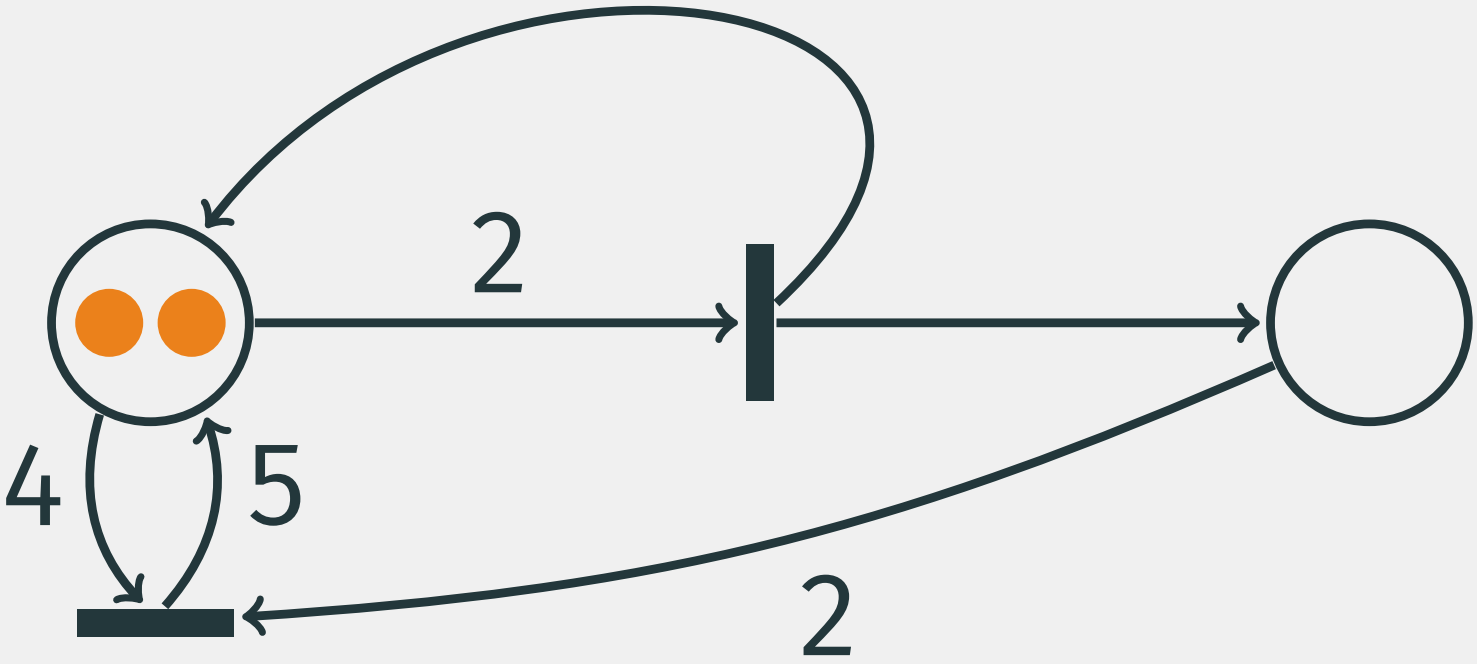
Places

(Discrete) Petri nets



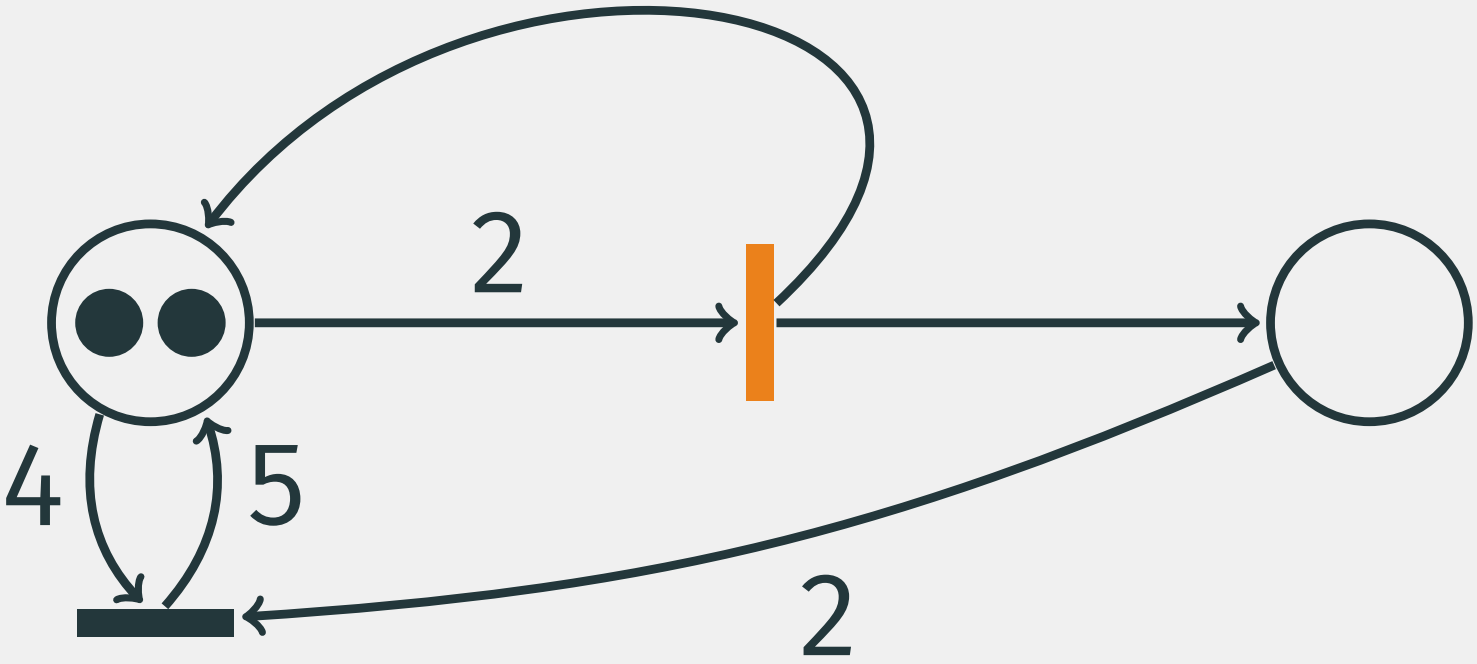
Transitions

(Discrete) Petri nets



Marking

(Discrete) Petri nets



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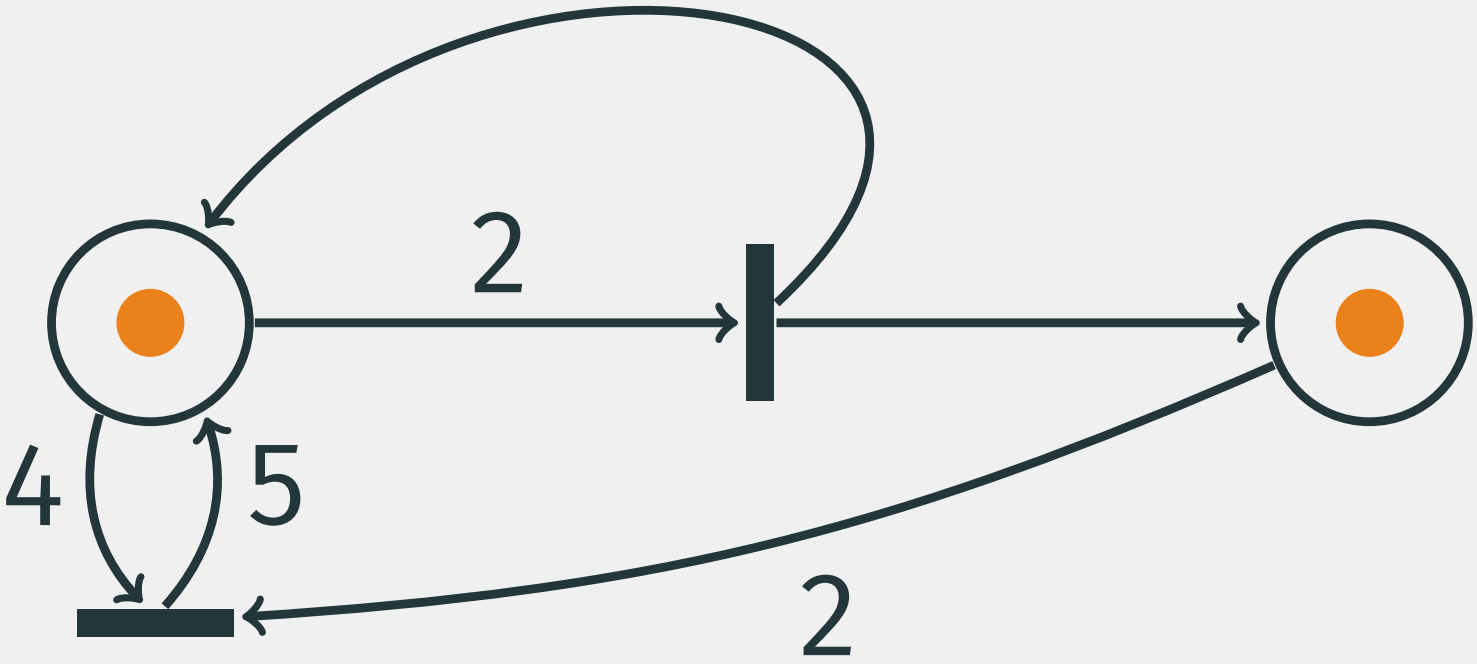

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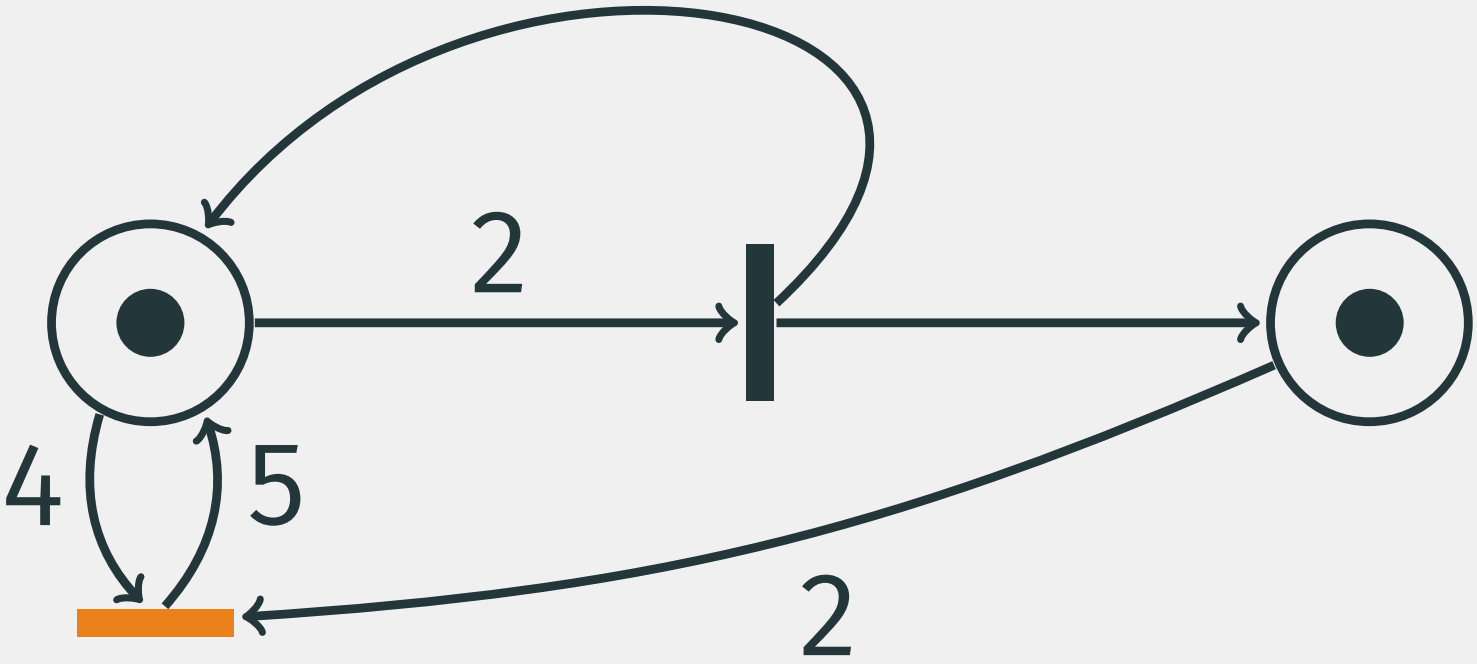

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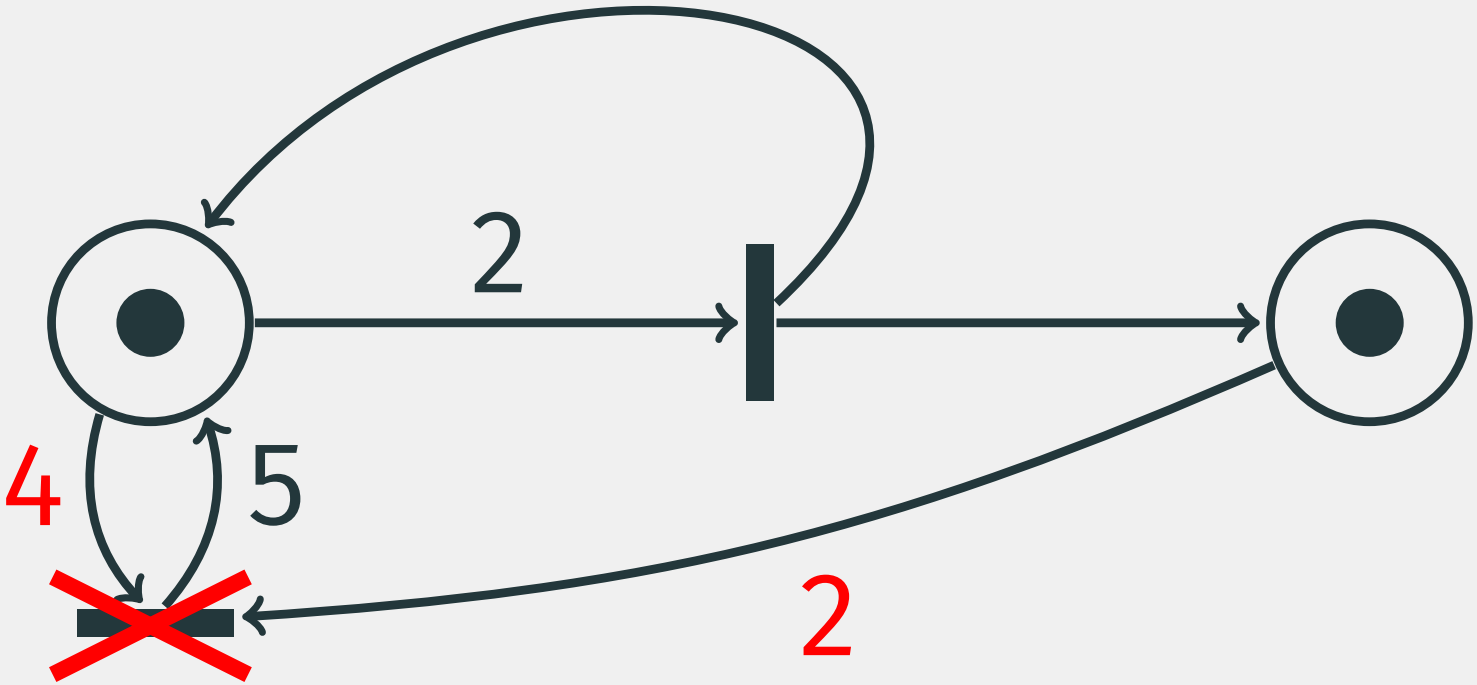

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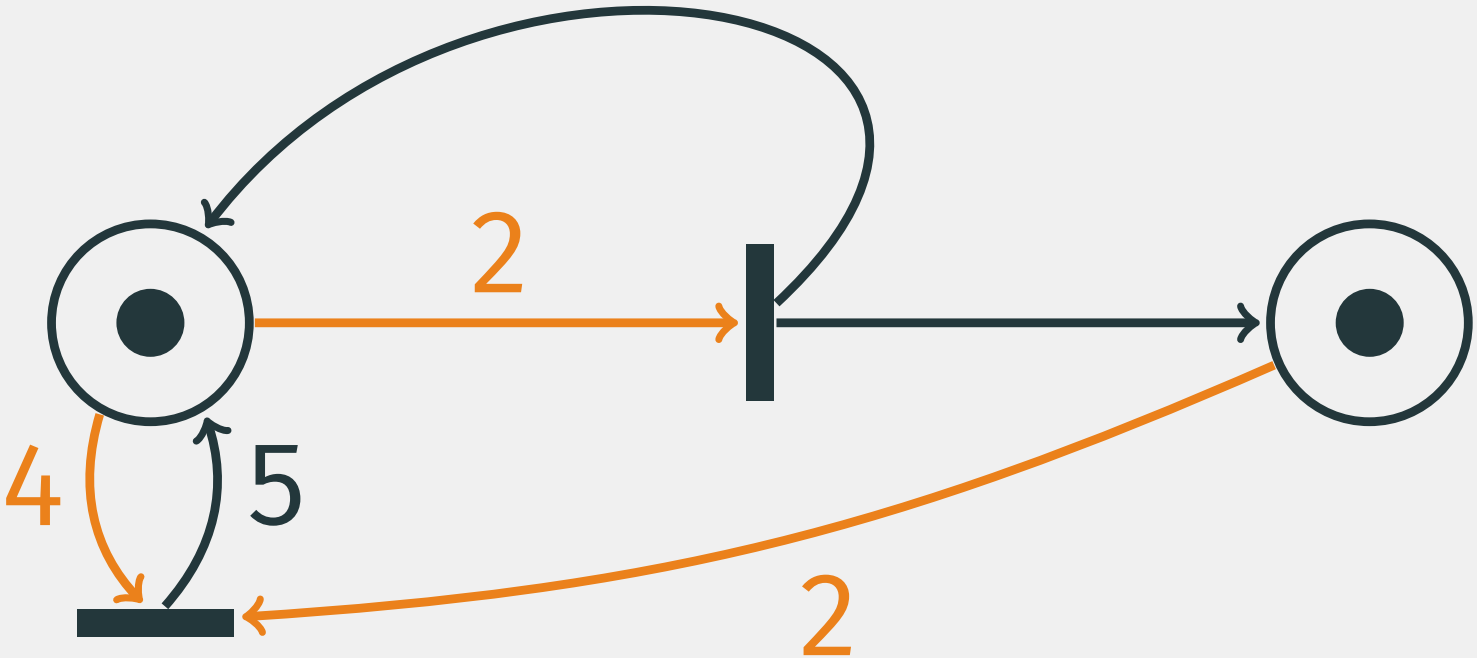
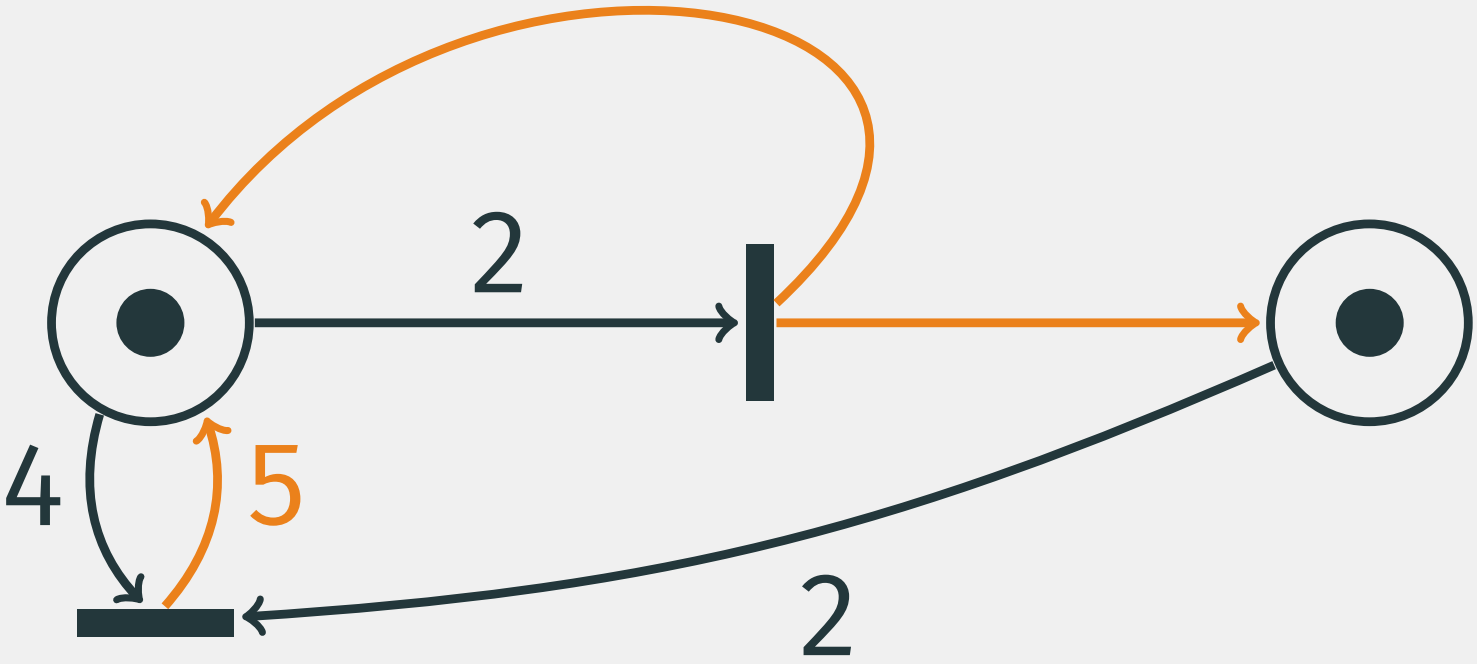


$$Pre = \begin{pmatrix} 4 & 2 \\ 2 & 0 \end{pmatrix}$$

(Discrete) Petri nets



$$Post = \begin{pmatrix} 5 & 1 \\ 0 & 1 \end{pmatrix}$$

Verifying safety with Petri nets

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

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Verifying safety with Petri nets

Lamport mutual exclusion "1-bit algorithm"

Verifying safety with Petri nets

```
while True:
  x = True
  while y: pass
  # critical section
  x = False
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  while True:
★   y = True
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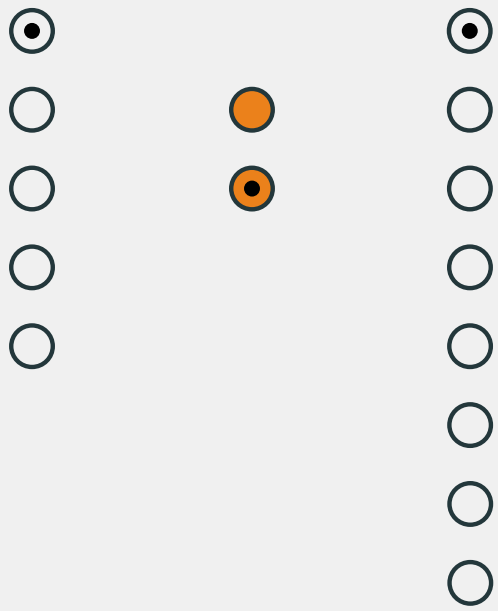

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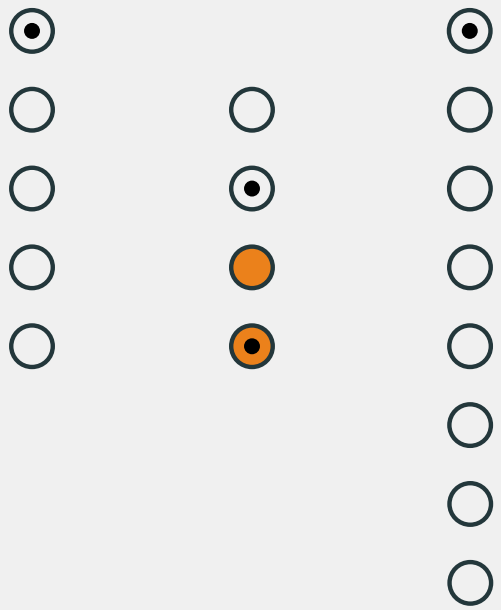

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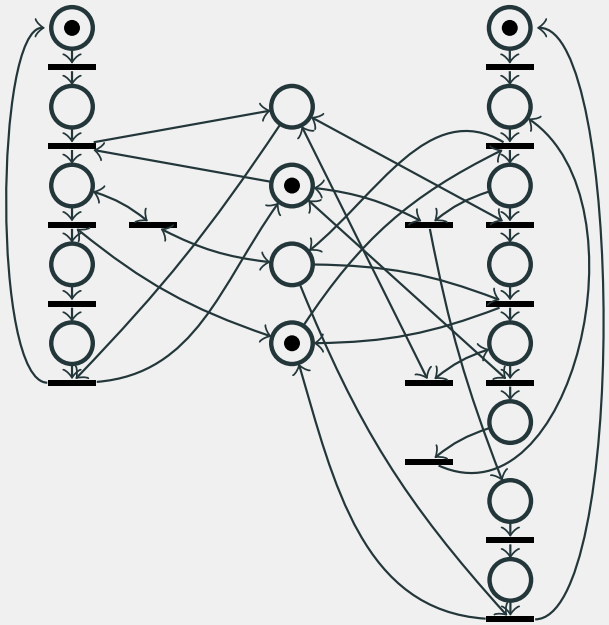

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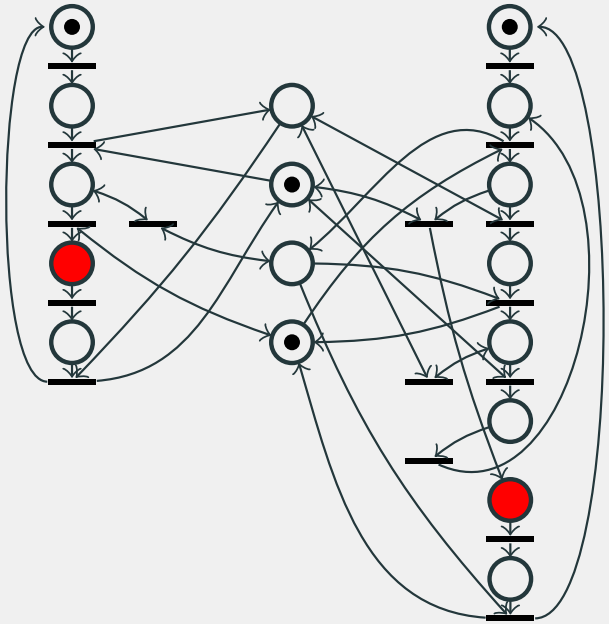

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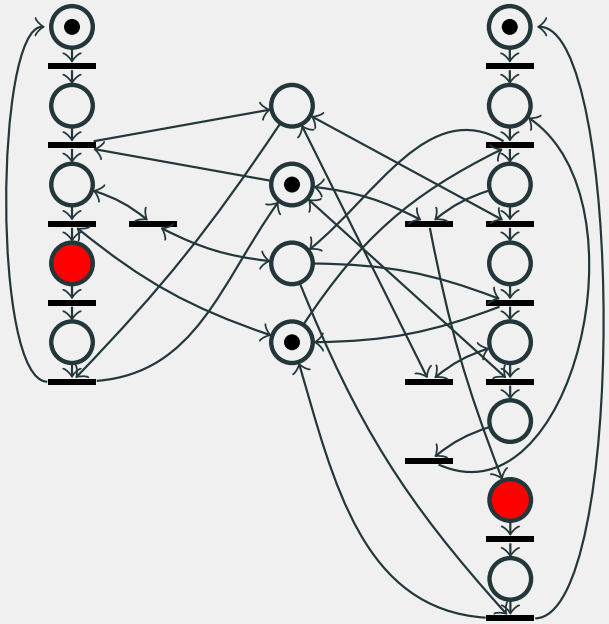
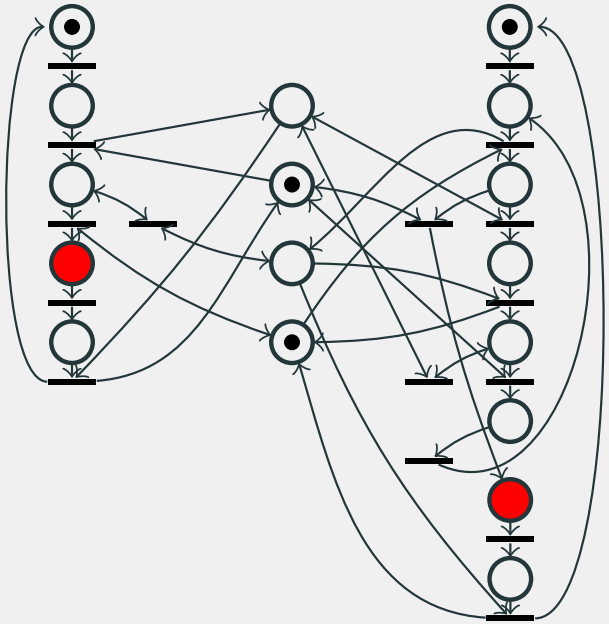


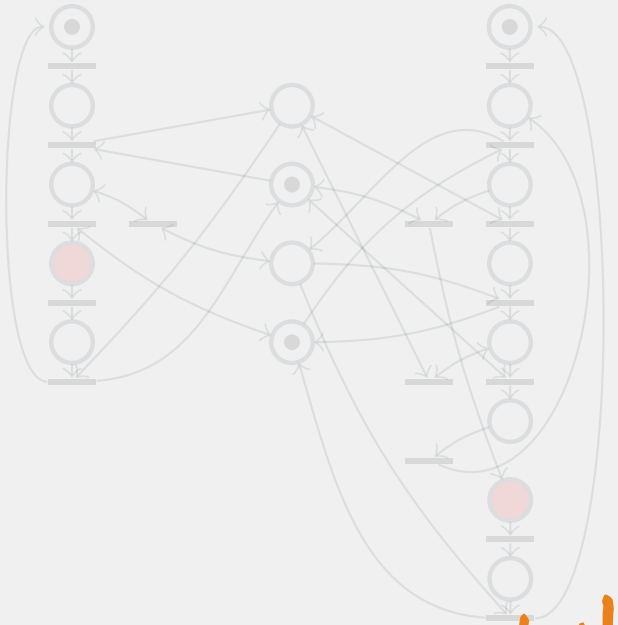
Processes at both critical sections $\iff$ each  ≥ 1

Verifying safety with Petri nets



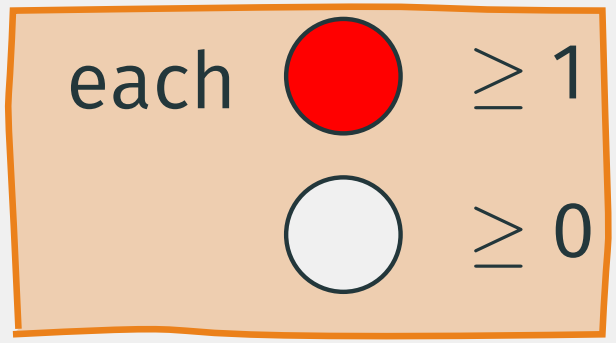
Processes at both critical sections $\iff$ each  ≥ 1,  ≥ 0

Verifying safety with Petri nets



Coverability problem

Processes at both critical sections $\iff$



Coverability problem

Problem

Input:	Petri net $\mathcal{N}$, initial marking $\boldsymbol{m}_0$, target marking $\boldsymbol{m}$
Question:	Is some $\boldsymbol{m}' \geq \boldsymbol{m}$ reachable from $\boldsymbol{m}_0$ in $\mathcal{N}$?

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EXPSPACE-complete

Lipton STOC'76, Rackoff TCS'78

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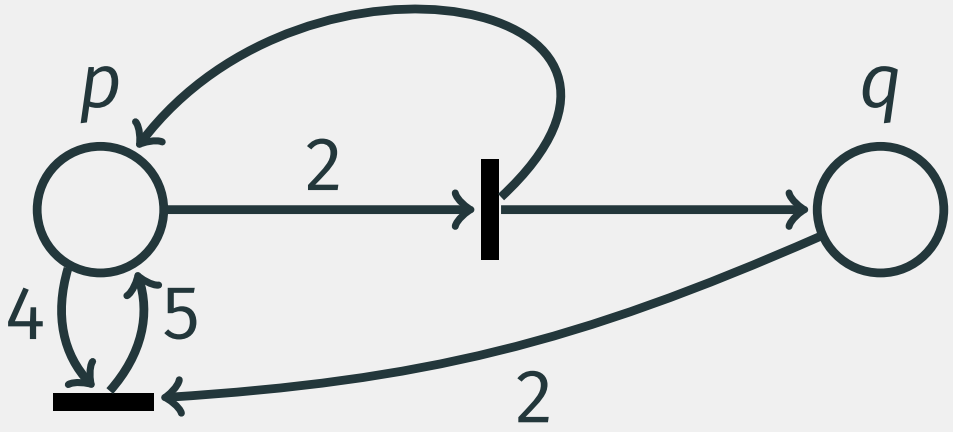
EXPSPACE-complete

Lipton STOC'76, Rackoff TCS'78

How can we be more efficient?

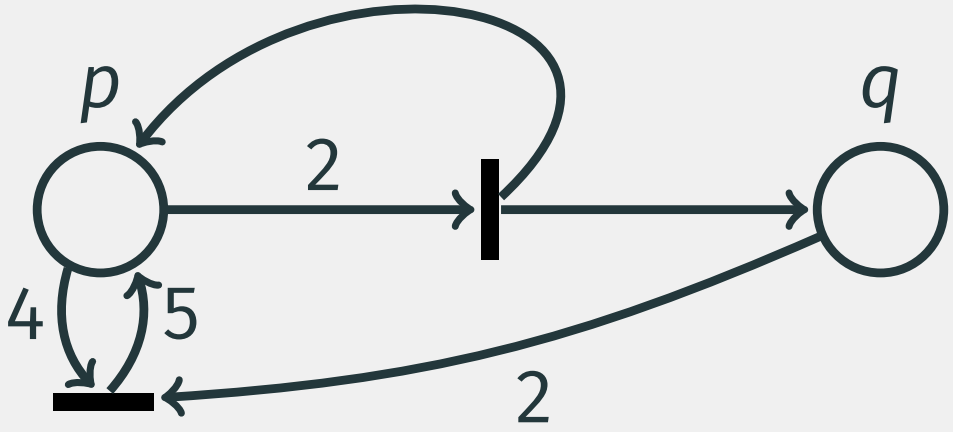
Over-approximating coverability: state equation



$\boldsymbol{m}_0$ can cover $\boldsymbol{m}$ $\Longrightarrow$

$$\exists \boldsymbol{v} \geq \mathbf{0} \text{ s.t. } \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v} \geq \boldsymbol{m}$$

Over-approximating coverability: state equation



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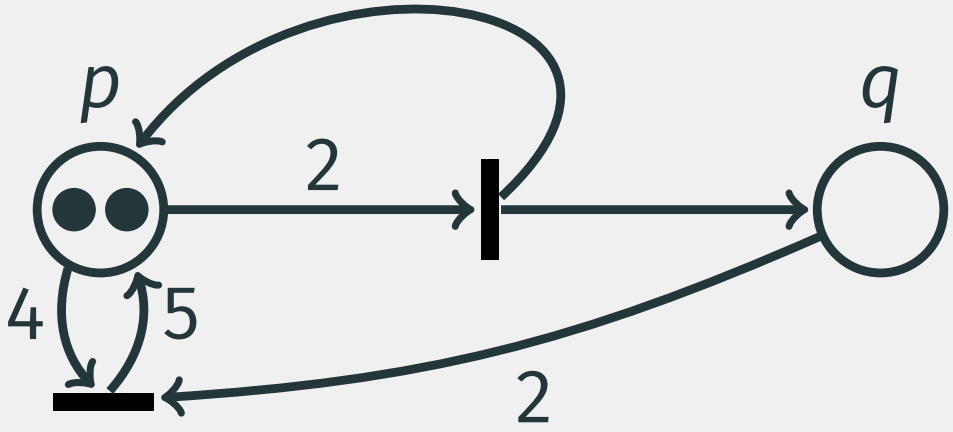
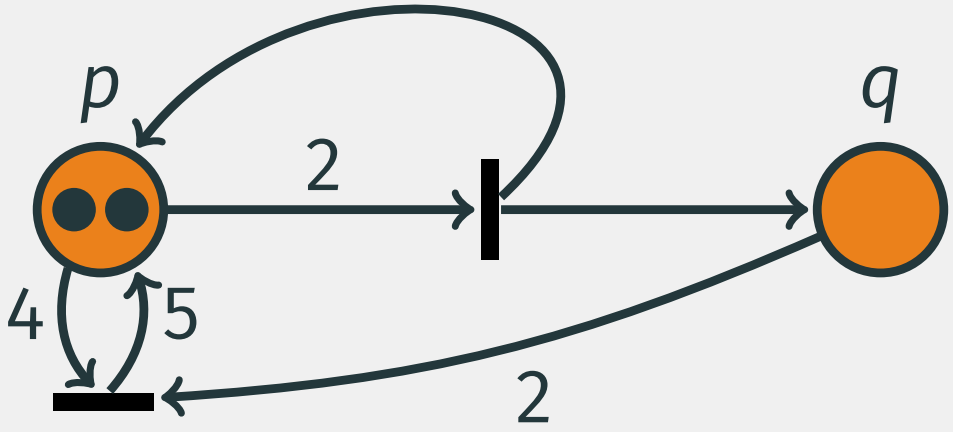


$(2 \;\; 0)$ can cover $\boldsymbol{m} \implies$

$$2 + x - y \geq \boldsymbol{m}(p)$$
$$0 - 2x + y \geq \boldsymbol{m}(q)$$

Over-approximating coverability: state equation



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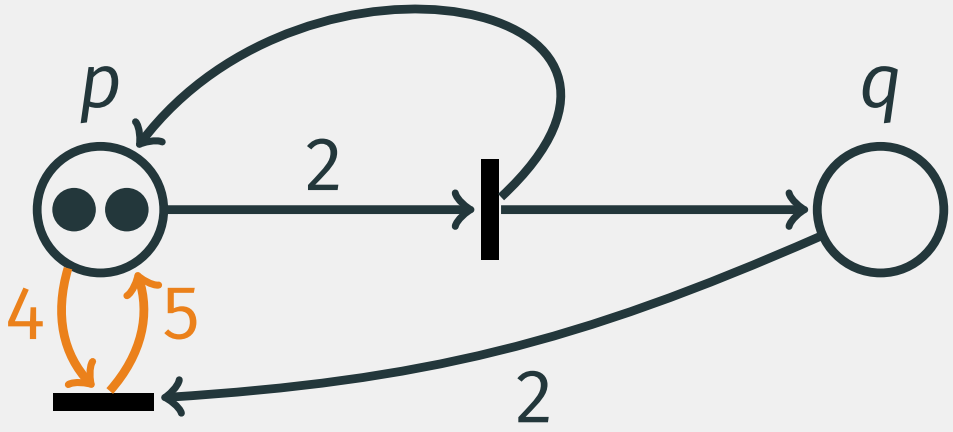

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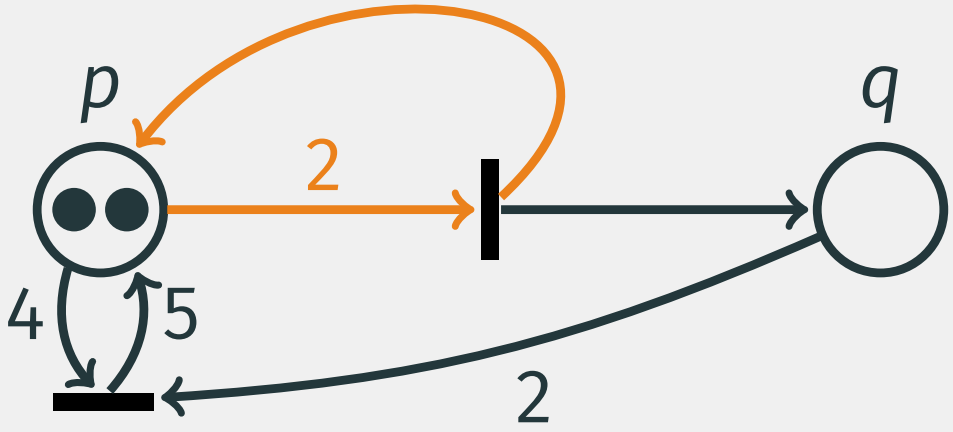

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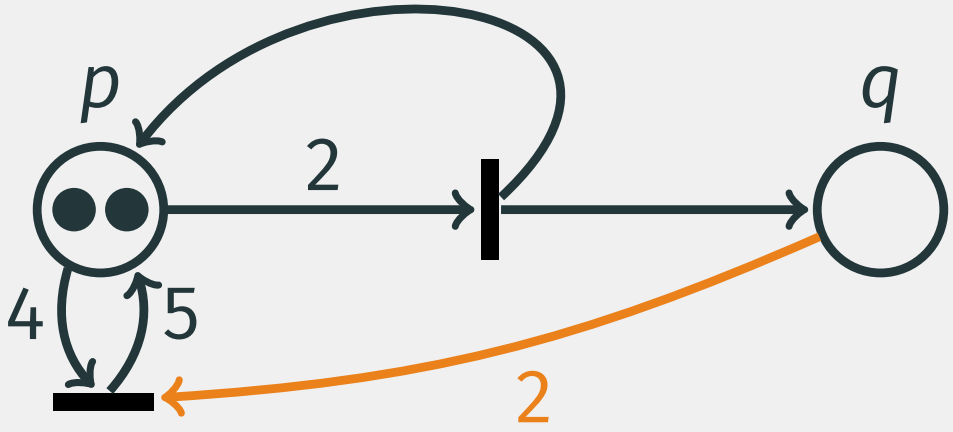

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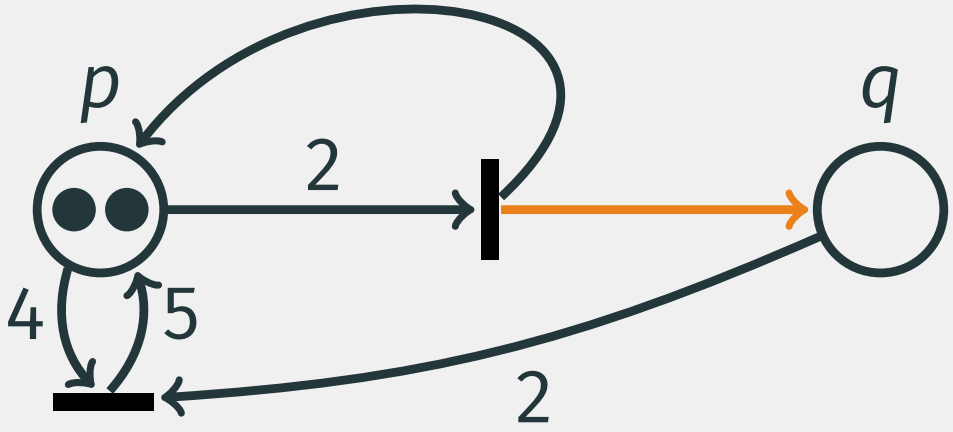

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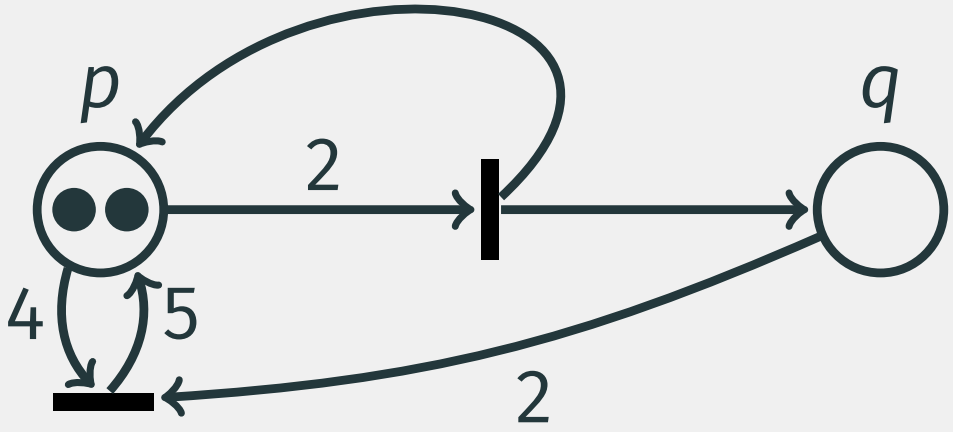
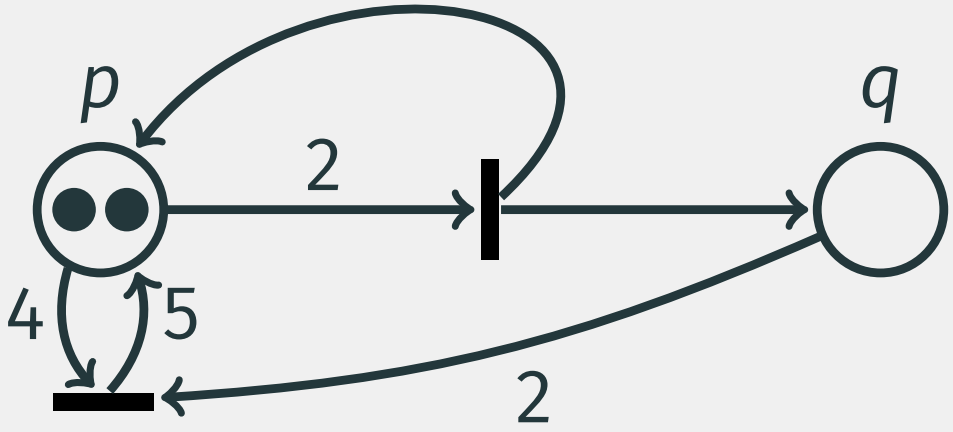


$(2 \;\; 0)$ can cover $(0 \;\; 3) \Longrightarrow$

$$\begin{aligned} 2 + x - y &\geq 0 \\ 0 - 2x + y &\geq 3 \end{aligned}$$

Over-approximating coverability: state equation

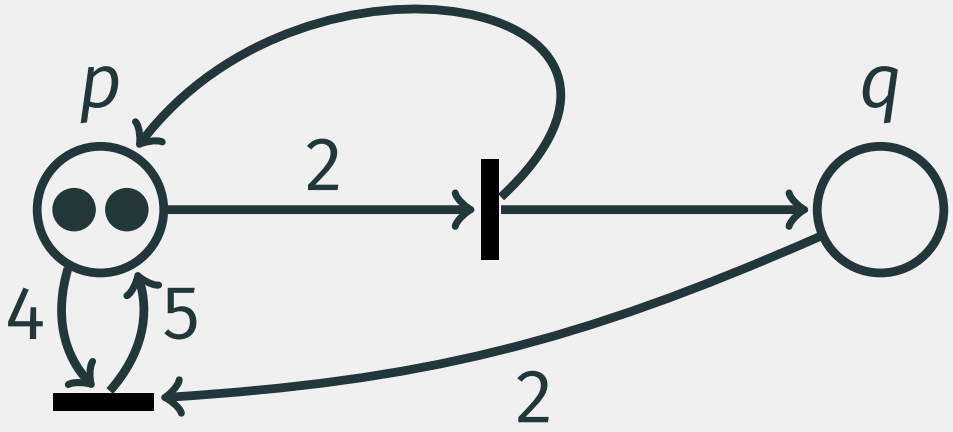


$(2\ \ 0)$ can cover $(0\ \ 3) \Longrightarrow$

$$\begin{aligned} 2 + x - y &\geq 0 \\ 0 - 2x + y &\geq 3 \end{aligned}$$

✗

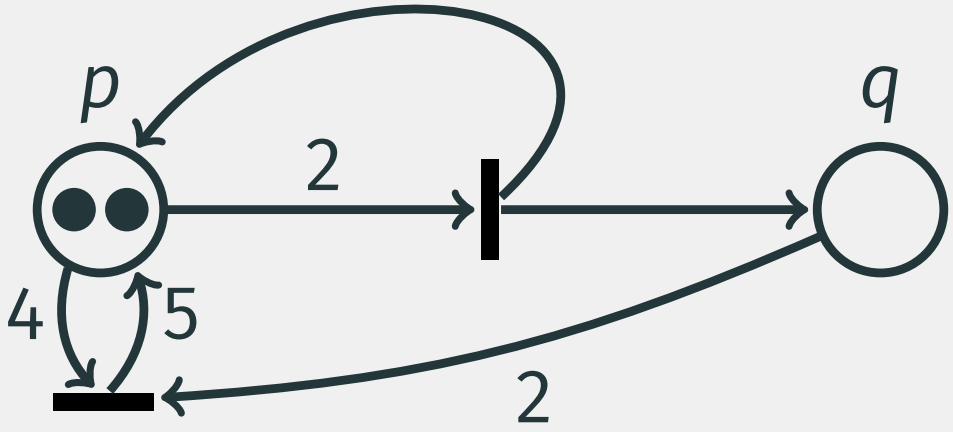
Over-approximating coverability: state equation



$(2 \;\; 0)$ can cover $(0 \;\; 2) \Longrightarrow$

$$\begin{aligned} 2 + x - y &\geq 0 \\ 0 - 2x + y &\geq 2 \end{aligned}$$

Over-approximating coverability: state equation



$(2 \;\; 0)$ can cover $(0 \;\; 2) \Longrightarrow$

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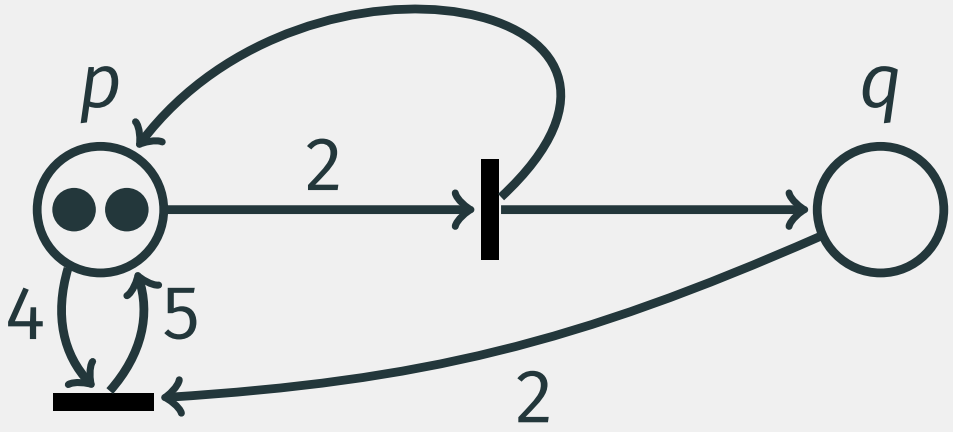
✓

Over-approximating coverability: state equation



$(2\ \ 0)$ can cover $(0\ \ 2) \Longrightarrow$

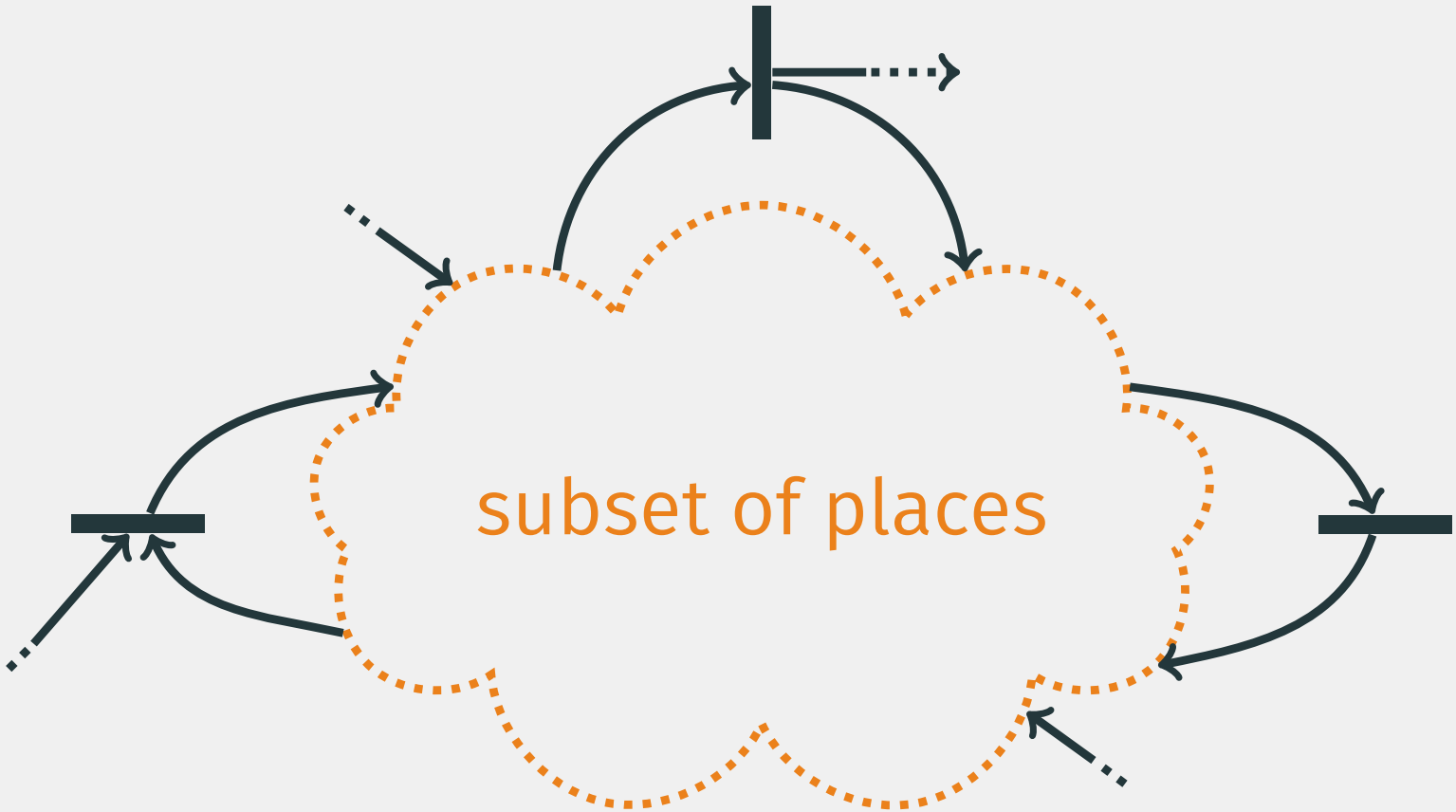
Yet, not coverable...

$$2 + 0 - 2 \geq 0$$

$$0 - 0 + 2 \geq 2$$

✓

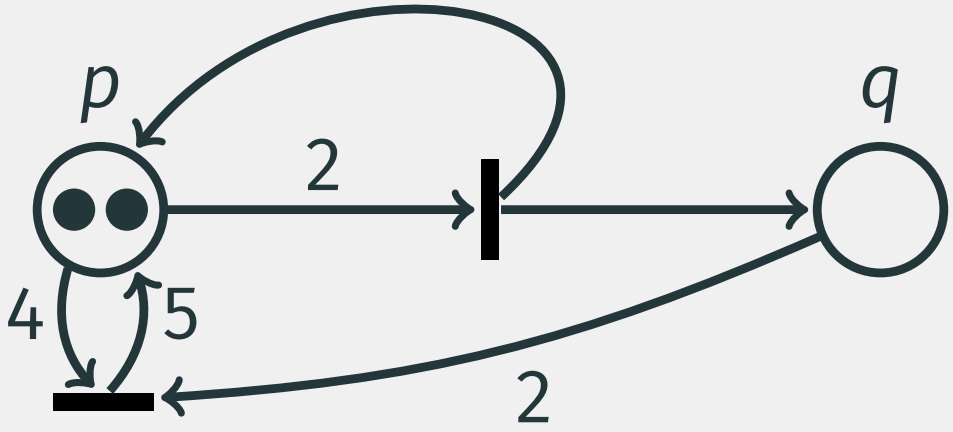
Traps



Traps

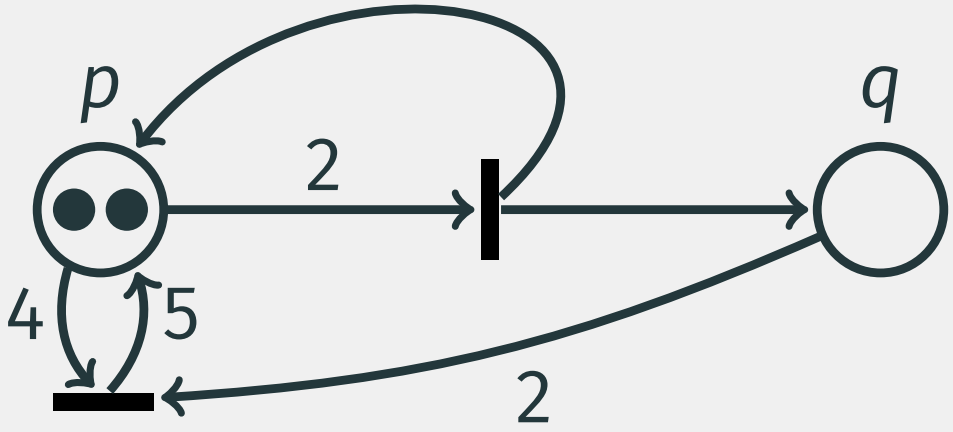
Once trap has tokens, it will always have tokens

Over-approximating coverability: trap constraints



$(2\ \ 0)$ can cover $(0\ \ 2)$?

Over-approximating coverability: trap constraints



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$$2 + \quad x - y \geq 0$$

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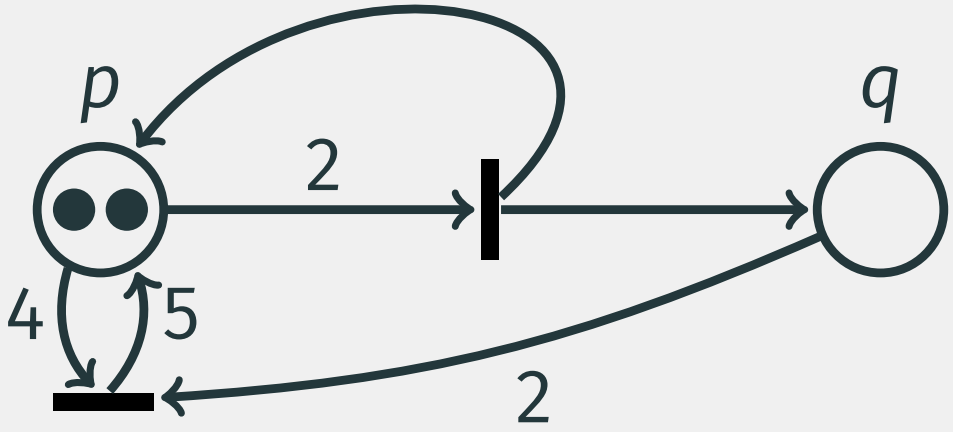
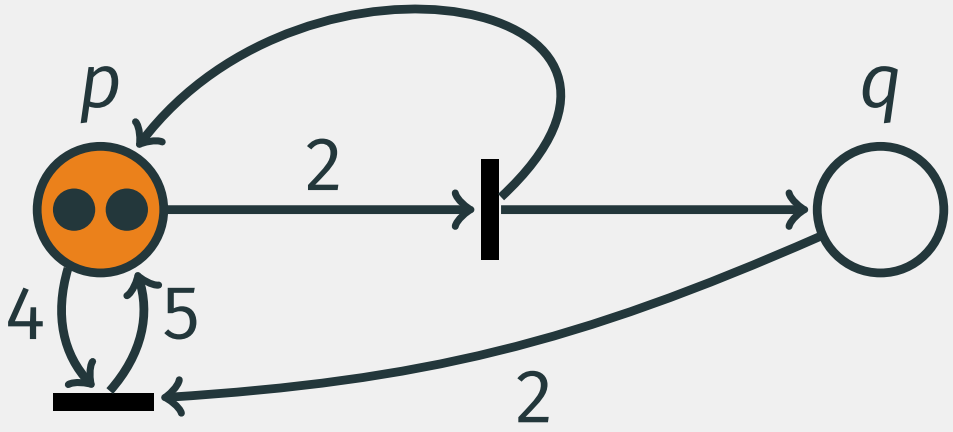


$(2 \;\; 0)$ can cover $(0 \;\; 2)$?

$$\begin{aligned} 2 + \quad x - y &= 0 \\ 0 - 2x + y &= 2 \end{aligned}$$

Over-approximating coverability: trap constraints

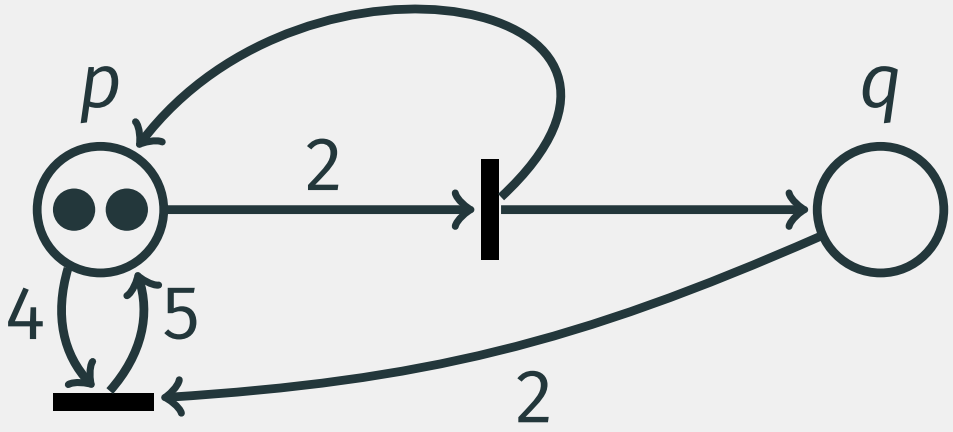


$(2\ \ 0)$ can cover $(0\ \ 2)$?

$$2 + \quad x - y = 0$$

$$0 - 2x + y = 2$$

Over-approximating coverability: trap constraints

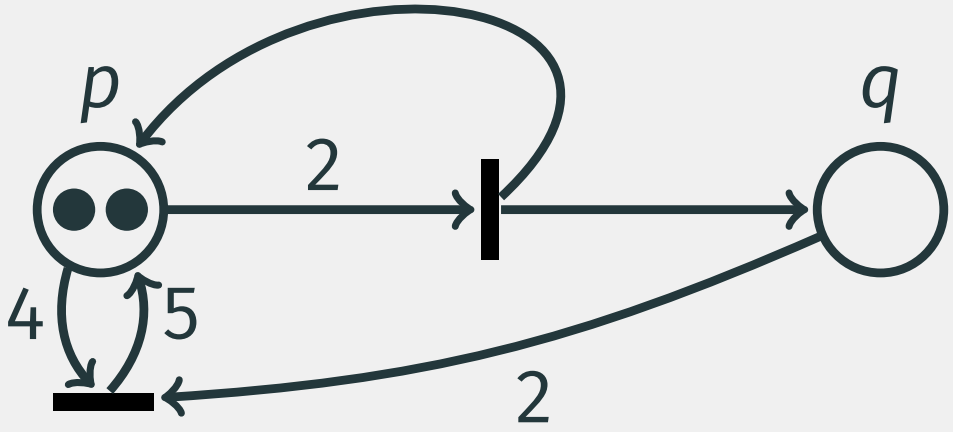


$(2 \;\; 0)$ can cover $(0 \;\; 2)$?

State equation ✓

Trap constraints ✗

Over-approximating coverability: trap constraints

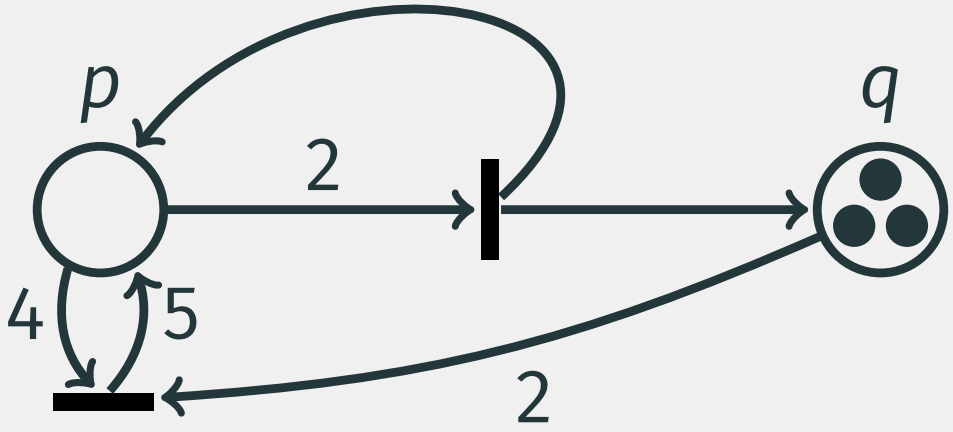


$(2\ \ 0)$ can cover $(0\ \ 2)$? No!

State equation ✓

Trap constraints ✗

Over-approximating coverability: trap constraints



$(0\ \ 3)$ can cover $(1\ \ 1)$?

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$$3 - 2x + y \geq 1$$

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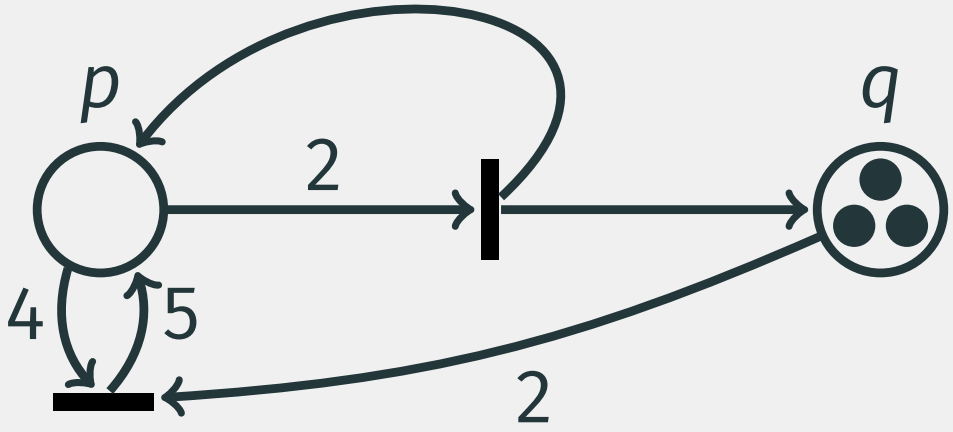

$(0 \;\; 3)$ can cover $(1 \;\; 1)$?

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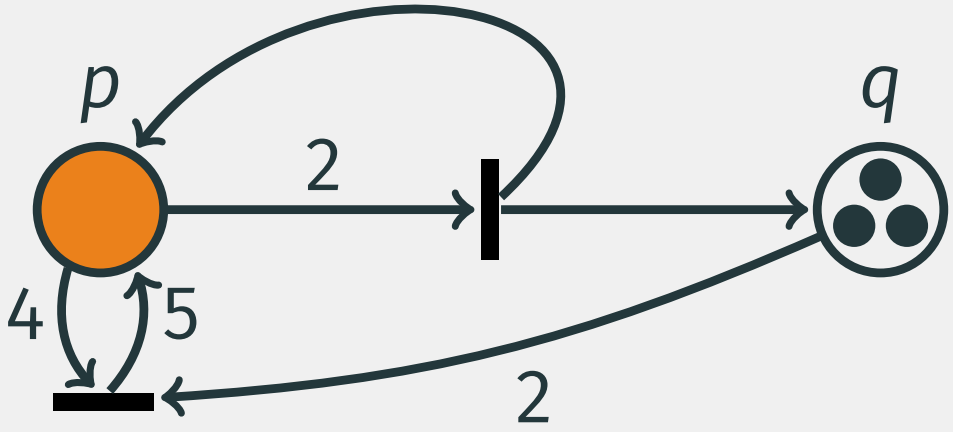
$$3 - 2 \cdot 1 + 0 \geq 1$$

Over-approximating coverability: trap constraints



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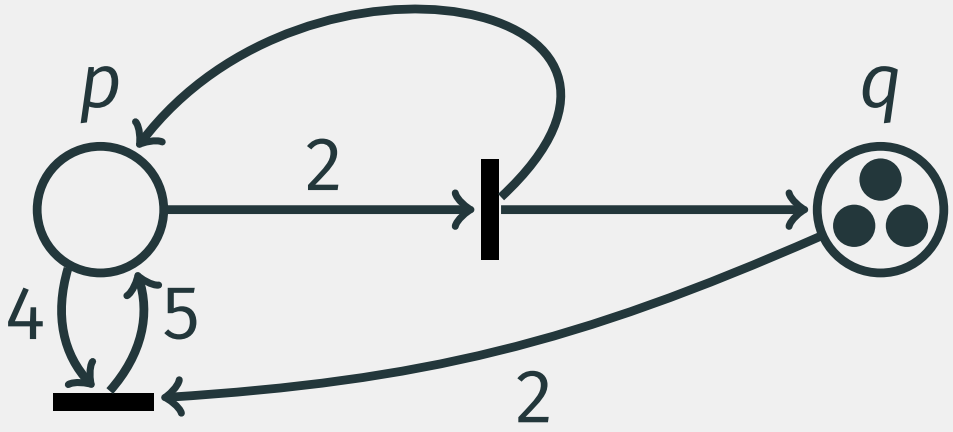
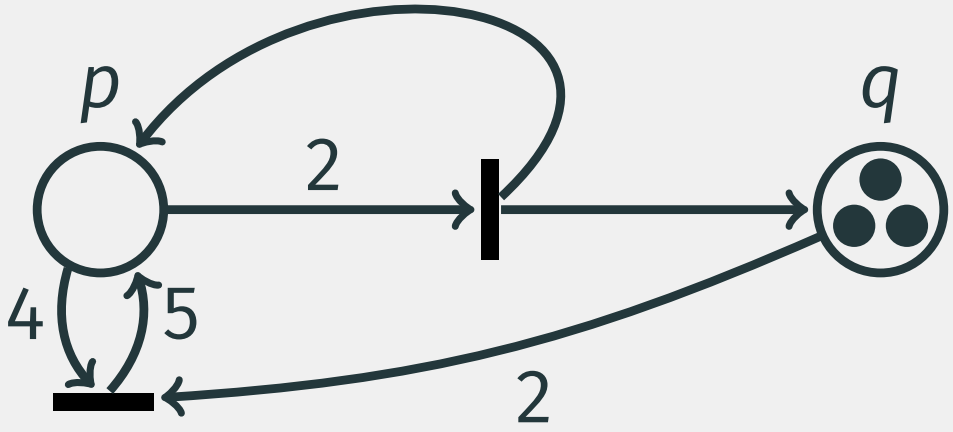


$(0\ \ 3)$ can cover $(1\ \ 1)$?

State equation ✓

Trap constraints ✓

Over-approximating coverability: trap constraints



(0 3) can cover (1 1)?

Yet, not coverable...

State equation ✓

Trap constraints ✓

Deciding safety with over-approximations

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

$$\Downarrow$$

$\boldsymbol{m}_0, \boldsymbol{m}$ satisfy

state equation and trap constraints

Deciding safety with over-approximations

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$\Uparrow$

$\boldsymbol{m}_0, \boldsymbol{m}$ do not satisfy

state equation or trap constraints

Deciding safety with over-approximations

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

$\Uparrow$

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Efficient in practice!

Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic CAV'14

Deciding safety with over-approximations

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$\Uparrow$

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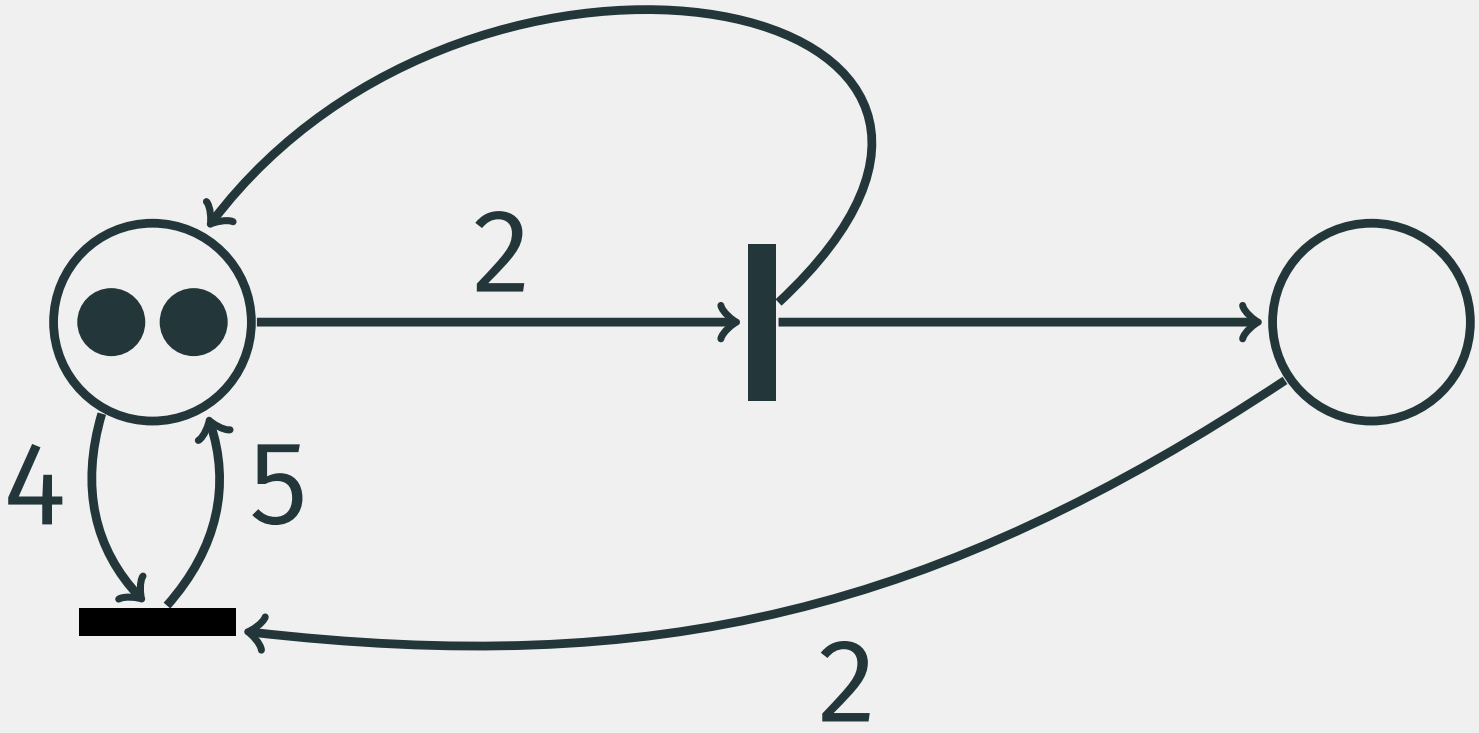
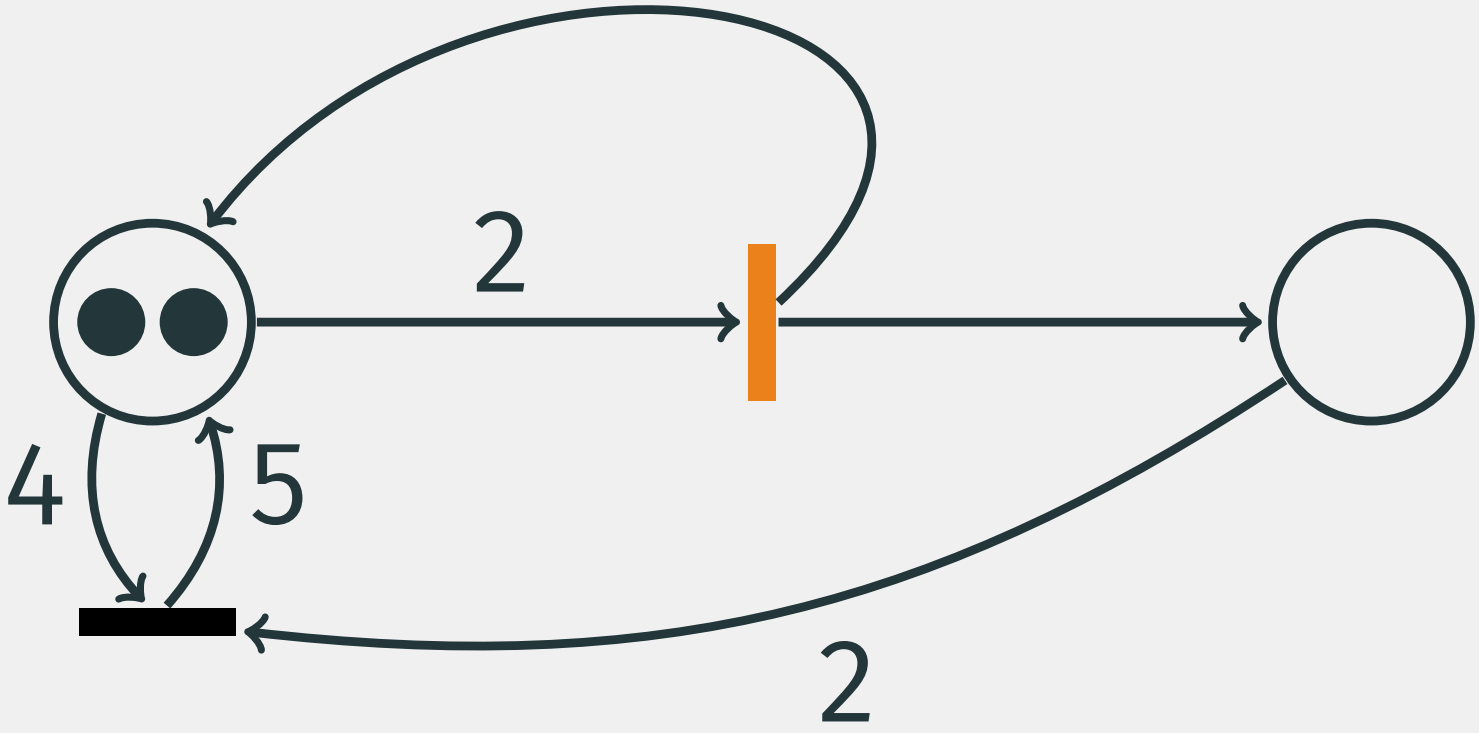
Any finer approximation, yet efficient?

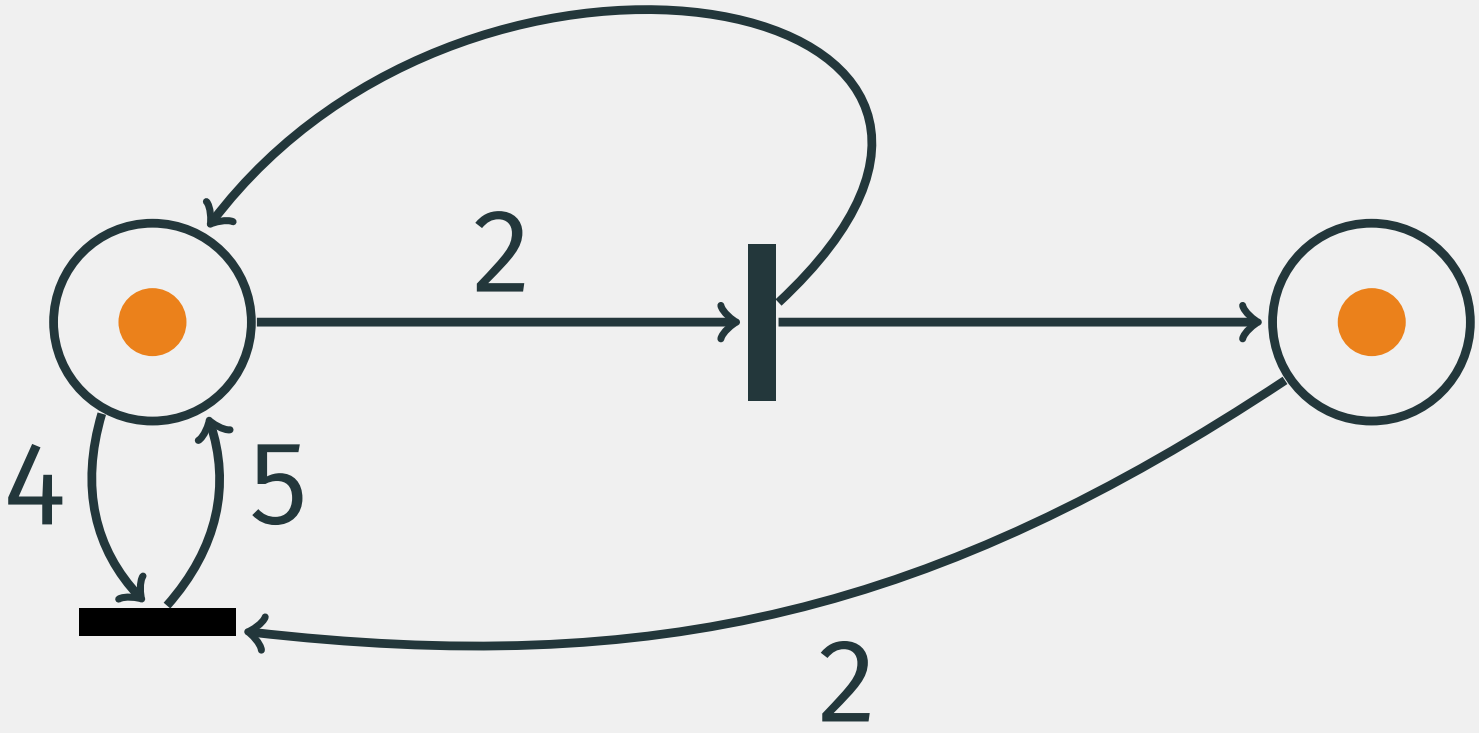
(Discrete) Petri nets



(Discrete) Petri nets



(Discrete) Petri nets



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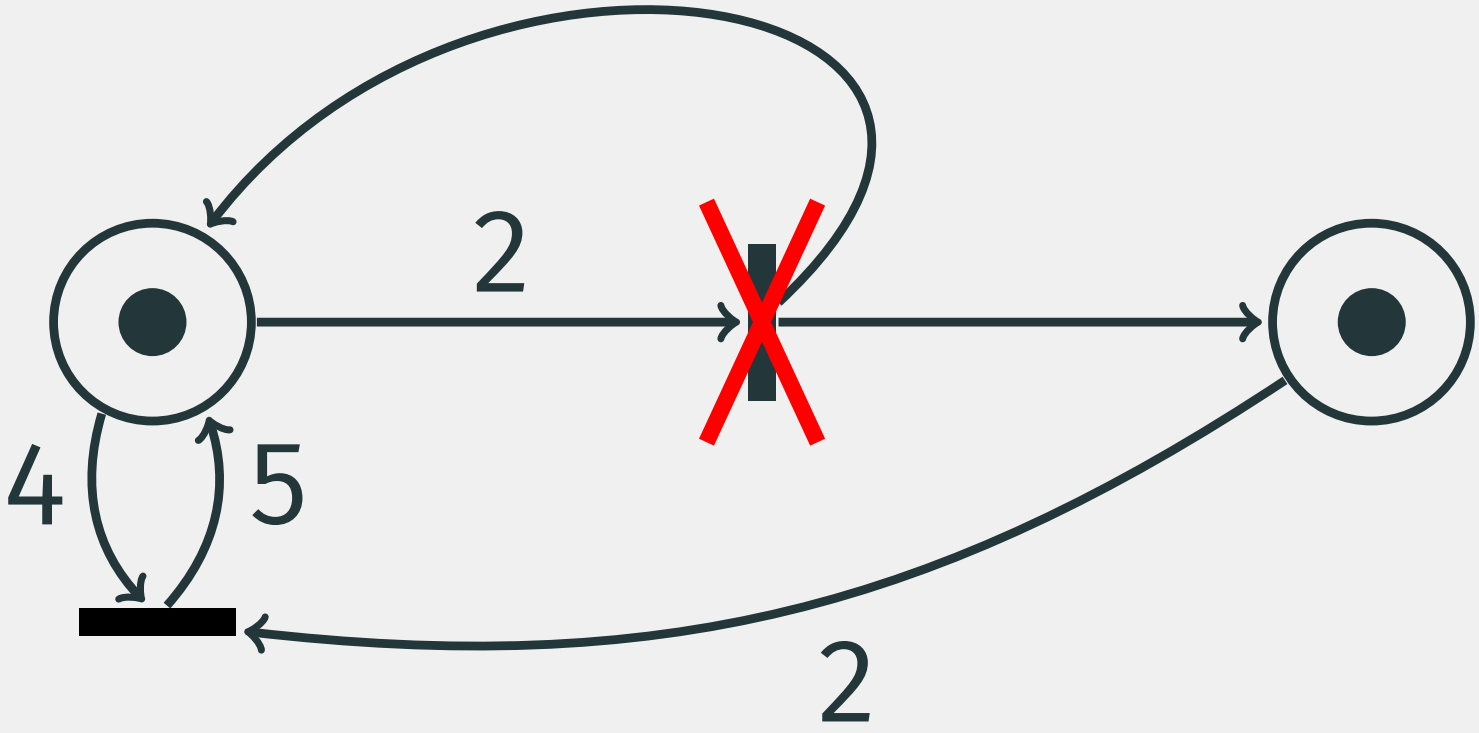
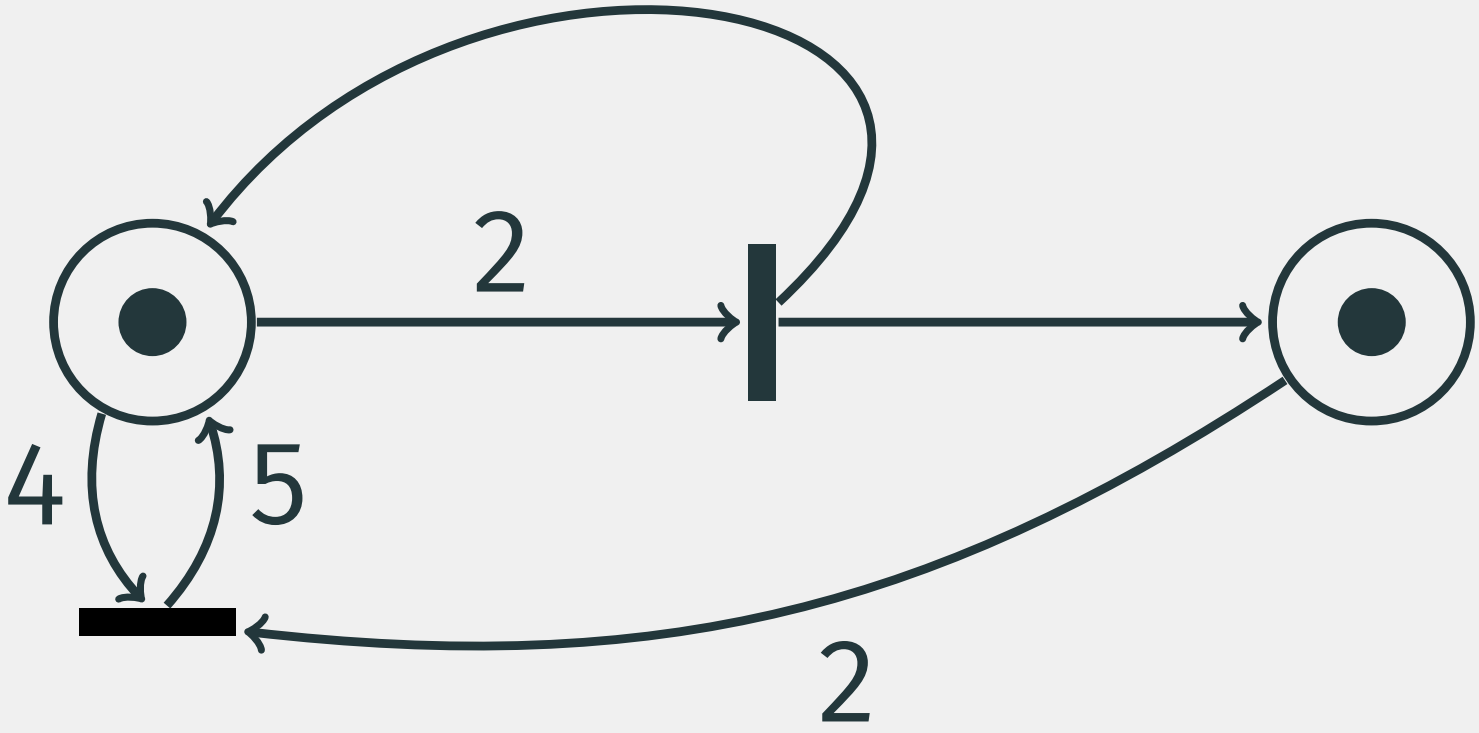
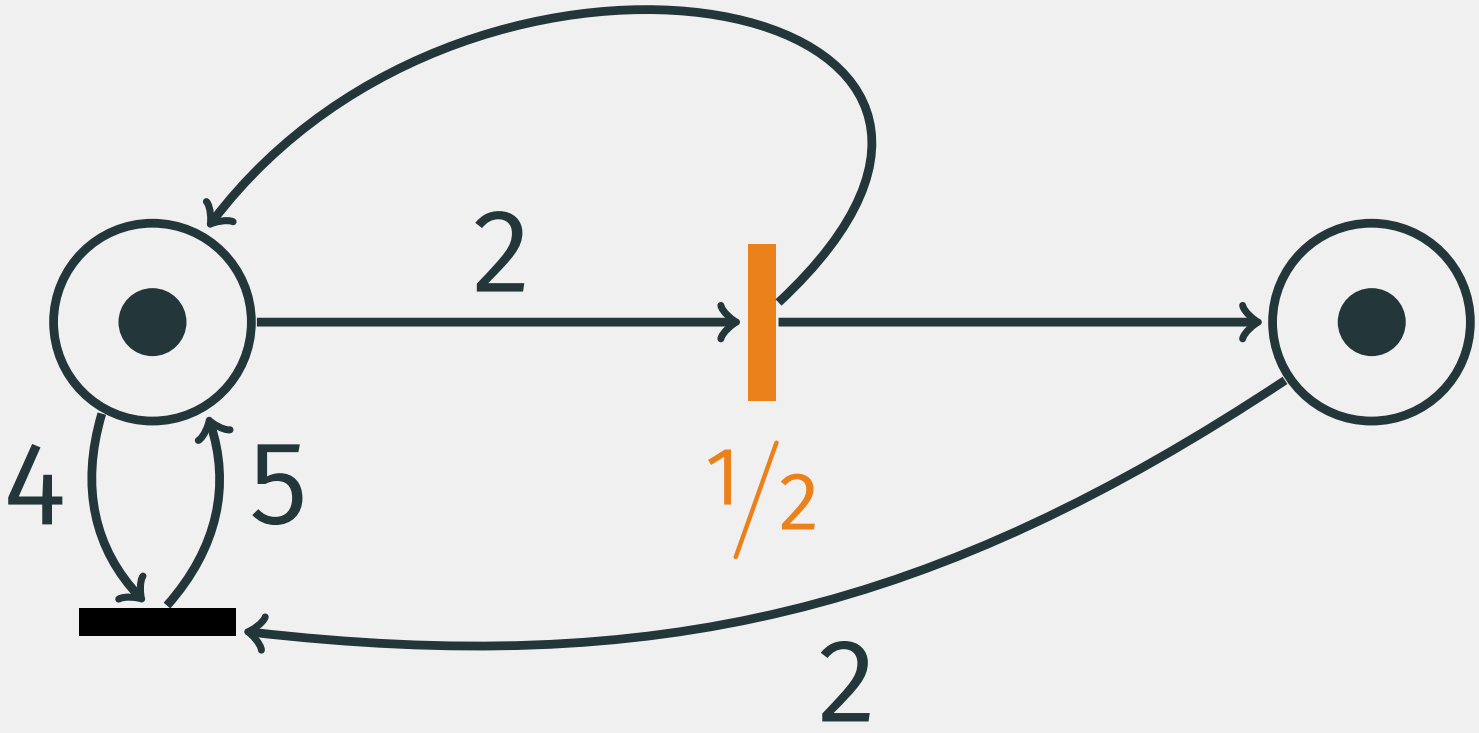
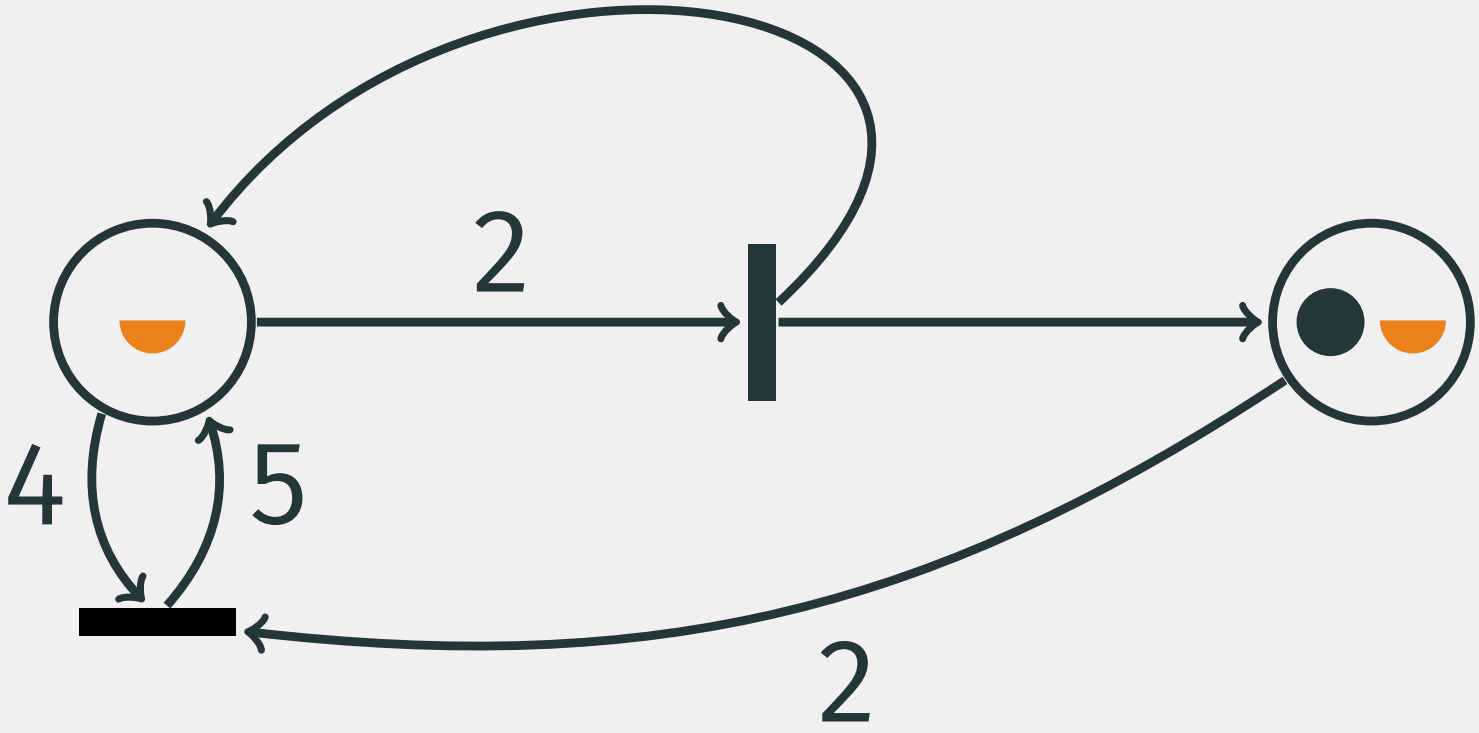



2
4
5
2


2
4
5
1/2
2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4
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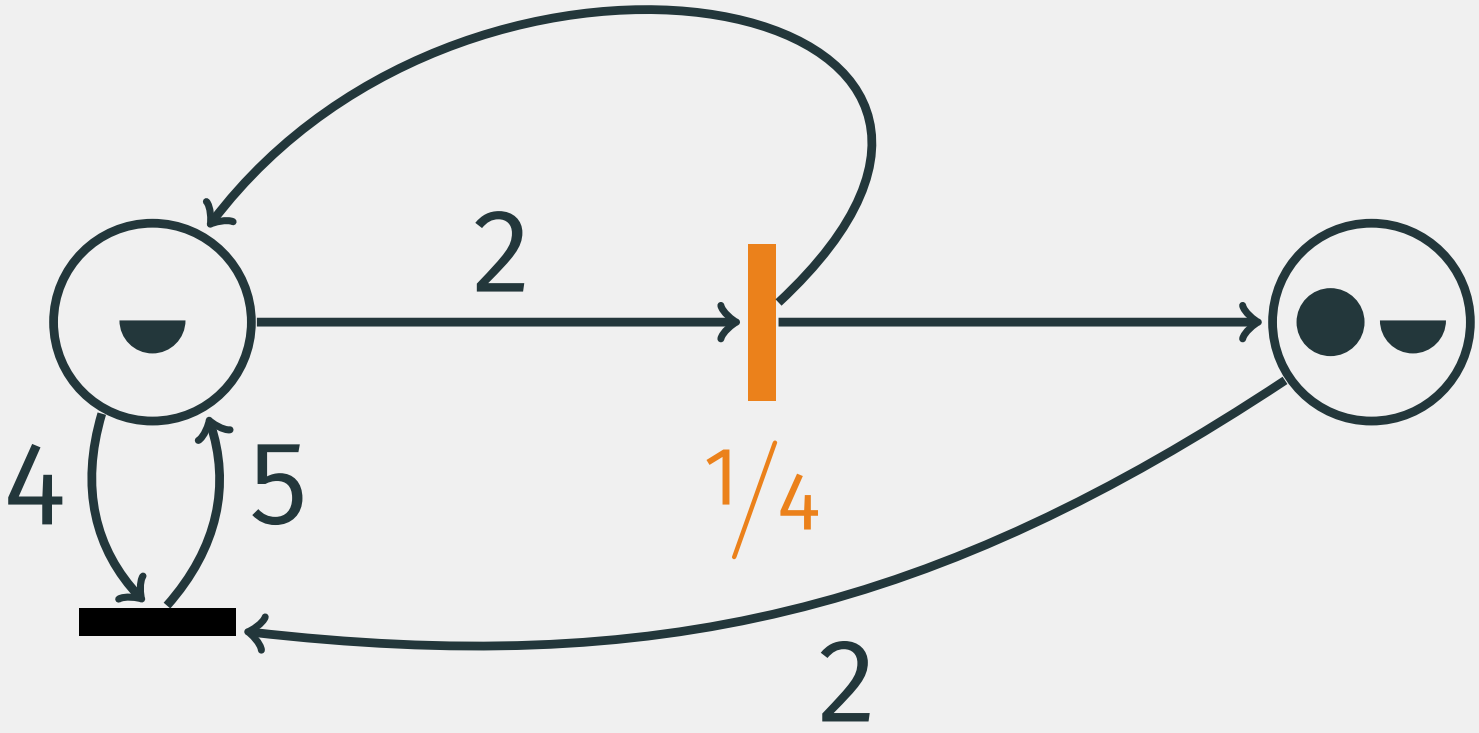
2
4
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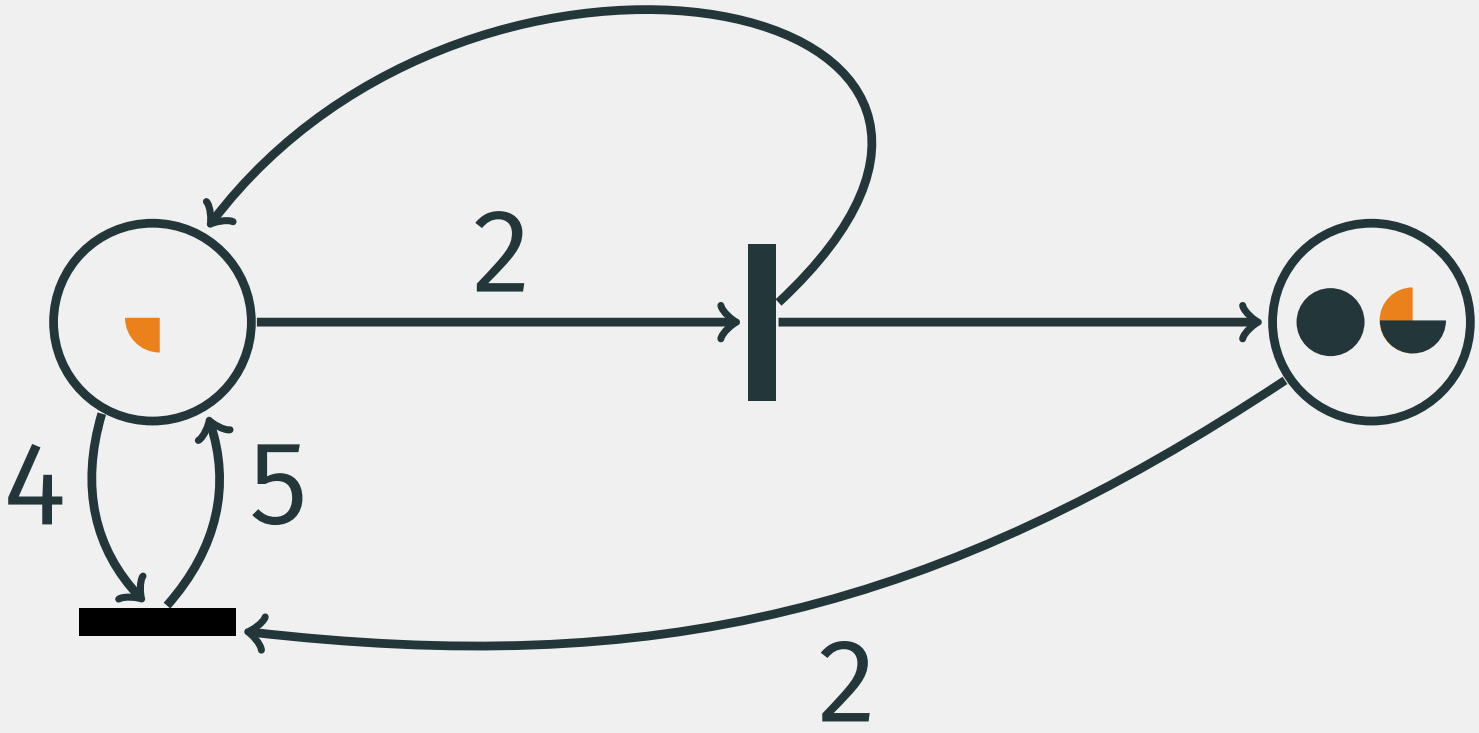
1/4
2


2
4
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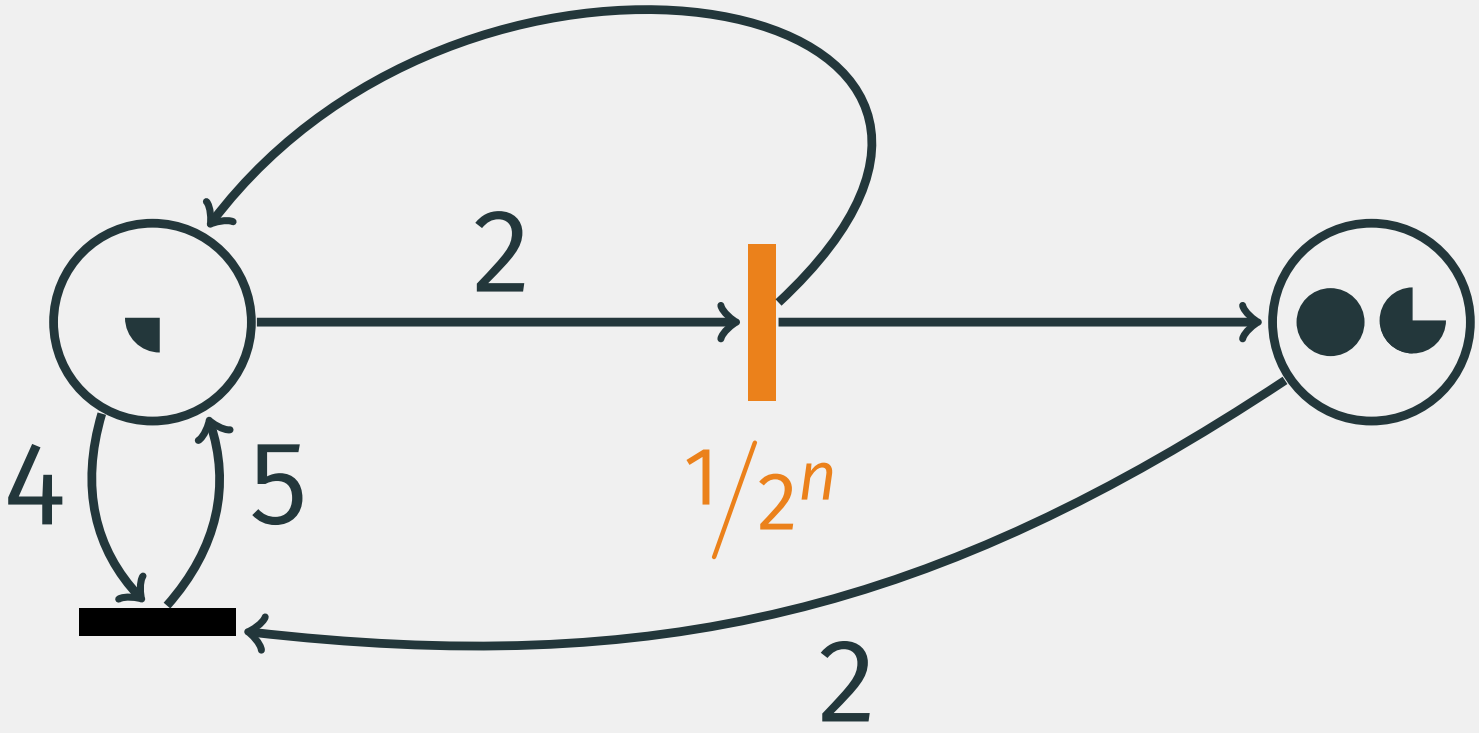
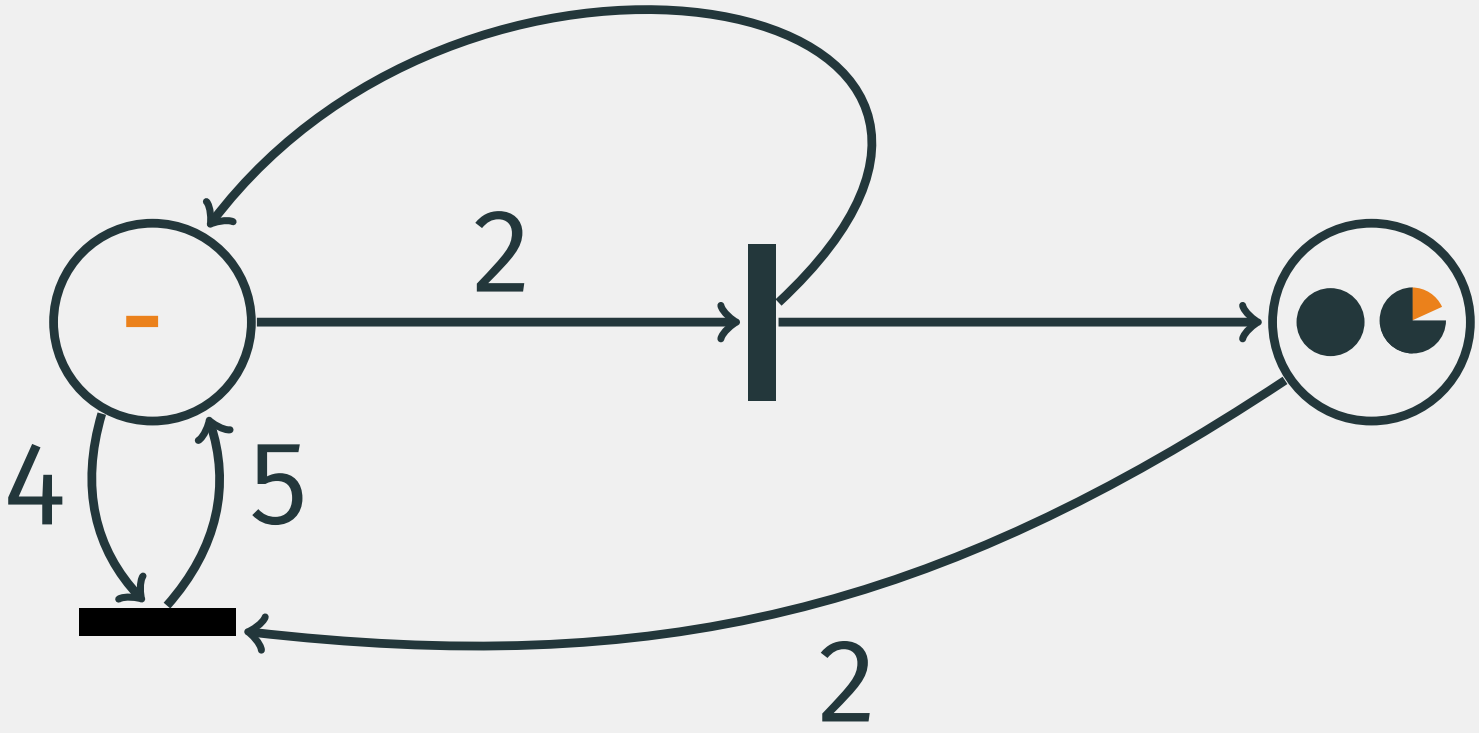

2
4
5
$1/2^n$
2


2
4
5
2

Continuity to over-approximate coverability

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

$$\Downarrow$$

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Continuity to over-approximate coverability

$\boldsymbol{m}$ is coverable from $\boldsymbol{m}_0$

EXPSPACE

$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

PTIME

$\boldsymbol{m}_0$ and $\boldsymbol{m}$ satisfy
state equation & trap constraints

Esparza & Melzer FMSD'00
Esparza, Ledesma-Garza, Majumdar, Meyer & Niksic CAV'14

PTIME / NP / coNP

Continuity to over-approximate coverability

$\boldsymbol{m}$ is not coverable from $\boldsymbol{m}_0$

Safety

$\boldsymbol{m}$ is not $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$

Coverability in continuous Petri nets

Fix some continuous Petri net $(P, T, \mathbf{Pre}, \mathbf{Post})$

m is $\mathbb{Q}$-coverable from m_0 iff... **Fraca & Haddad** **Fundam. Inf.'15**

Coverability in continuous Petri nets

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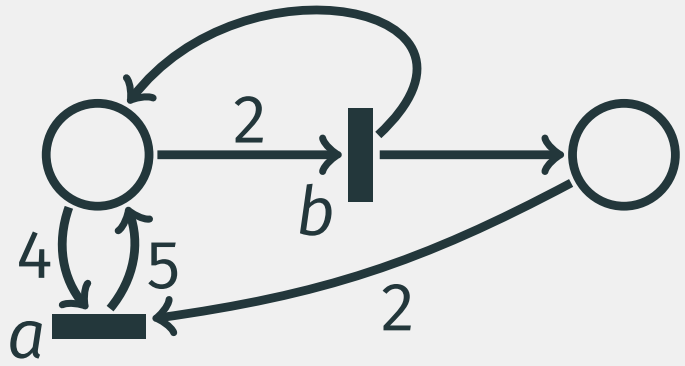

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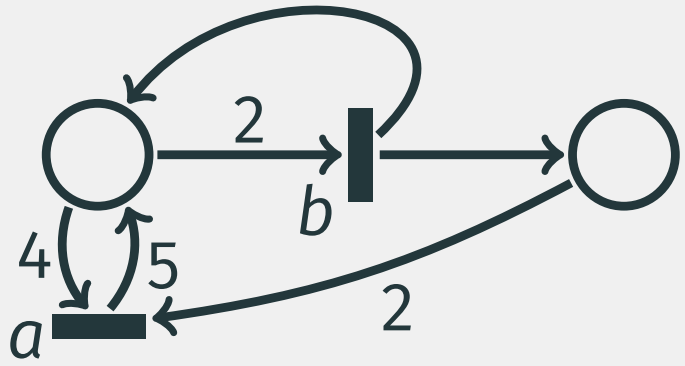

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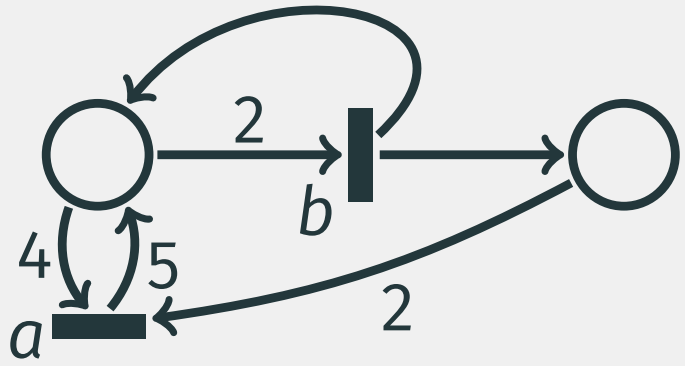

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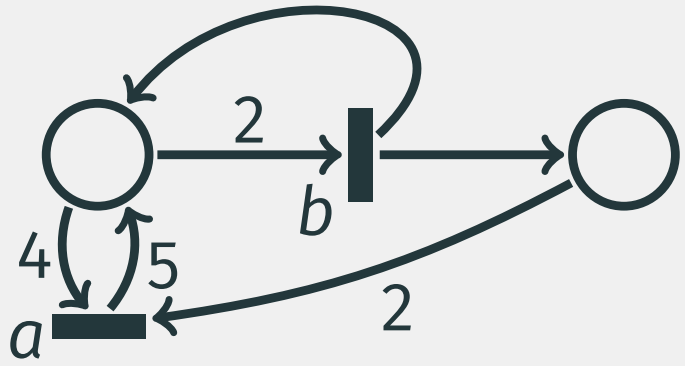

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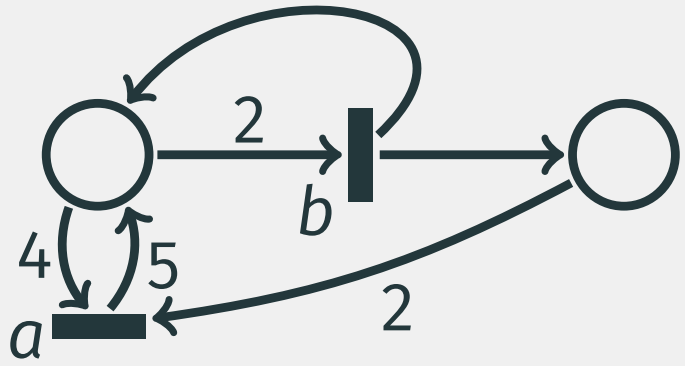

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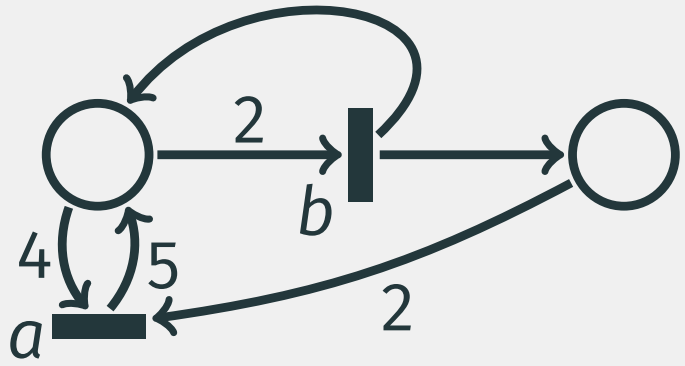

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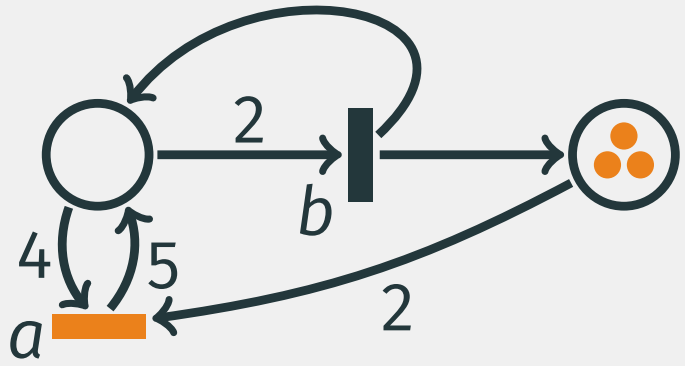
Coverability in continuous Petri nets



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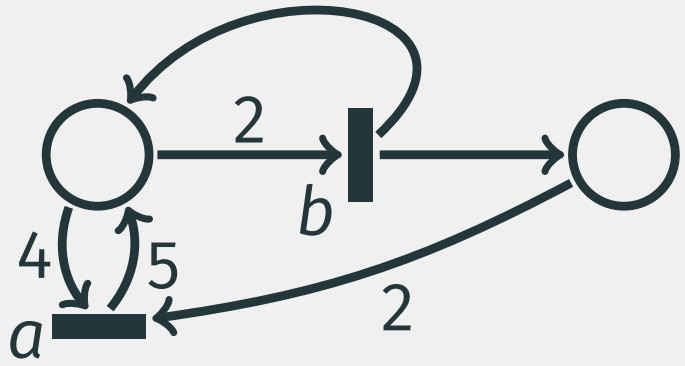

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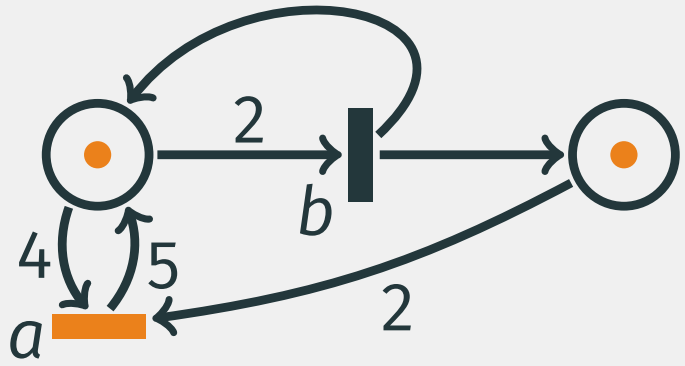

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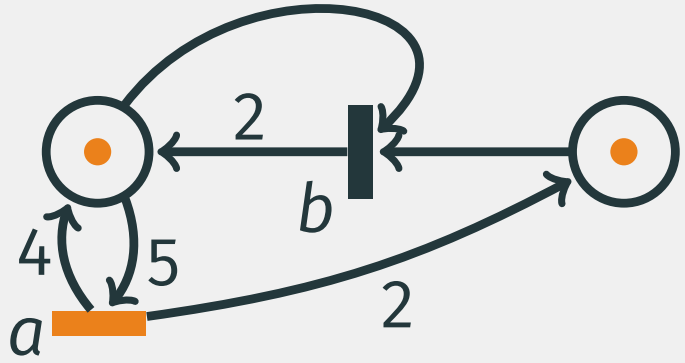

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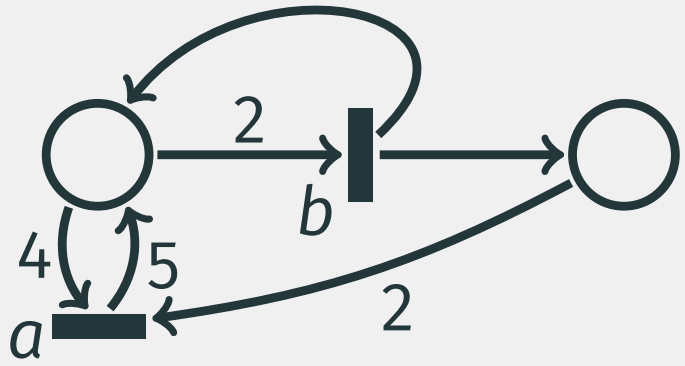

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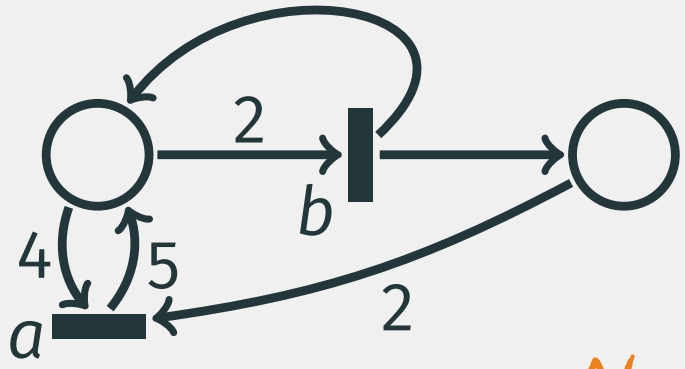

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Not $\mathbb{Q}$-coverable from

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Coverability in continuous Petri nets

Polynomial time!

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Coverability in continuous Petri nets

Logical characterization B., Finkel, Haase & Haddad TACAS'16

$\mathbb{Q}$-coverability can be encoded in a linear size formula of existential FO$(\mathbb{Q}_{\geq 0}, +, <)$

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Even better approximation

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- $\boldsymbol{m}' = \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$ Straightforward
- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
- some execution to $\boldsymbol{m}'$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

Coverability in continuous Petri nets

Logical characterization B., Finkel, Haase & Haddad TACAS'16

$\mathbb{Q}$-coverability can be encoded in a linear size formula of

$$\text{existential FO}(\mathbb{Q}_{\geq 0}, +, <)$$

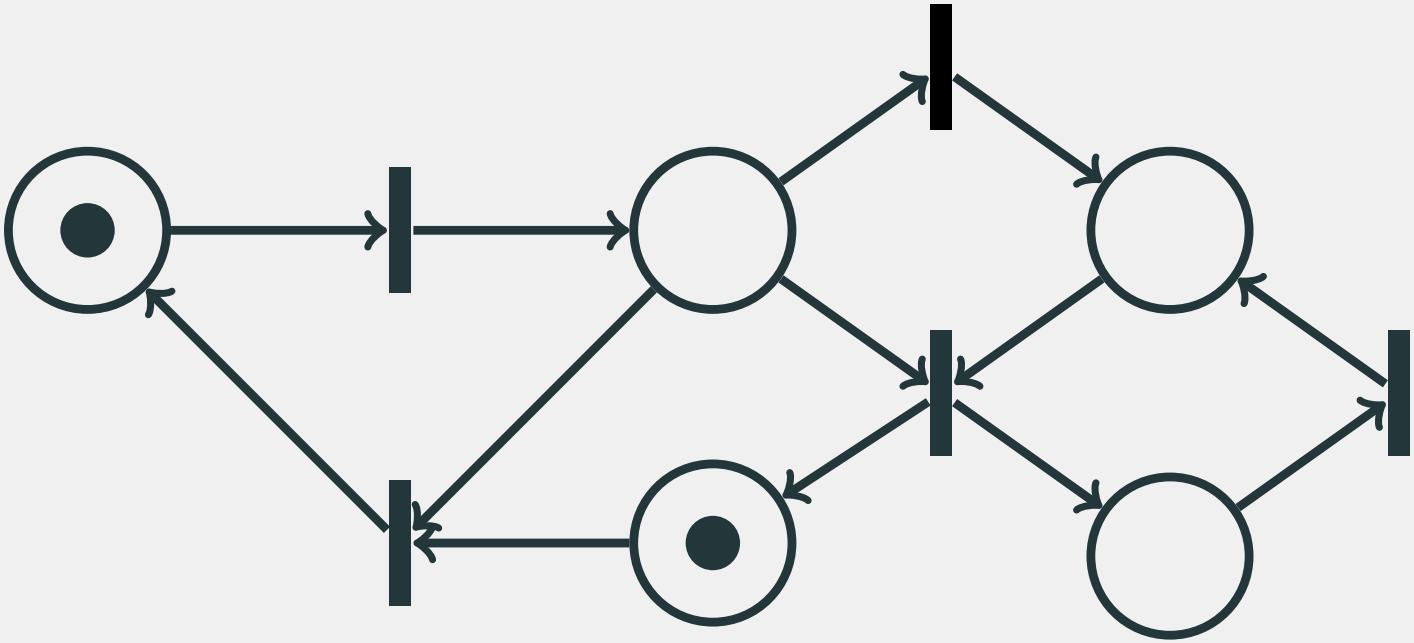
$\boldsymbol{m}$ is $\mathbb{Q}$-coverable from $\boldsymbol{m}_0$ iff... Fraca & Haddad Fundam. Inf.'15

there exist $\boldsymbol{m}' \geq \boldsymbol{m}$ and $\boldsymbol{v} \in \mathbb{Q}_{\geq 0}^{T}$ such that

- $\boldsymbol{m}' = \boldsymbol{m}_0 + (\mathbf{Post} - \mathbf{Pre}) \cdot \boldsymbol{v}$
- some execution from $\boldsymbol{m}_0$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$
- some execution to $\boldsymbol{m}'$ fires exactly $\{t \in T : \boldsymbol{v}_t > 0\}$

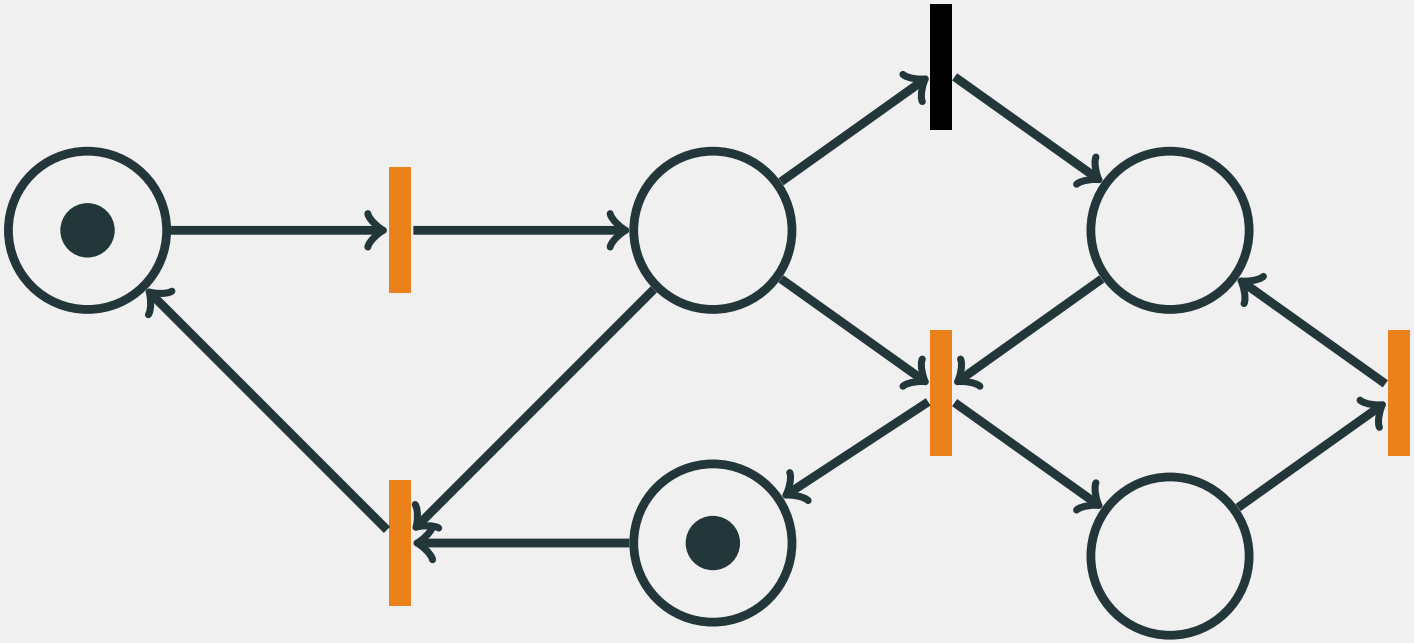
More subtle

Encoding the firing set conditions



Testing whether some transitions can be fired from initial marking

Encoding the firing set conditions



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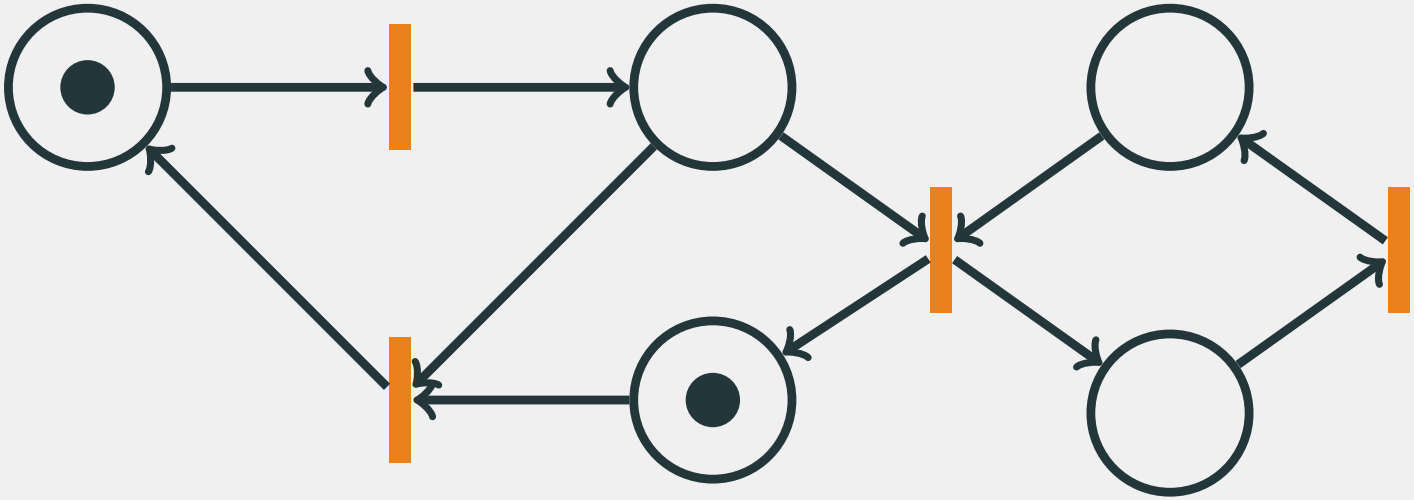

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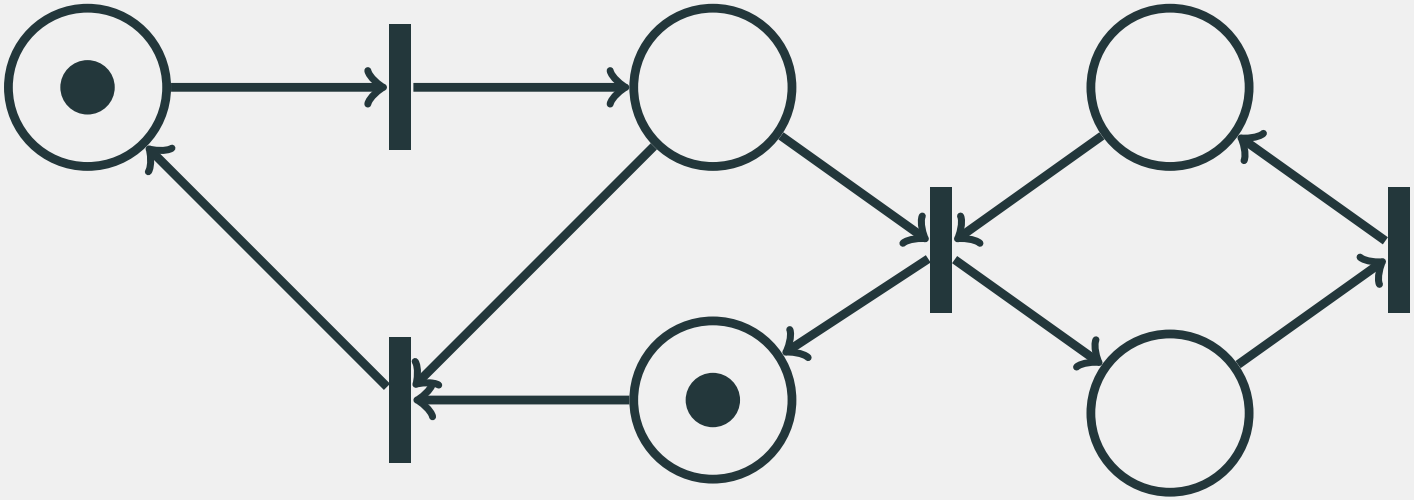
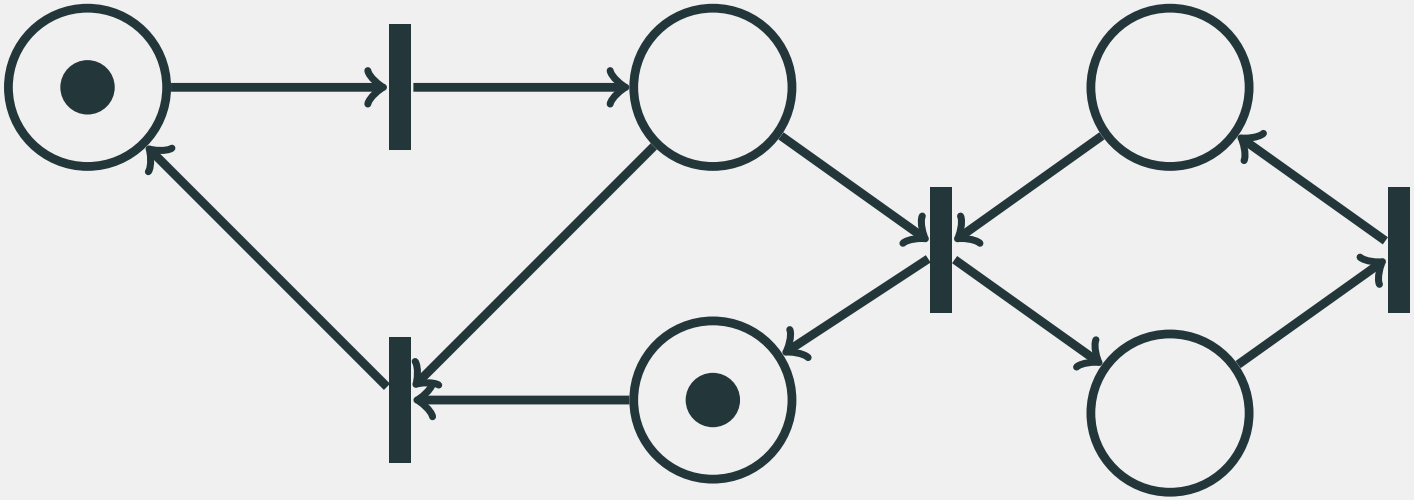


Simulate a "breadth-first" transitions firing

Encoding the firing set conditions

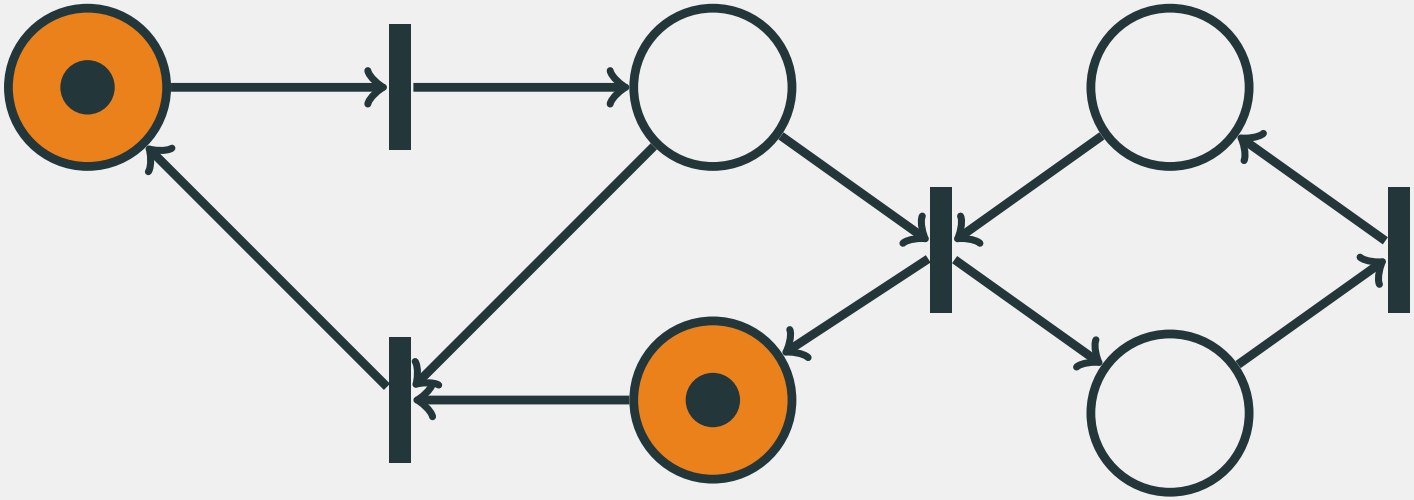


Simulate a "breadth-first" transitions firing

by numbering places/transitions

Verma, Seidl & Schwentick CADE'05

Encoding the firing set conditions



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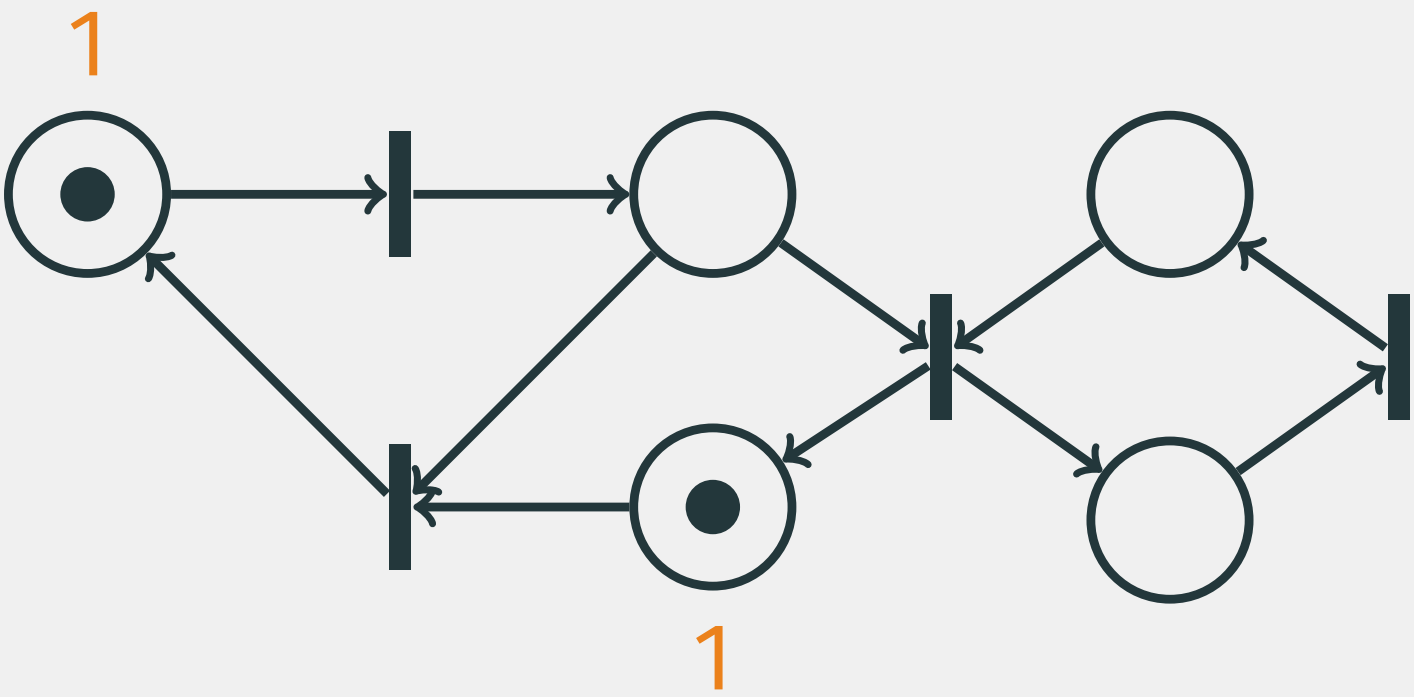

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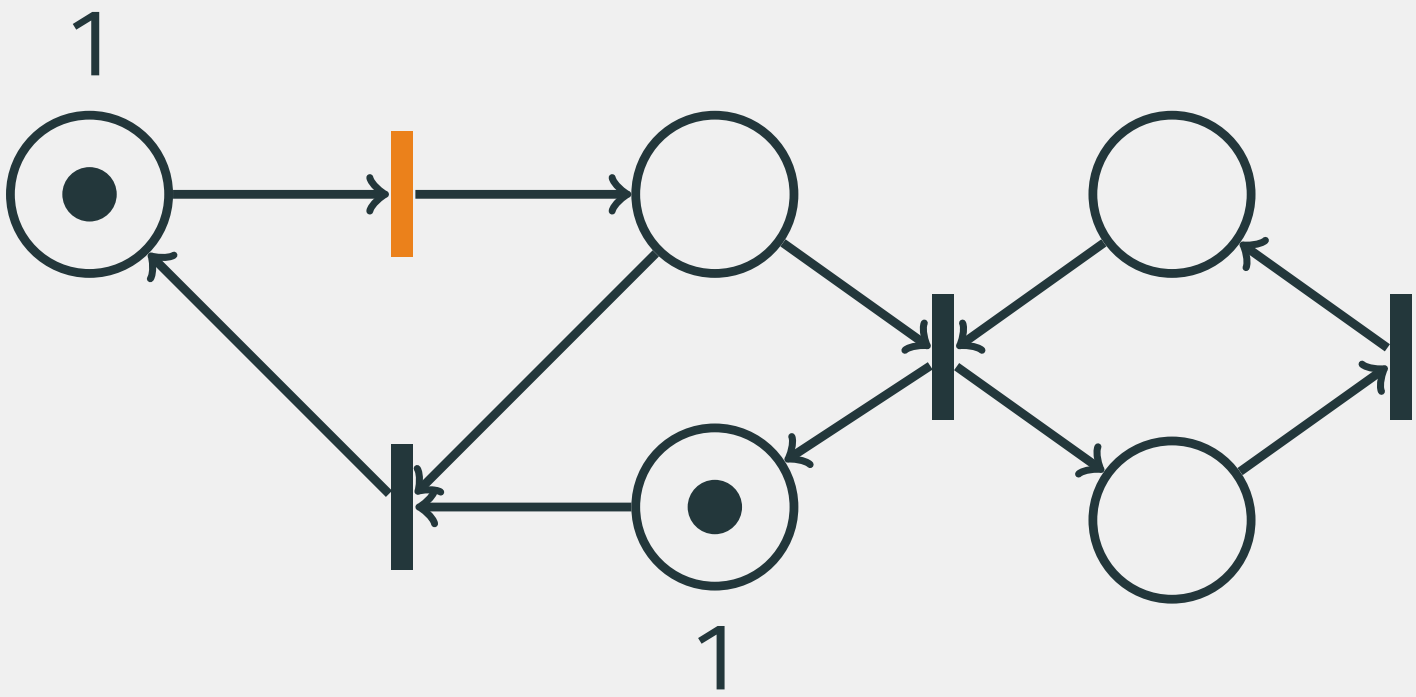

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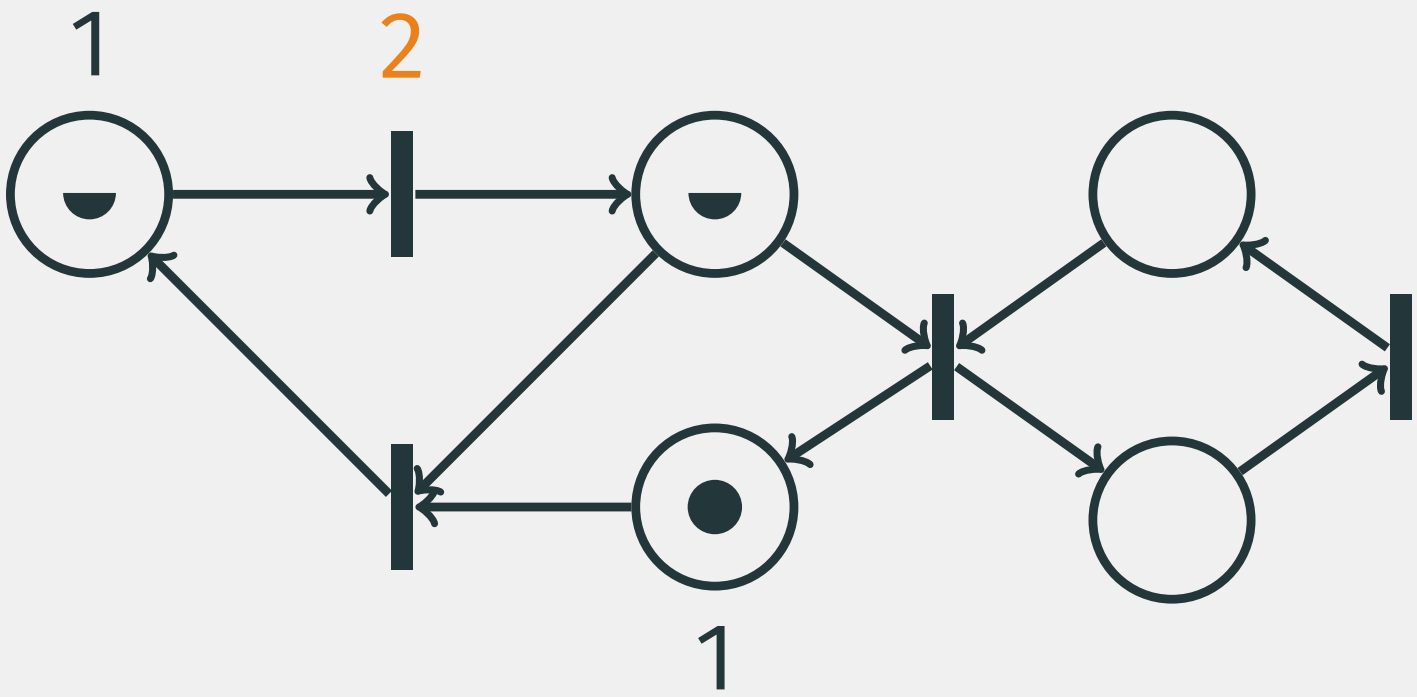

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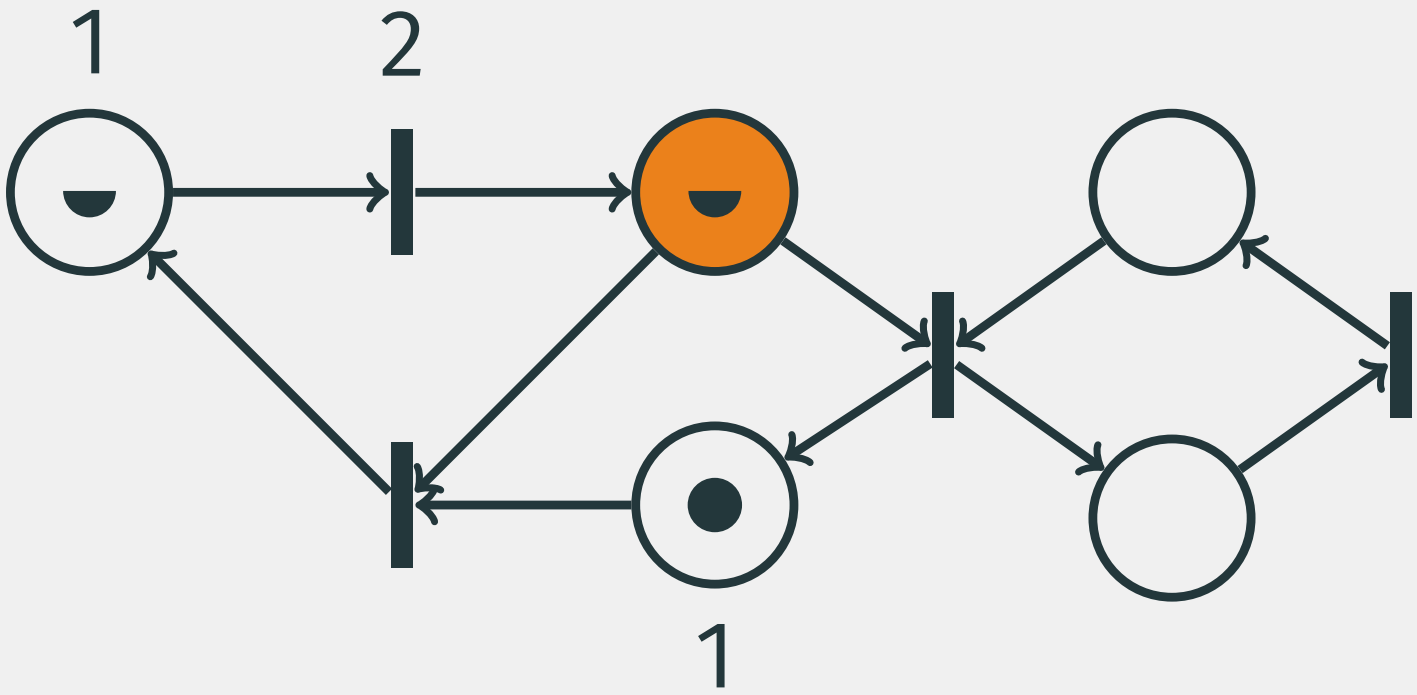

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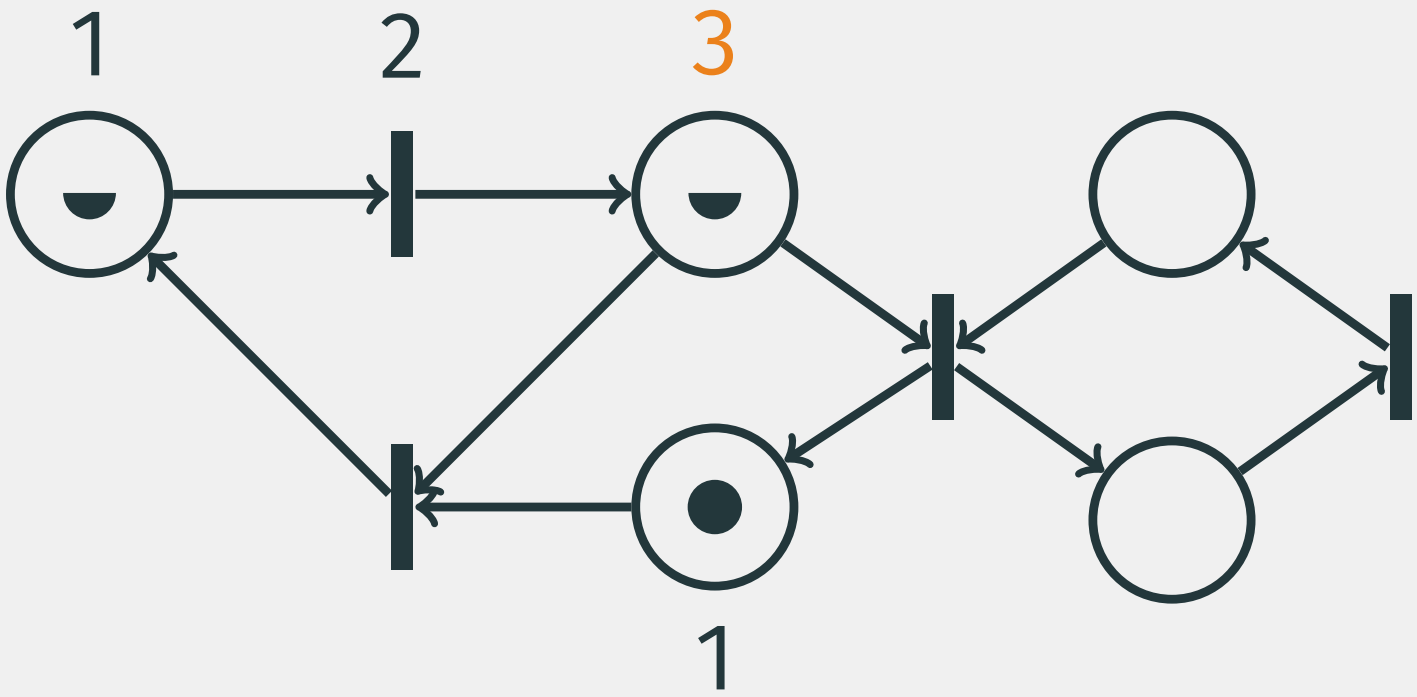

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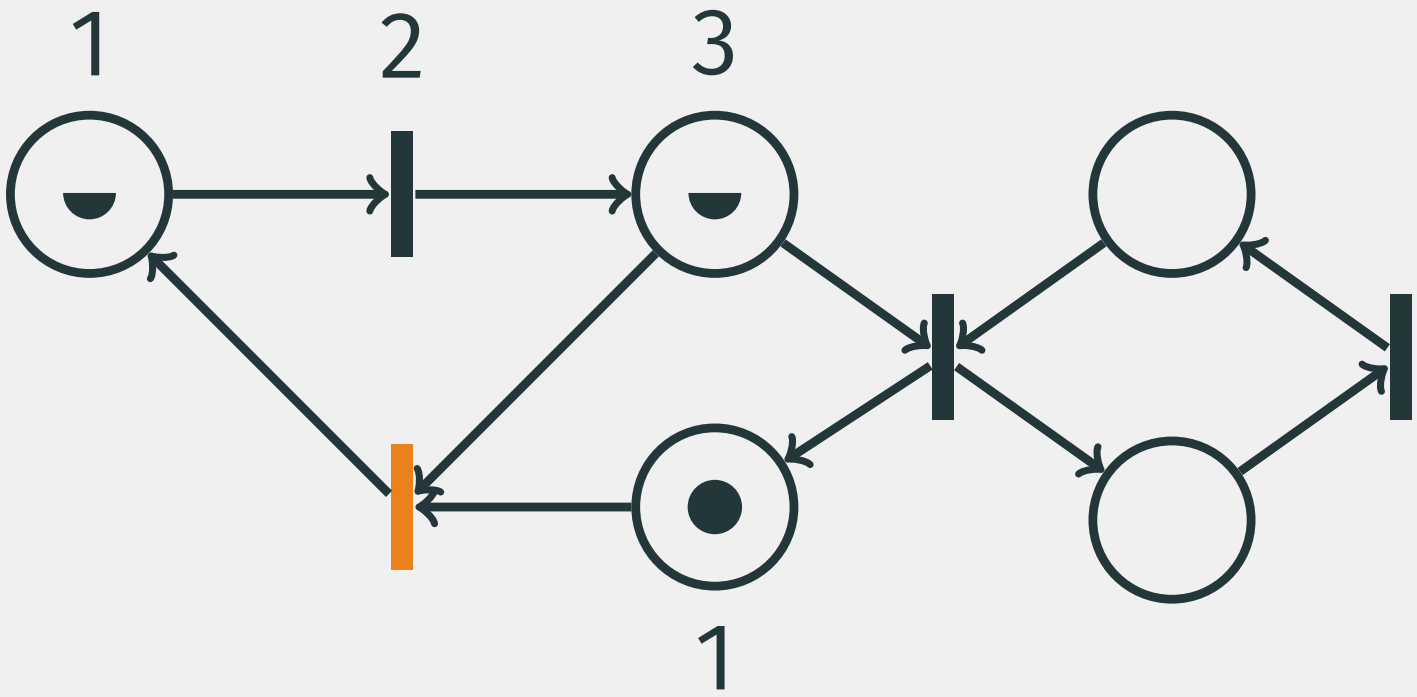

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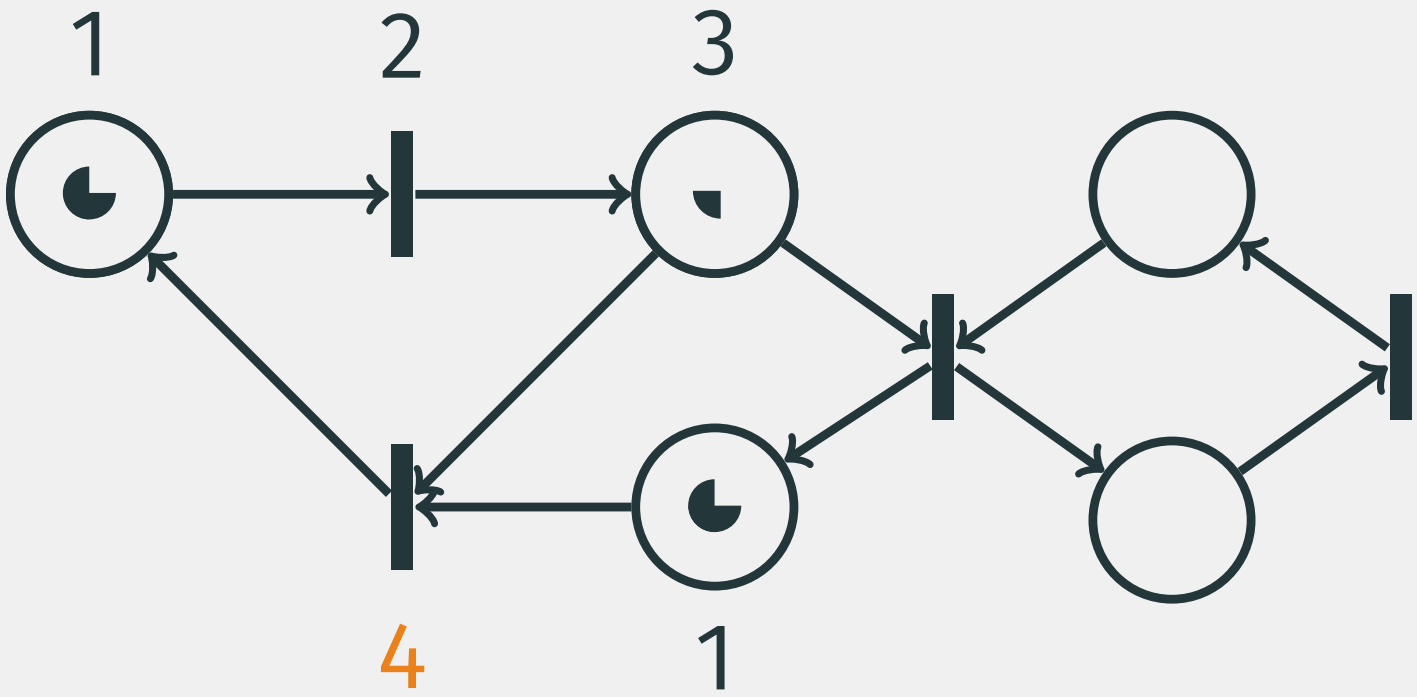

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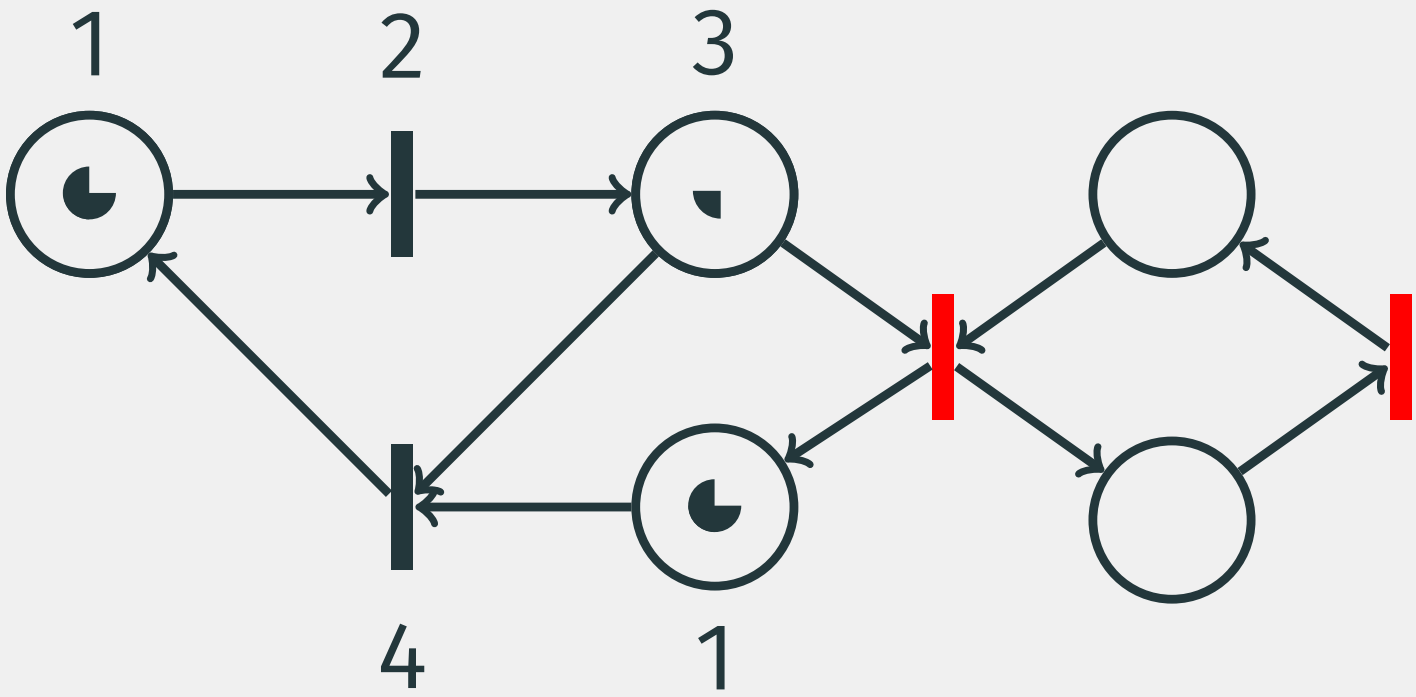

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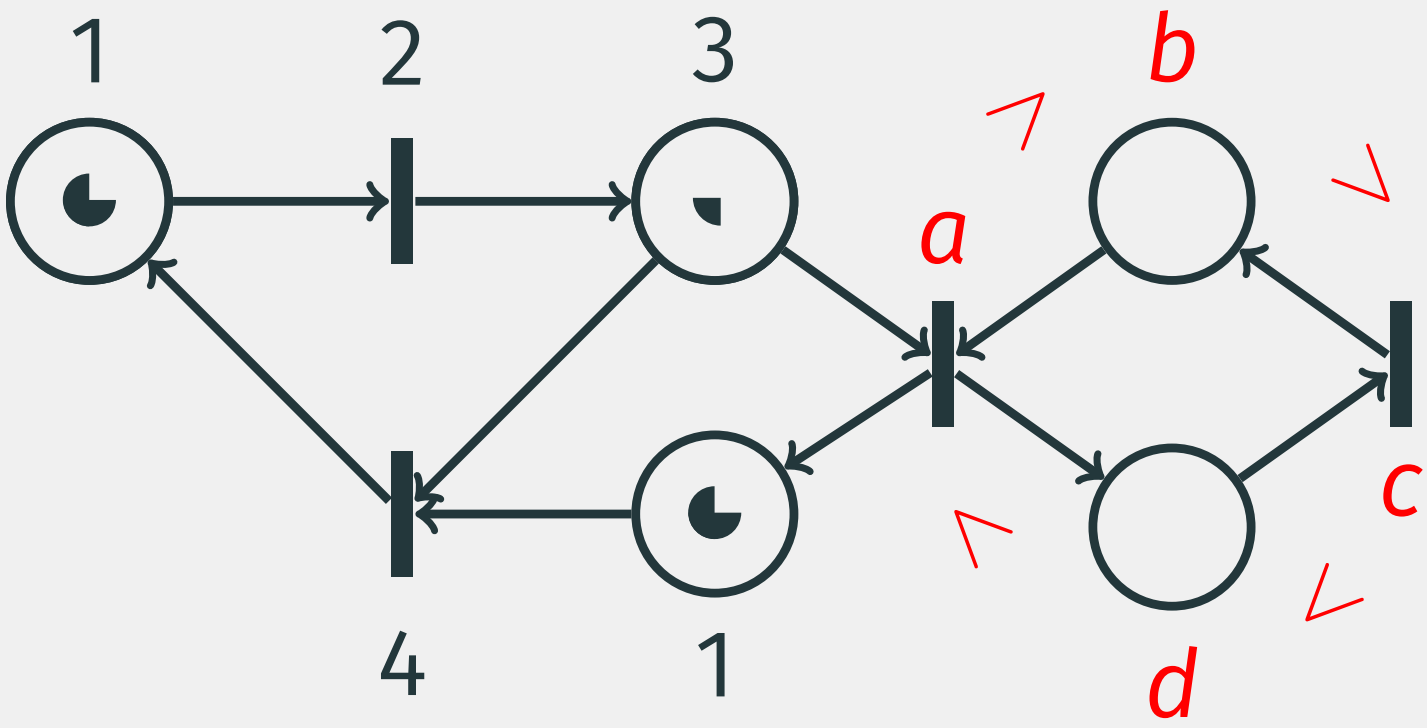

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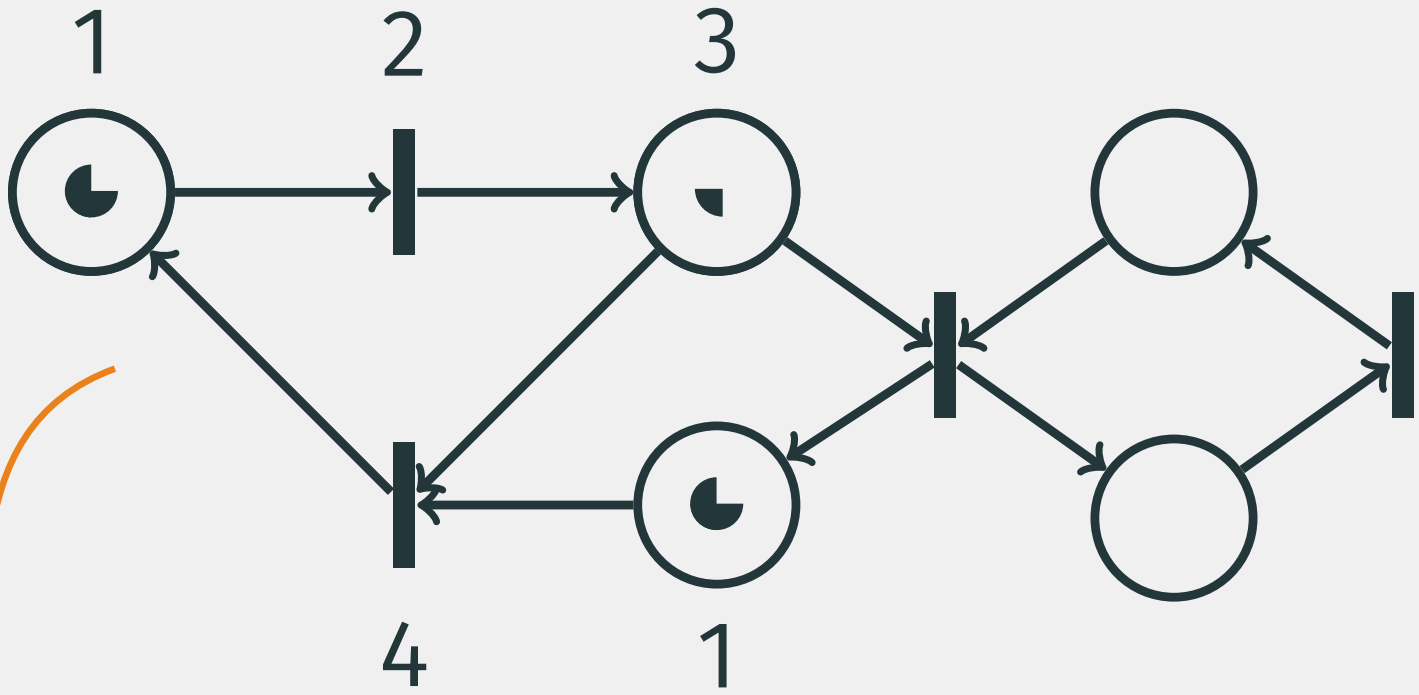

$$\varphi(\boldsymbol{x}) = \exists \boldsymbol{y} : \bigwedge_{p \in P} \boldsymbol{y}(p) > 0 \rightarrow \bigwedge_{t \in {}^\bullet p} \boldsymbol{y}(t) < \boldsymbol{y}(p) \cdots$$

From $\mathbb{Q}$-coverability to a complete algorithm

$\mathbb{Q}$-coverability: efficient but incomplete...

Combine approaches!

From $\mathbb{Q}$-coverability to a complete algorithm

Forward algorithm — **Karp & Miller** JCSS'69

- Build reachability tree from initial marking
- "Accelerate" loops

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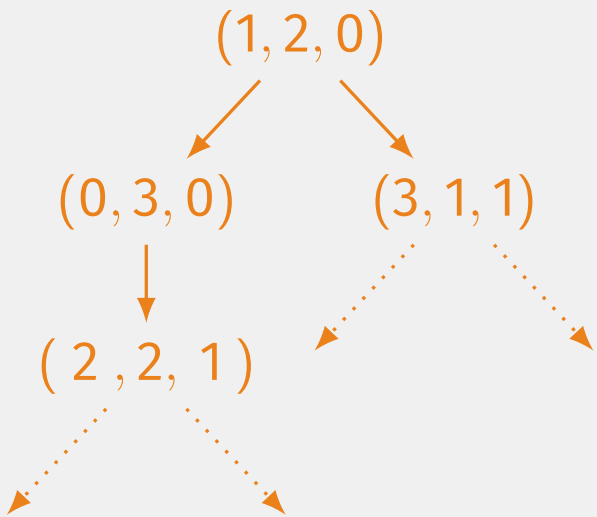

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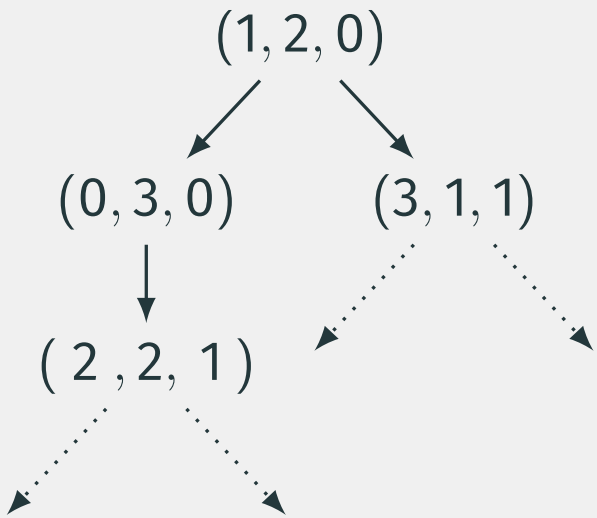

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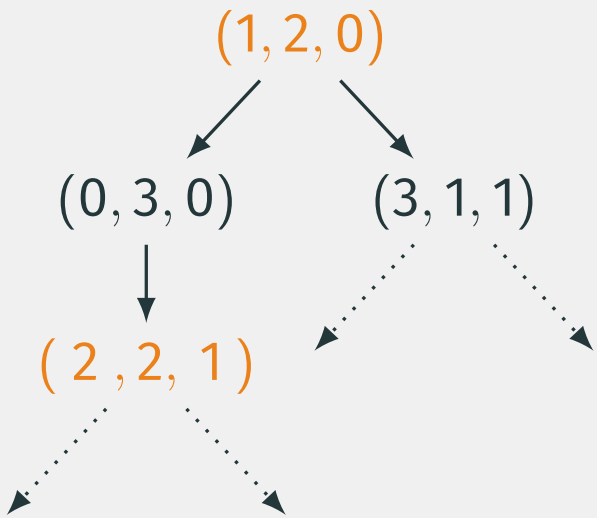

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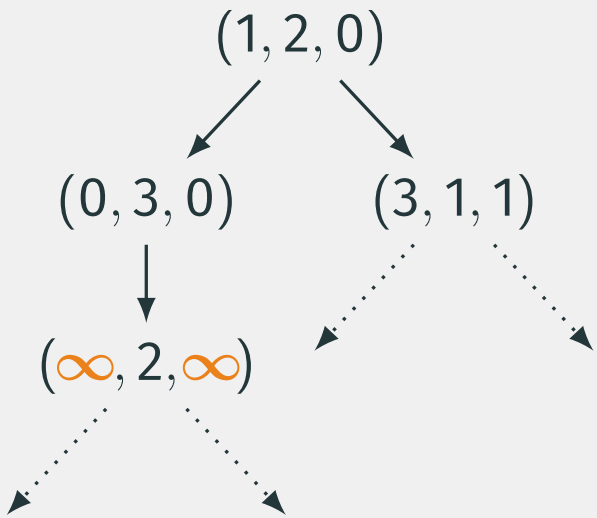

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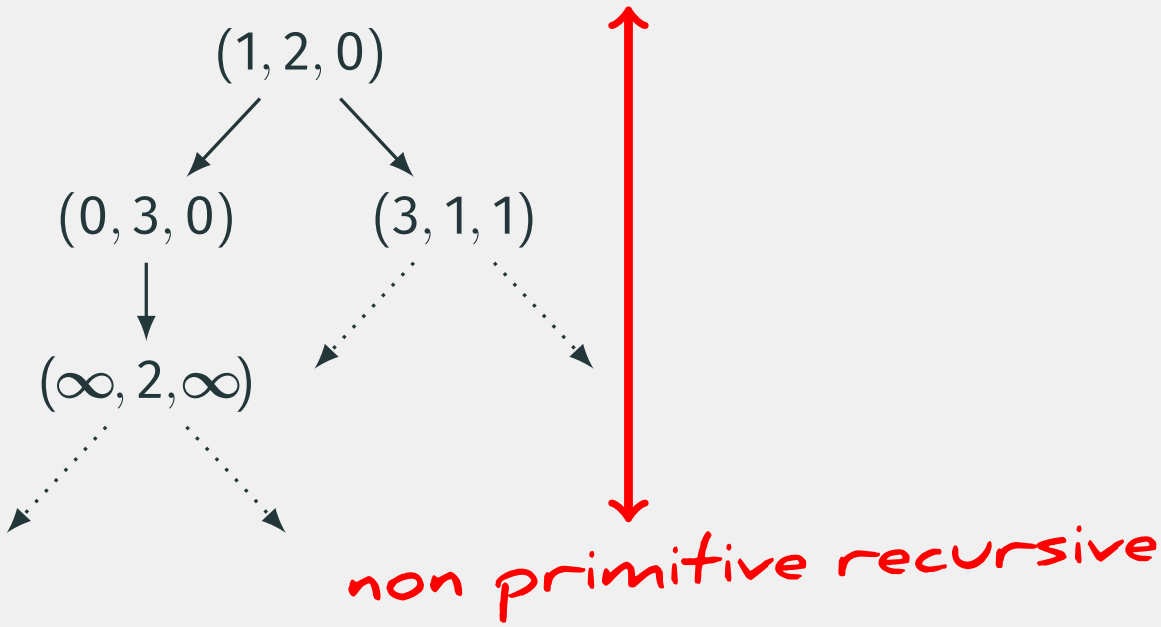

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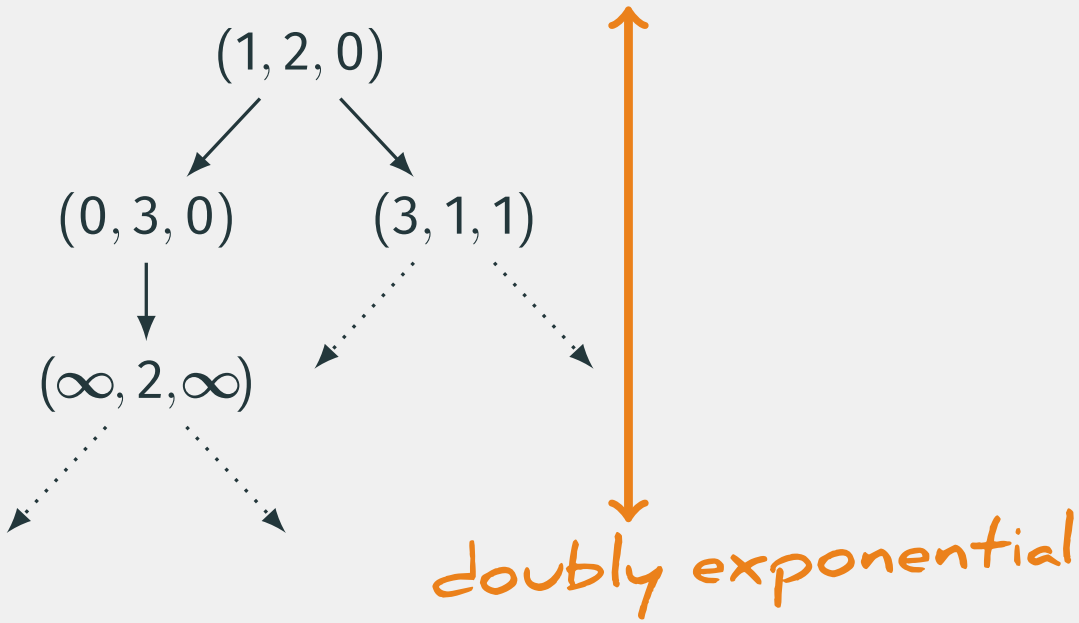

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Backward algorithm — **Arnold & Latteux** Calcolo'78, **Abdulla *et al.*** LICS'96

- Start from upward closure of target marking
- Compute predecessors of current markings

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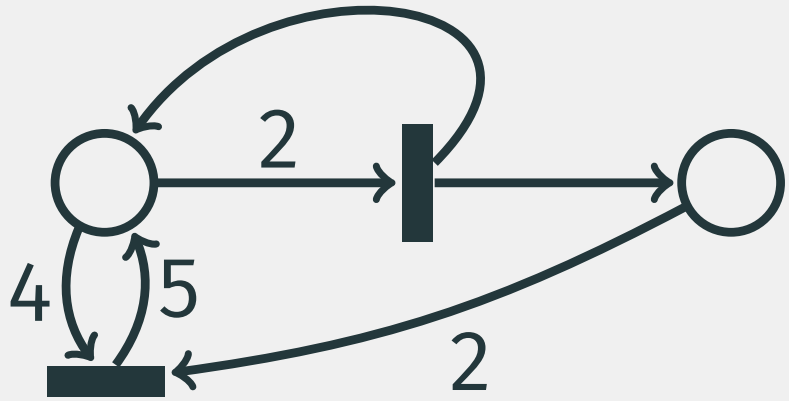

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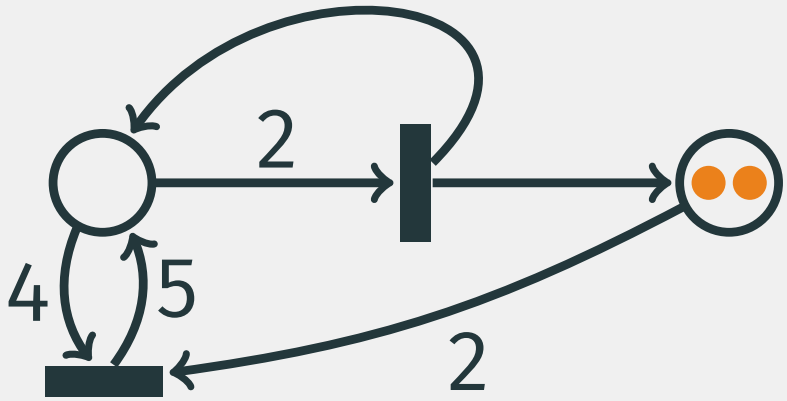
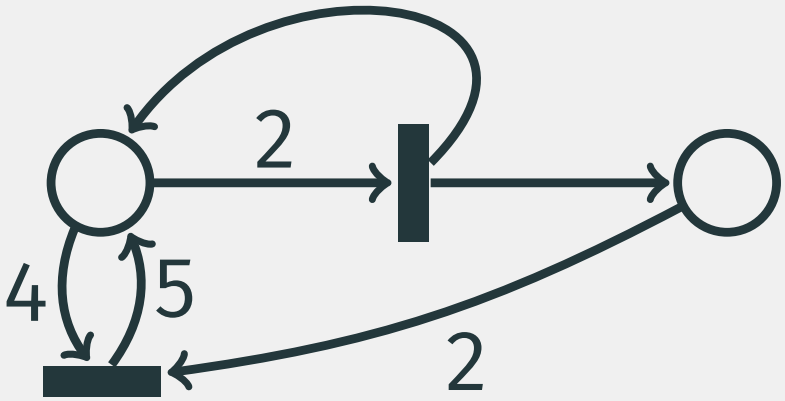
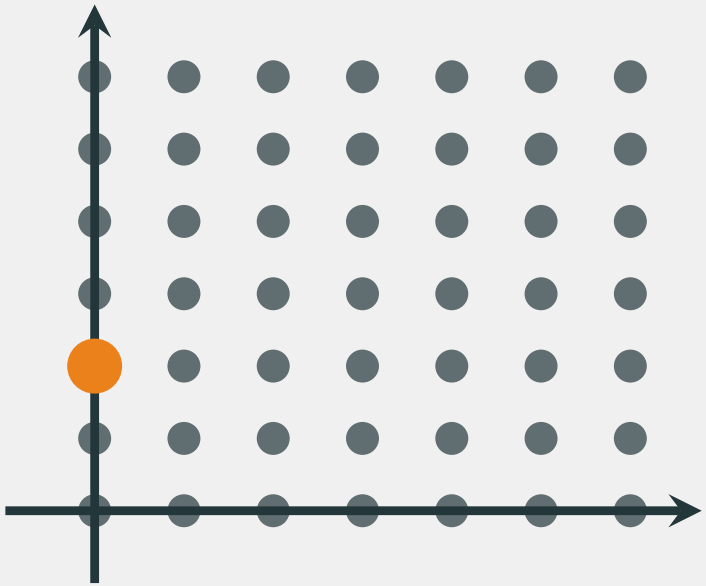


What initial markings may cover $(0, 2)$?

Backward algorithm





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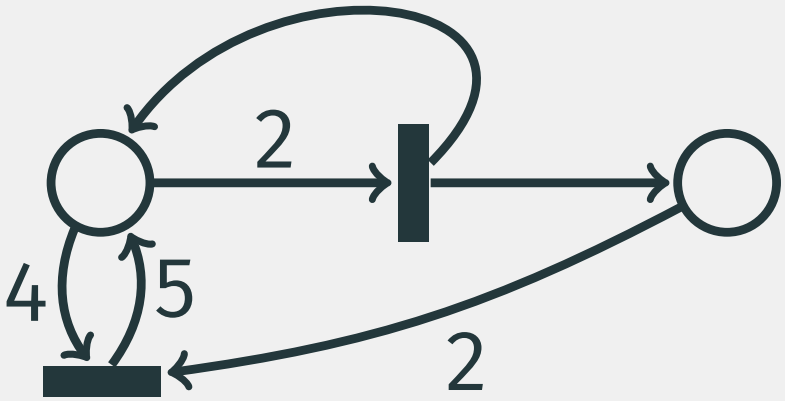

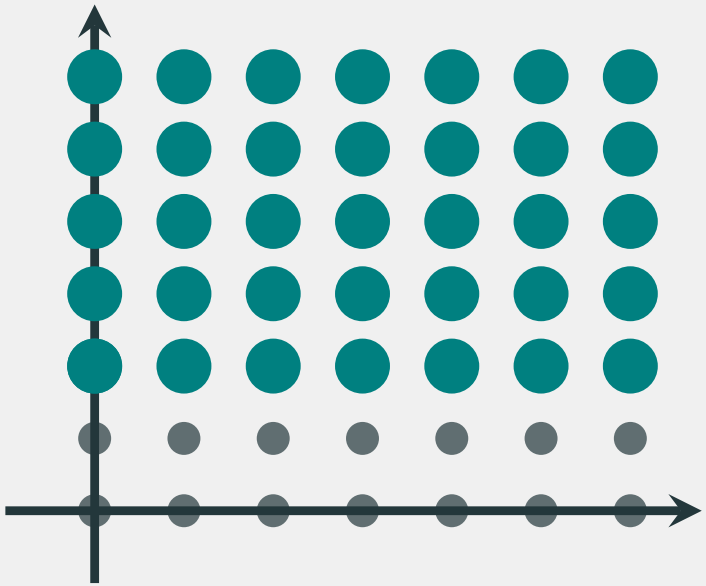

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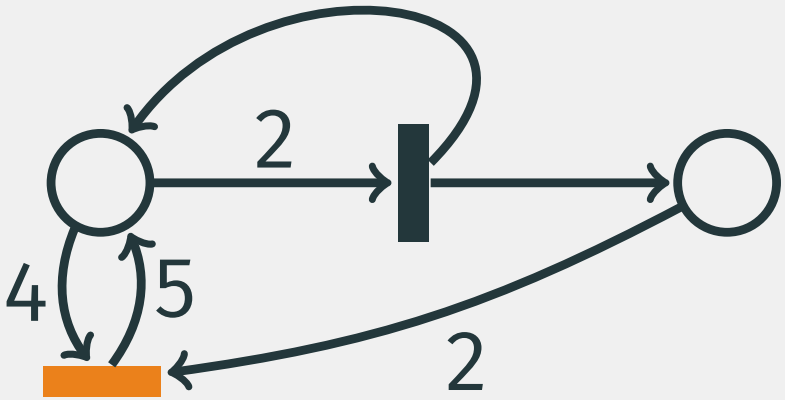

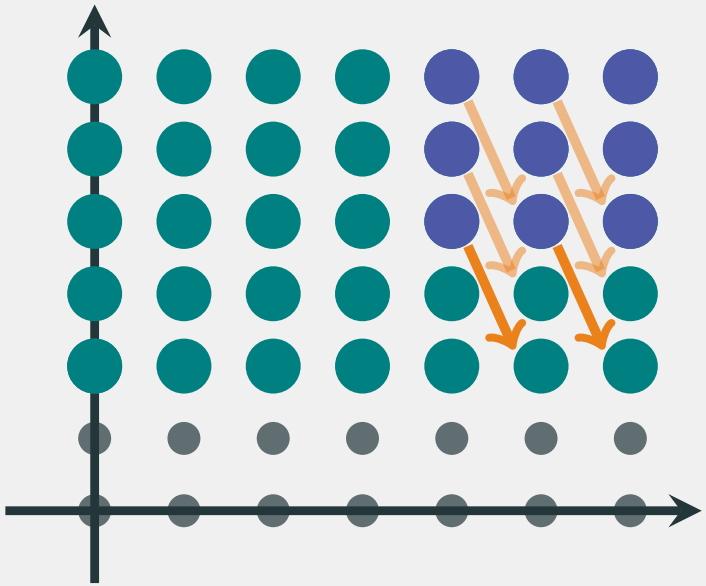

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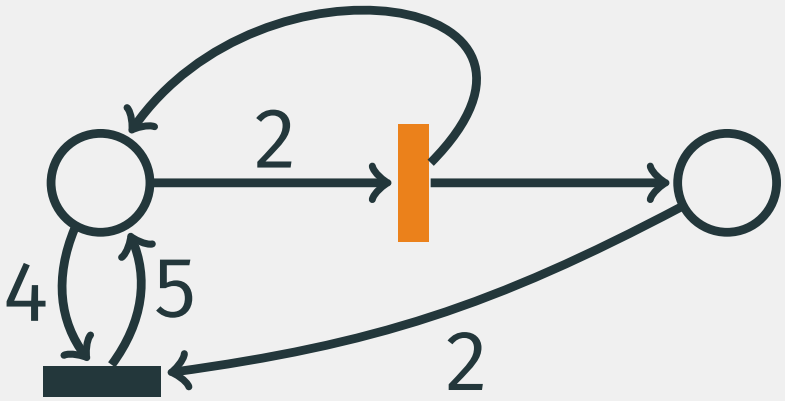

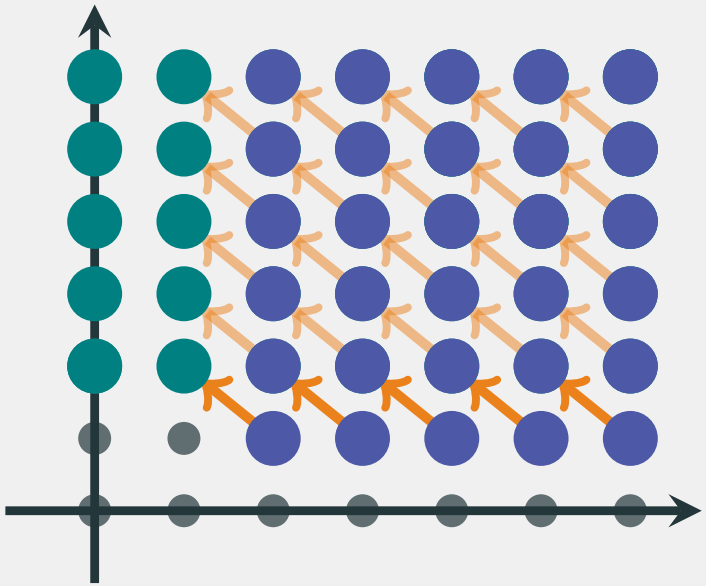

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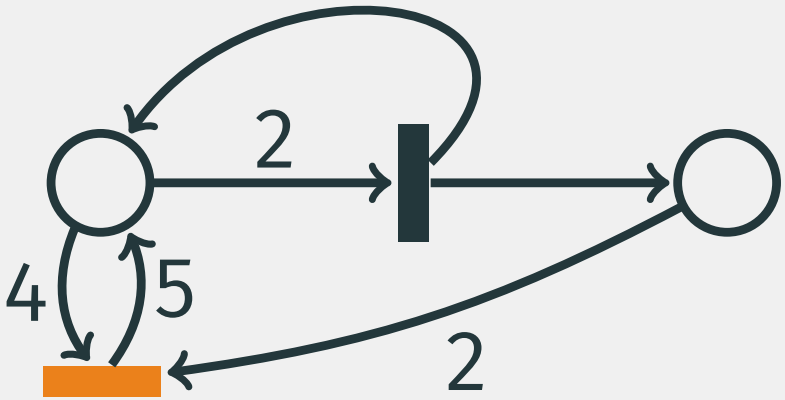

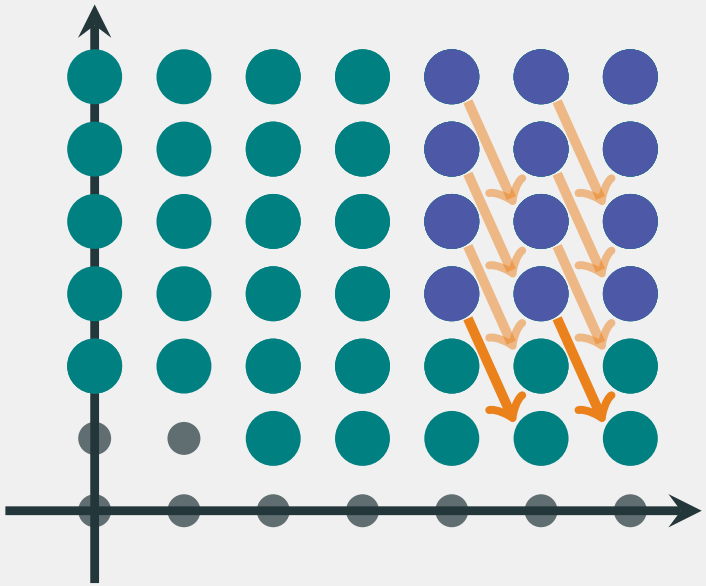

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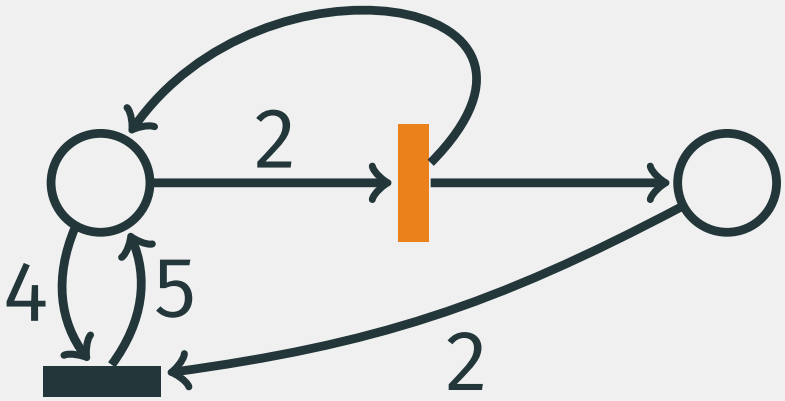

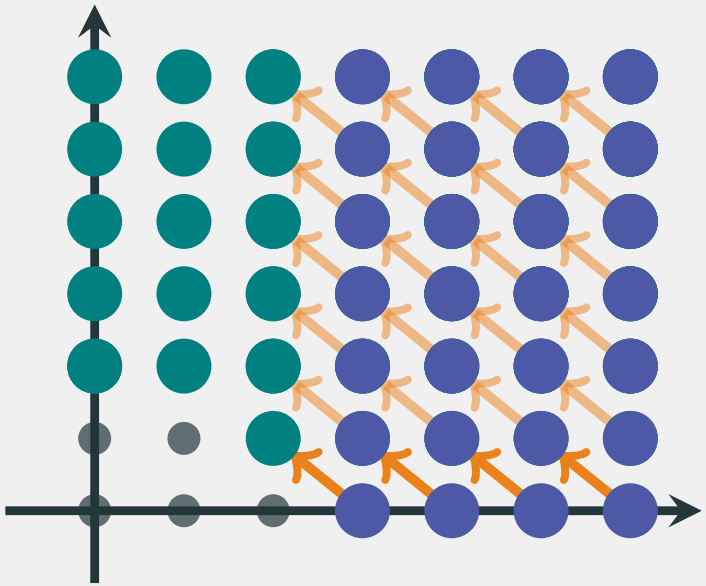

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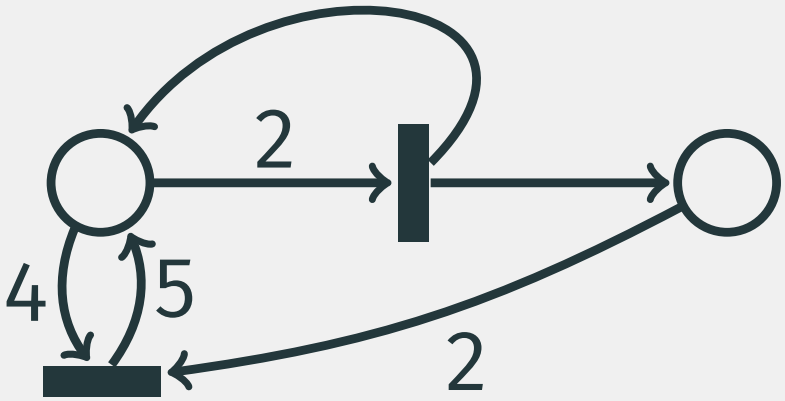
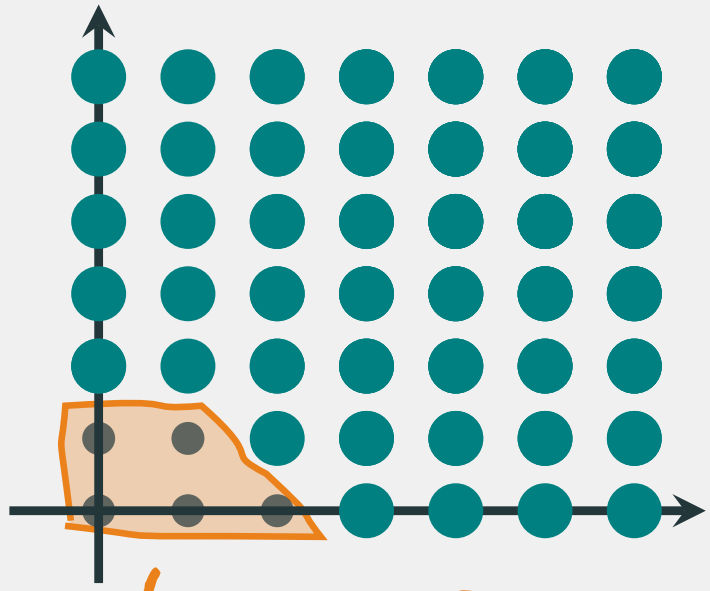
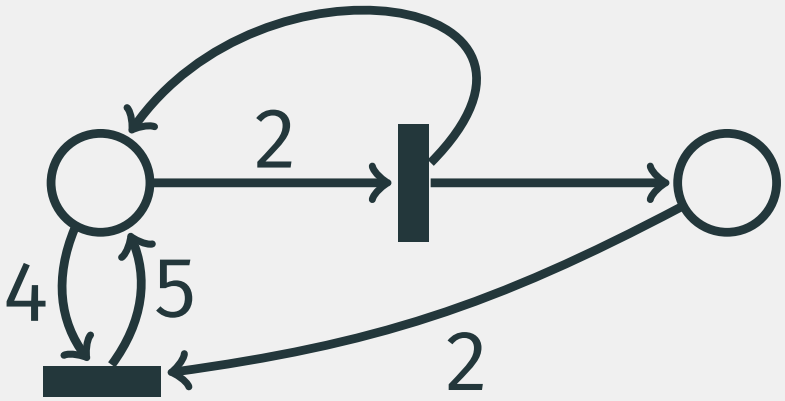
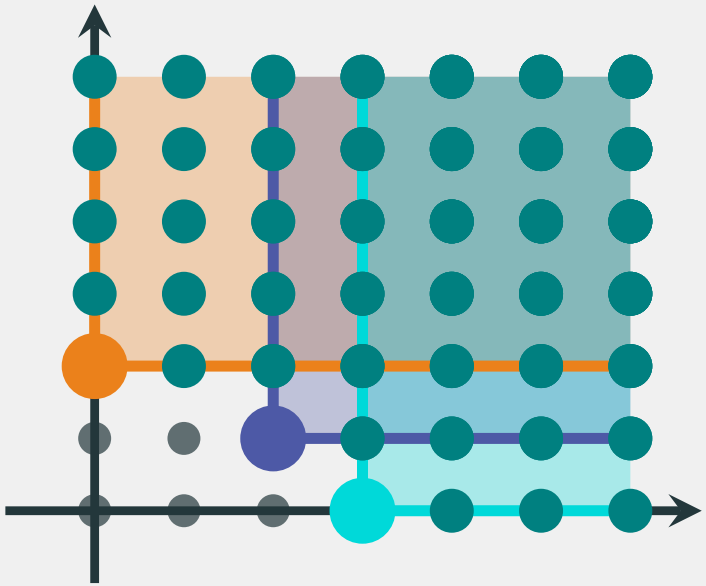




Cannot cover target marking

Backward algorithm

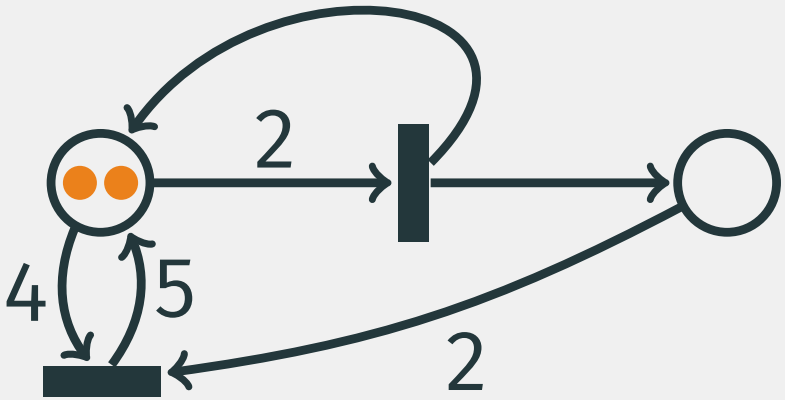


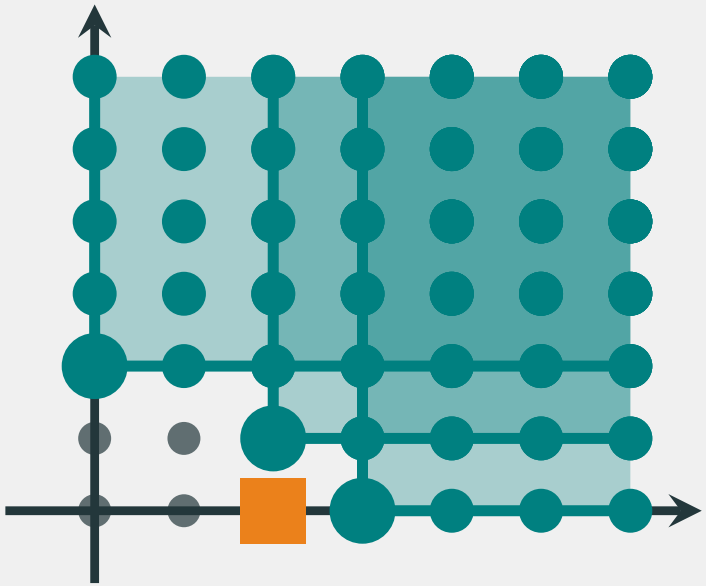


Basis size may become doubly exponential

Bozzelli & Ganty RP'11

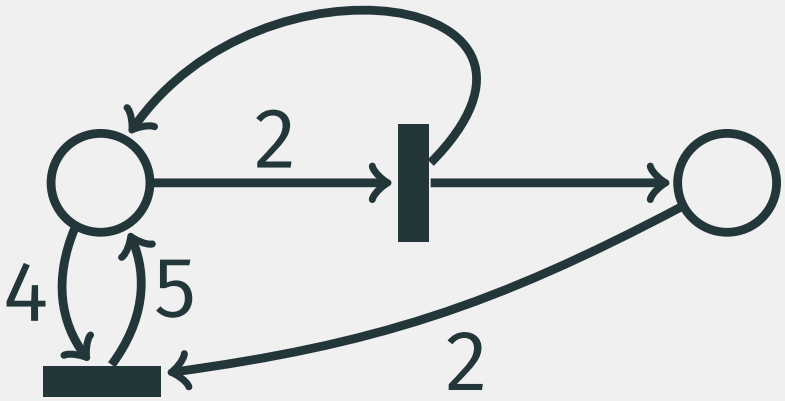
Backward algorithm

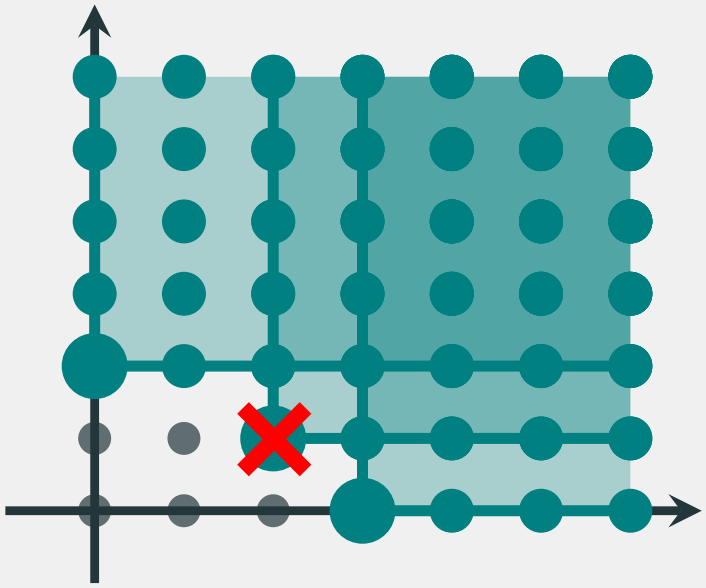




We only care about some initial marking...

Backward algorithm





We only care about some initial marking...

Prune basis with $\mathbb{Q}$-coverability!

Backward coverability modulo $\mathbb{Q}$-coverability

```
if target marking m is not ℚ-coverable:
  return False
```

Backward coverability modulo $\mathbb{Q}$-coverability

if target marking $\boldsymbol{m}$ is not $\mathbb{Q}$-coverable:

  return False

$X = \{\text{target marking } \boldsymbol{m}\}$

while (initial marking $\boldsymbol{m}_0$ not covered by X):

  $B =$ markings obtained from X one step backward

  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$

  if $B = \emptyset$: return False

  $\varphi(\boldsymbol{x}) = \varphi(\boldsymbol{x}) \ \wedge \ \bigwedge_{\text{pruned } \boldsymbol{b}} \boldsymbol{x} \not\geq \boldsymbol{b}$

  $X = X \cup B$

return True

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  $B = B \setminus \{\boldsymbol{b} \in B : \neg\varphi(\boldsymbol{b})\}$

  if $B = \emptyset$: return False

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Backward coverability modulo $\mathbb{Q}$-coverability

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$\mathbb{Q}$-coverability pruning (better than poly. time)

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SMT solver guidance

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- uses the MIST `.spec` format for counter machines
- supports dense/sparse matrices through NumPy/SciPy
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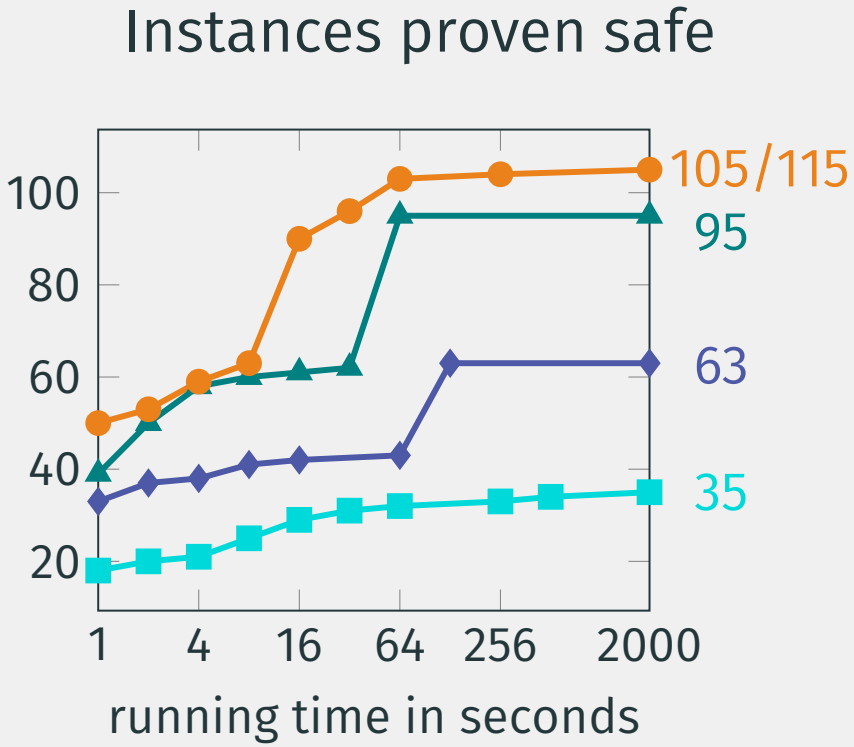
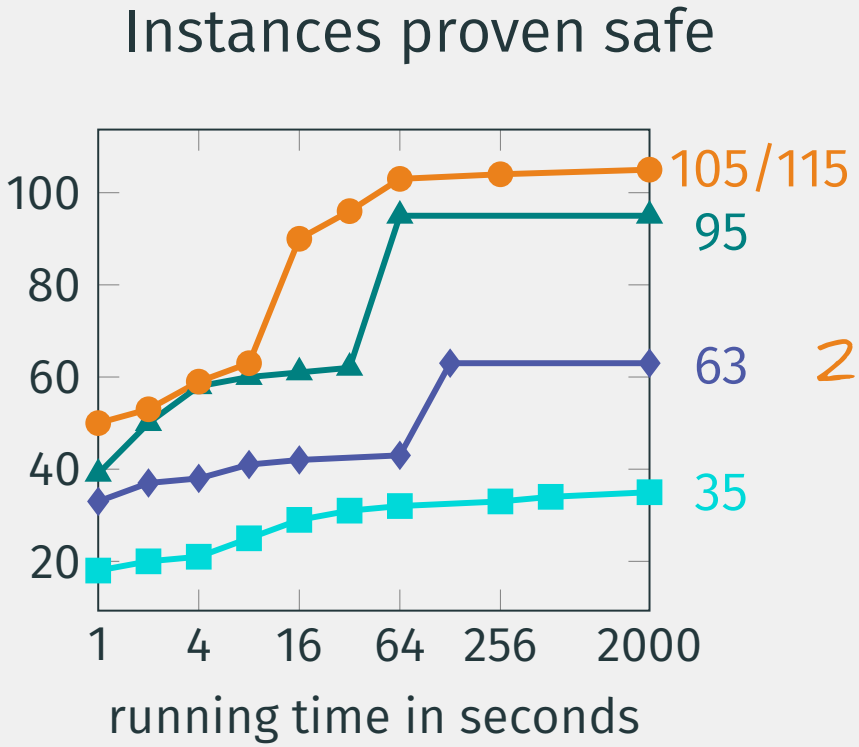

Instances proven safe
100
80
60
40
20
1
4
16
64
256
2000
running time in seconds
105/115
95
63
35
QCover
Petrinizer
BFC
MIST

Benchmarks



Largest nets proved safe:

21143 places
7150 trans.
42 secs.

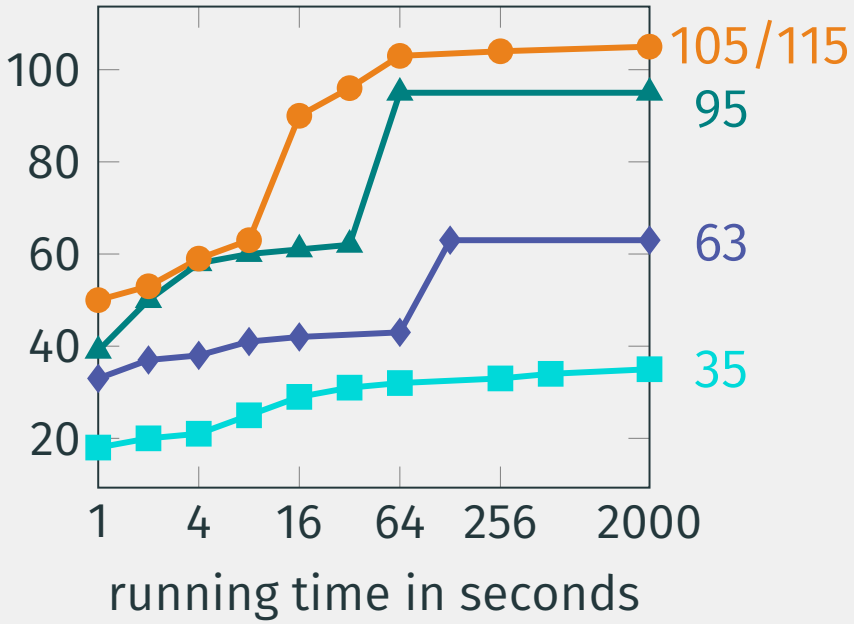
6690 places
11934 trans.
21 secs.

754 places
27370 trans.
3 secs.

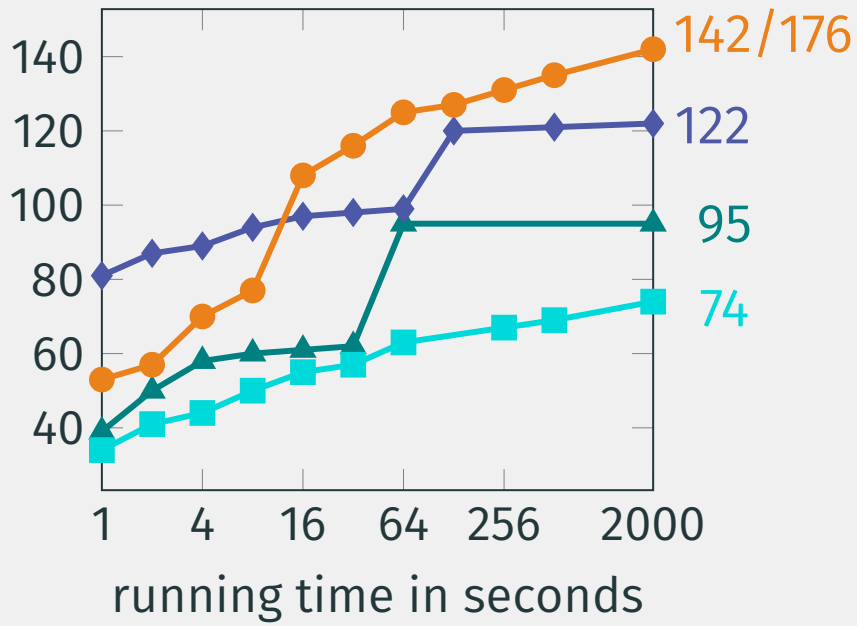
QCover Petrinizer BFC Mist

Benchmarks

Instances proven safe



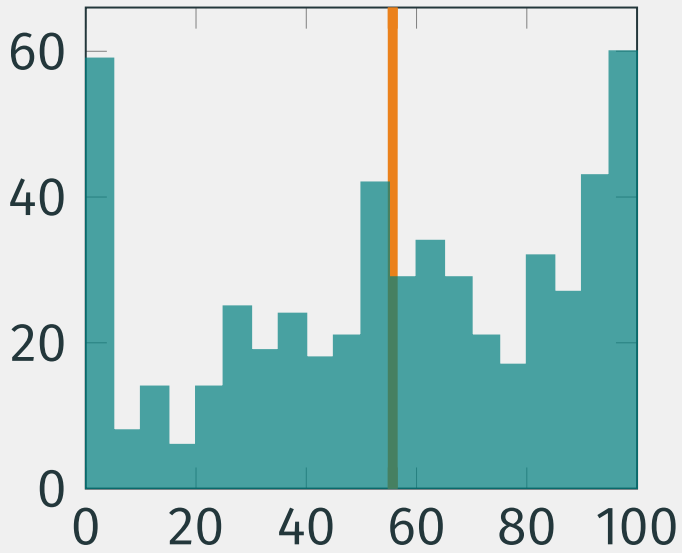
Instances proven safe or unsafe

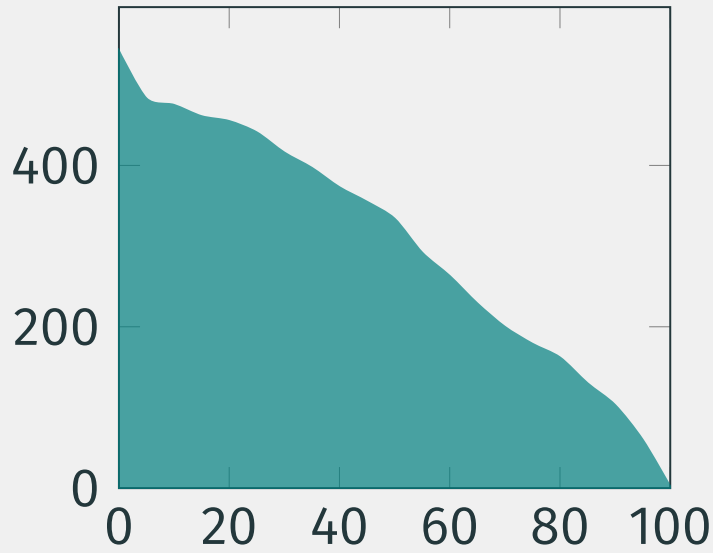


QCover Petrinizer BFC Mist

Benchmarks

Markings pruning efficiency across all iterations





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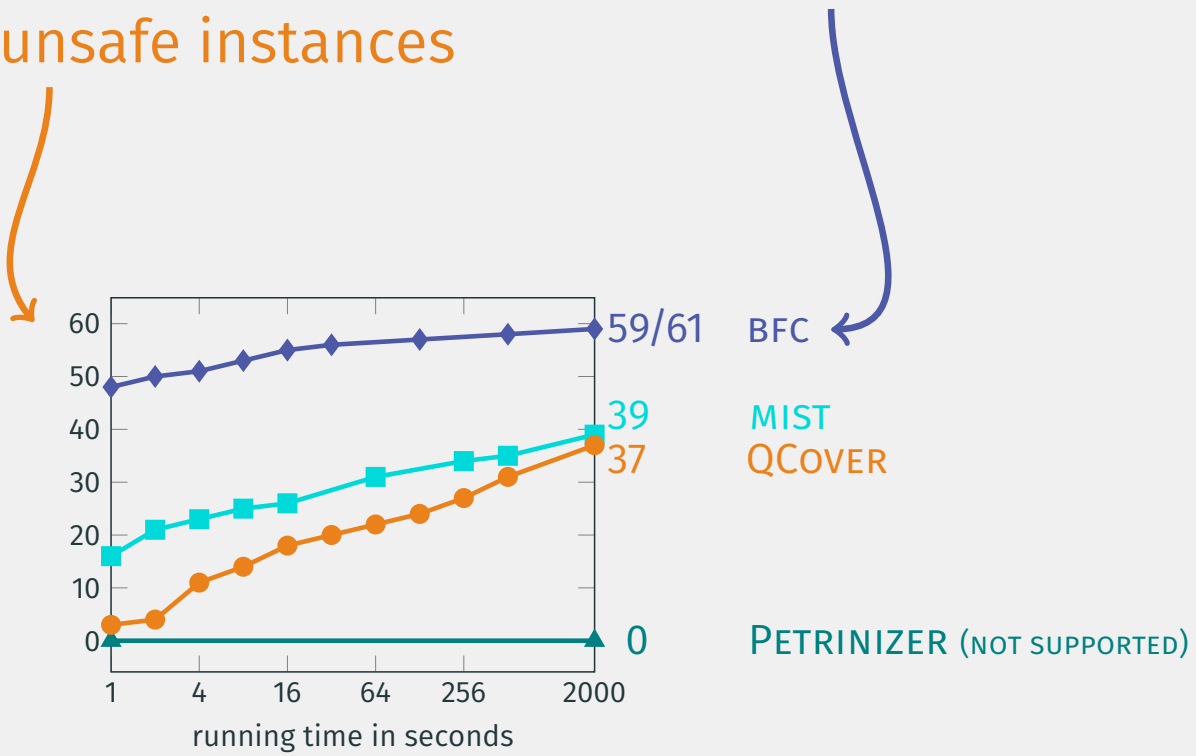

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- Use more efficient data structures, *e.g.* sharing trees

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- Use more efficient data structures, *e.g.* sharing trees

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- Extend to Petri nets with transfer/reset arcs

Thank you!
Vielen Dank!

Additional benchmarks

- 105/115 proved safe
- 101/115 proved safe with all constraints
- 83/115 proved safe with state equation only

- 142/176 verified with $\varphi \in \mathrm{FO}(\mathbb{Q}_{\geq 0}, +, <)$ pruning
- 132/176 verified with "polynomial time" algorithm pruning