Weighted Unranked Tree Automata over Valuation Monoids

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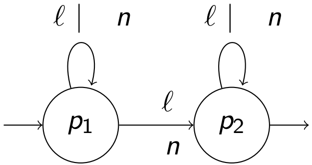
(joint work with Manfred Droste, Heiko Vogler)

Universität Leipzig

15.01.2016

Weighted word automata

- alphabet $\Sigma = \{n, \ell\}$, word $w = n\ell\ell$
- automaton $\mathcal{A}$:



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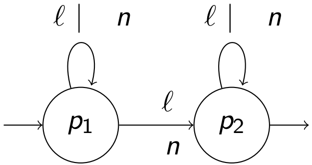

- successful runs on w:
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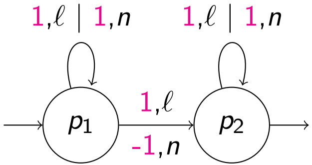

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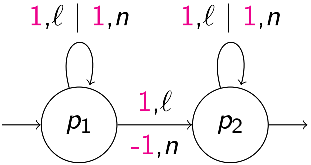

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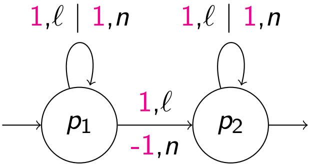

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$$\|\mathcal{A}\|(w) = \sum_{\text{succ. run } r \text{ on } w} \mathrm{wt}(r) = -1 + 1 + 1$$

Weight structure: semirings

Example

$(\mathbb{Z}, +, \cdot, 0, 1)$

Weight structure: semirings

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$(\mathbb{Z}, +, \cdot, 0, 1)$, $(\mathbb{R} \cup \{-\infty\}, \max, +, -\infty, 0)$, $(\{0,1\}, \vee, \wedge, 0, 1)$, ...

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$\mathbb{S} = (S, +, \cdot, \mathbb{0}, \mathbb{1})$

- $(K, +, \mathbb{0})$ and $(K, \cdot, \mathbb{1})$ monoids, i.e.:
  - $+, \cdot$ associative
  - $\forall x \in \mathbb{S}$: $\mathbb{0} + x = x + \mathbb{0} = x$, $\mathbb{1} \cdot x = x \cdot \mathbb{1} = \mathbb{1}$
- $\mathbb{0}$ multiplicative zero
- $\cdot$ distributive over $+$

Droste, Vogler (2009):

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Theorem

$\mathbb{K}$ *semiring,* S *tree series over* $\mathbb{K}$, *alphabet* Σ.

1. $S = [\![\varphi]\!]$ *for some syntactically restricted wMSO sentence* φ $\Rightarrow$ S *recognizable*
2. $S = \|\mathcal{M}\|$ *for some s-wuta* $\mathcal{M}$ $\Rightarrow$ S *wMSO definable*
3. $\mathbb{K}$ *commutative:* $S = [\![\varphi]\!]$ *for some syntactically restricted wMSO sentence* φ $\Leftrightarrow$ S *recognizable*

weighted unranked tree automata $\neq$ restricted weighted MSO weighted unranked tree automata $\neq$ restricted weighted MSO

open problem:

… ension of the syntactically …

definable and expressively equivalent to the class of wta.

2. Find a subclass of wta which is expressively equivalent to the syntactically restricted MSO(S, Σ)-logic.

3. Are there crucially different results if we replace our depth-first left-to-right order on trees

- here: tree valuation monoids (Droste, G., Märker, Meinecke 2011) as weight structure
  - based on ideas of Chatterjee, Doyen, Henzinger (2008)
  - generalized to (word) valuation monoids by Droste, Meinecke (2010)
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- define weighted unranked tree automata / weighted MSO logic over tree valuation monoids

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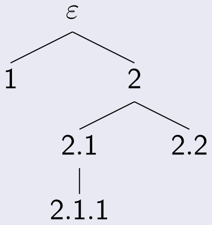
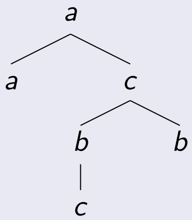
- study support of weighted unranked tree automata

Unranked Trees

tree domain



finite tree over unranked alphabet Σ



- U_Σ: set of all trees over set Σ

(Product) Tree Valuation Monoid

$\mathbb{D} = (D, +, \mathsf{Val}, \diamond, \mathbb{0}, \mathbb{1})$:

- commutative monoid $(D, +, \mathbb{0})$
- $\mathsf{Val}\colon U_D \to D$ with $\forall t \in T_D$:
  - $\mathsf{pos}(t) = \{\varepsilon\} \quad \to \quad \mathsf{Val}(t) = t(\varepsilon)$
  - $\mathbb{0} \in \mathsf{im}(t) \quad \to \quad \mathsf{Val}(t) = \mathbb{0}$
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- $\diamond\colon D^2 \to D$; $\forall d \in D : \mathbb{0} \diamond d = d \diamond \mathbb{0} = \mathbb{0} \ \wedge \ \mathbb{1} \diamond d = d \diamond \mathbb{1} = d$

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Example (Average)

$\mathbb{Q}_{\mathsf{avg}} = (\mathbb{Q} \cup \{-\infty, \infty\}, \max, \mathsf{avg}, \min, -\infty, \infty)$

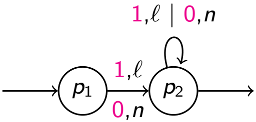
$$t \in U_{\mathbb{Q}_{\mathsf{avg}}} \qquad \curvearrowright \qquad \mathsf{avg}(t) = \frac{\sum_{u \in \mathsf{pos}(t)} t(u)}{|\mathsf{pos}(t)|}$$

Weighted Unranked Tree Automata (wuta)

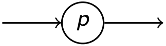
Example over $\mathbb{Q}_{\text{avg}}$ and $\Sigma = \{a\}$:

- $\mathcal{M}_{ex} = (Q, \mathcal{A}, \gamma)$ with $Q = \{n, \ell\}$, $\gamma(n) = 0$, $\gamma(\ell) = 1$,

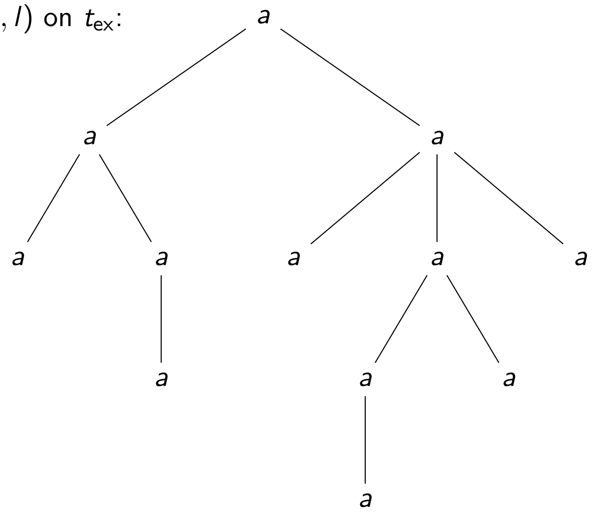
$\mathcal{A}_{n,a}$:



$\mathcal{A}_{\ell,a}$:



extended run (q, s, l) on t_{ex}:



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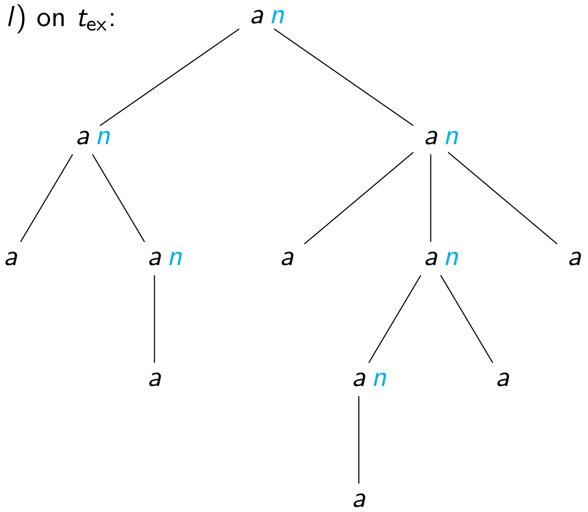

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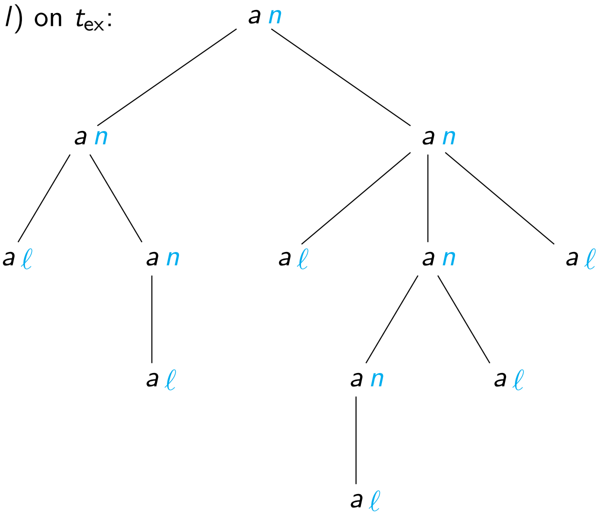

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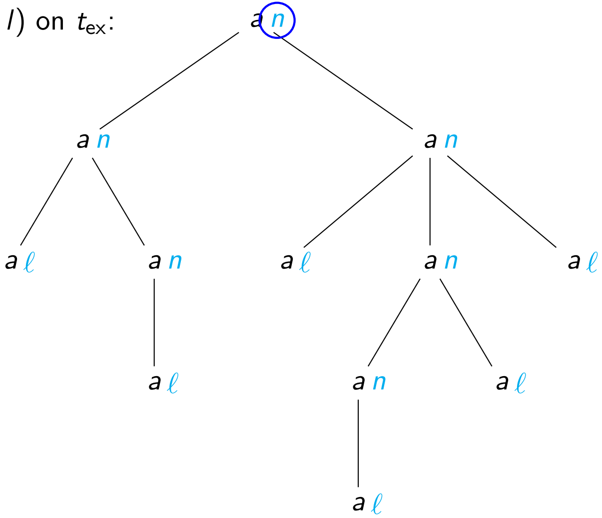

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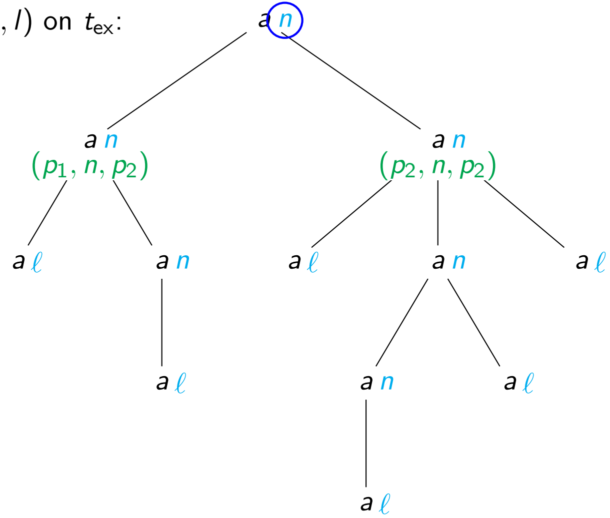

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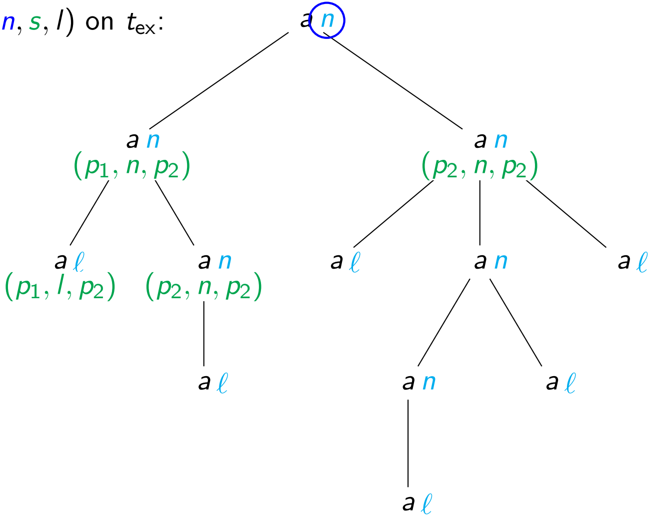

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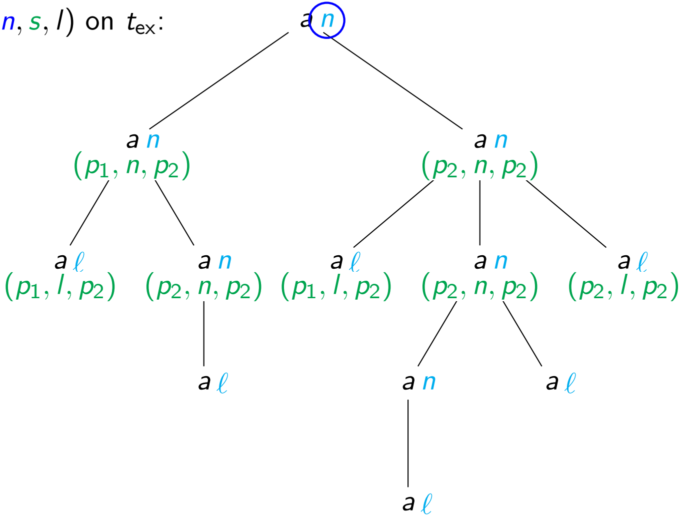

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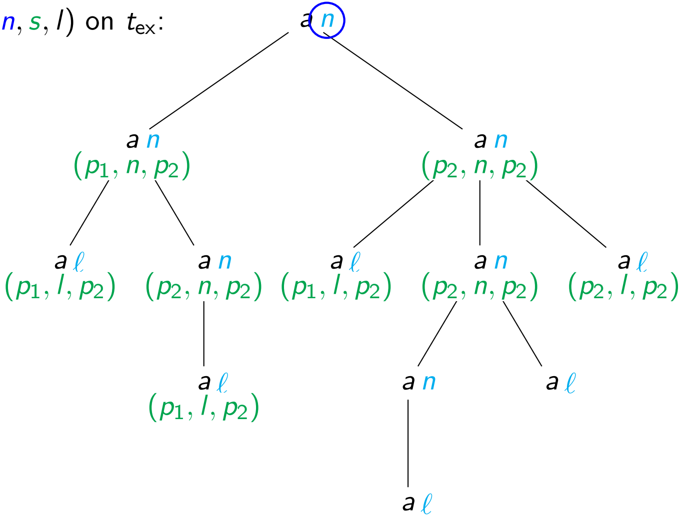

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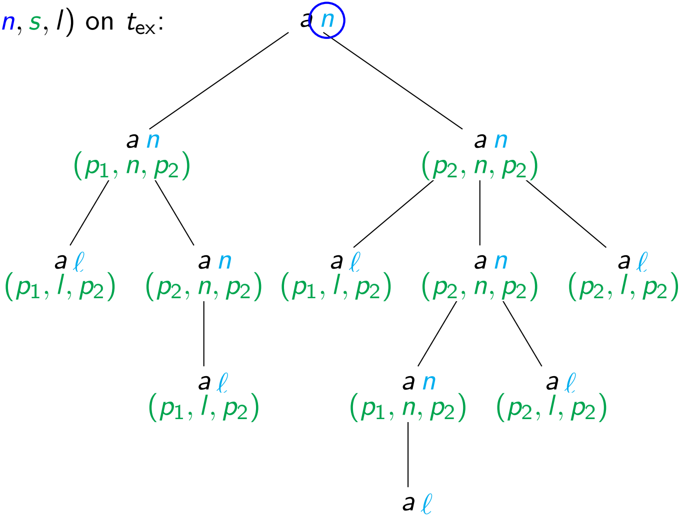

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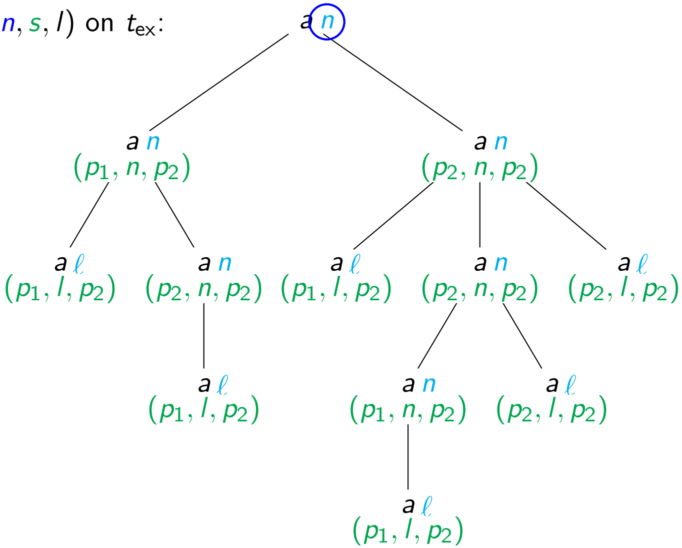

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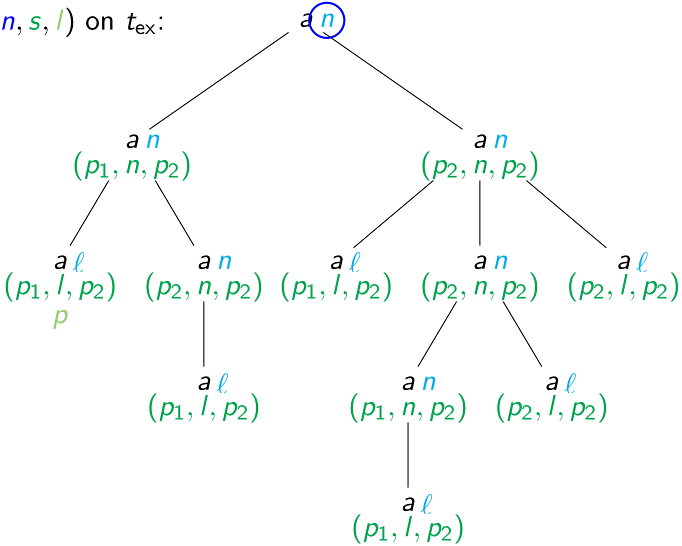

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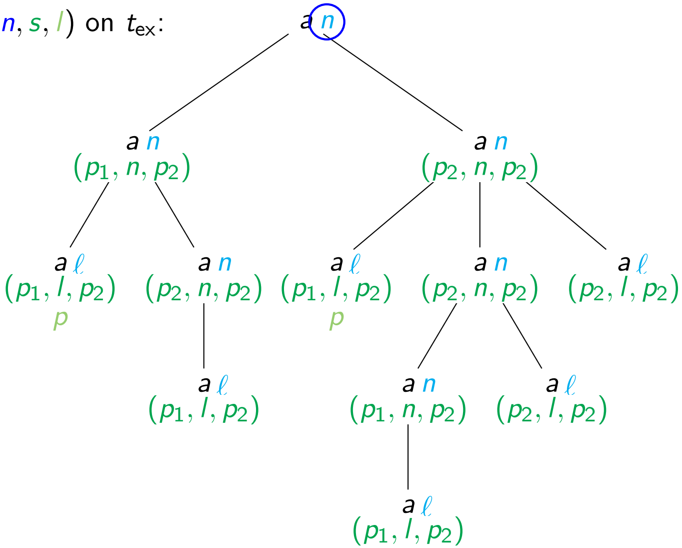

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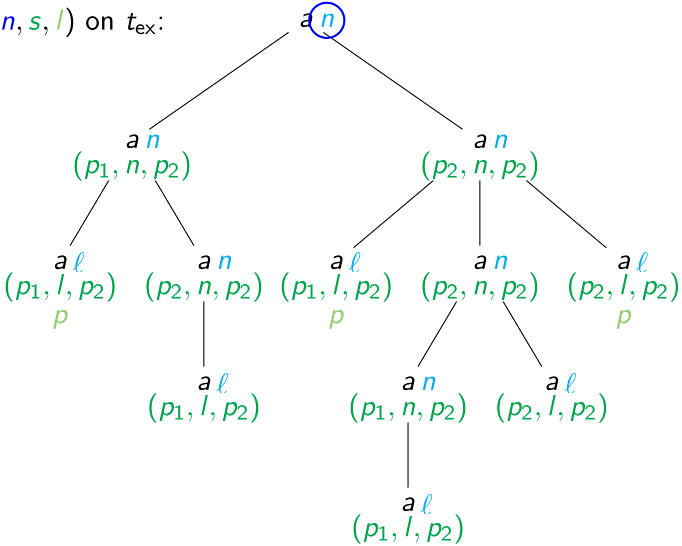

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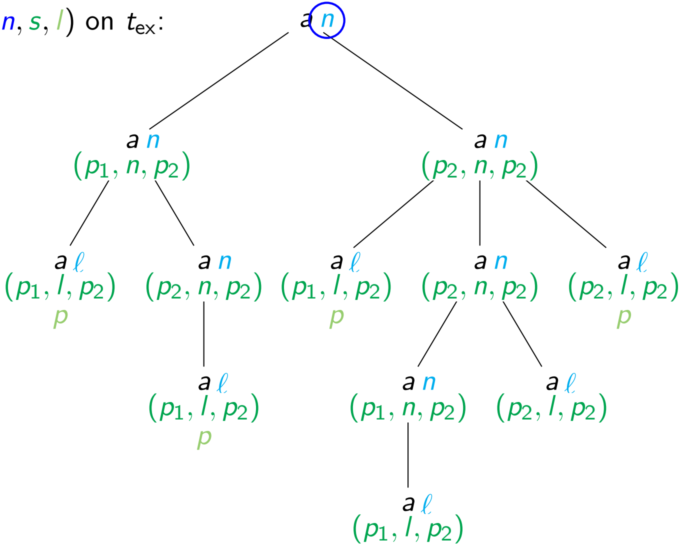

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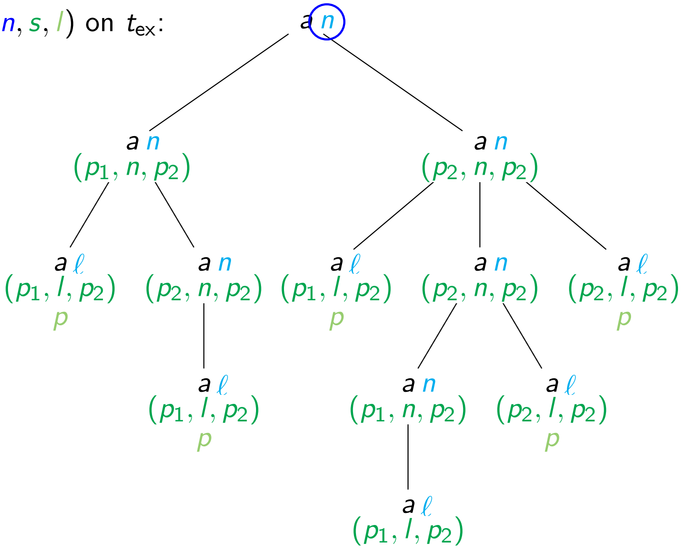

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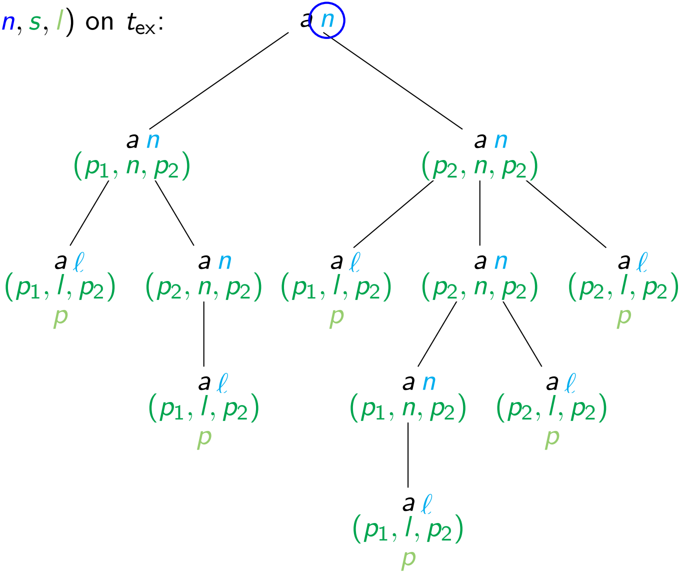

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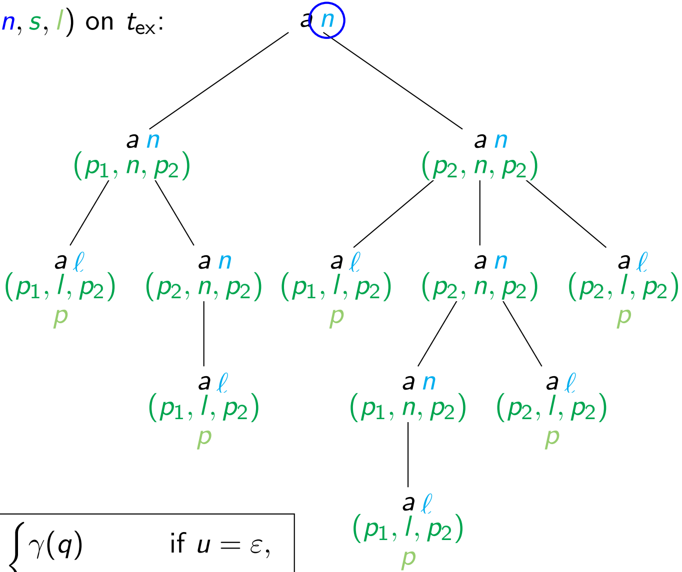

$$\text{wt}(q, s, l)_u = \begin{cases} \gamma(q) & \text{if } u = \varepsilon, \\ \mu_{\mathcal{A}}(s(u)) & \text{otherwise} \end{cases}$$

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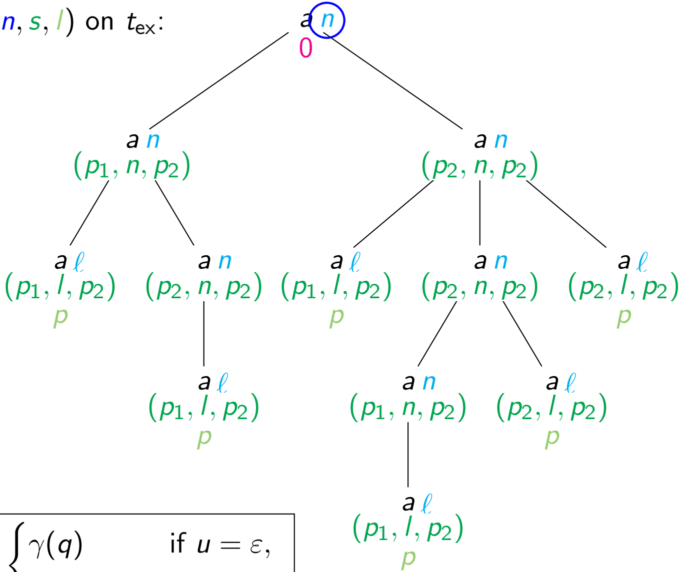

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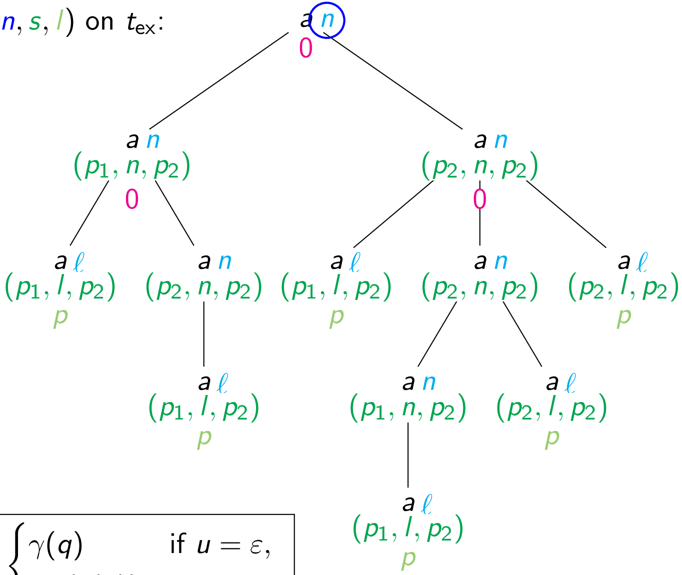

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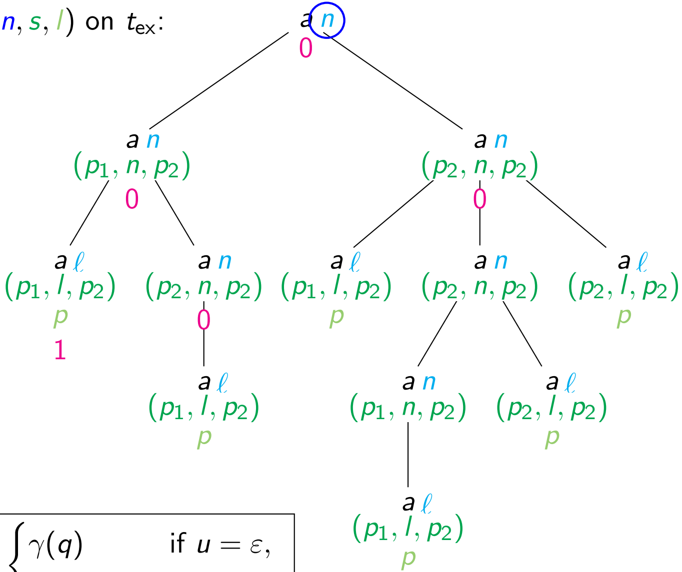

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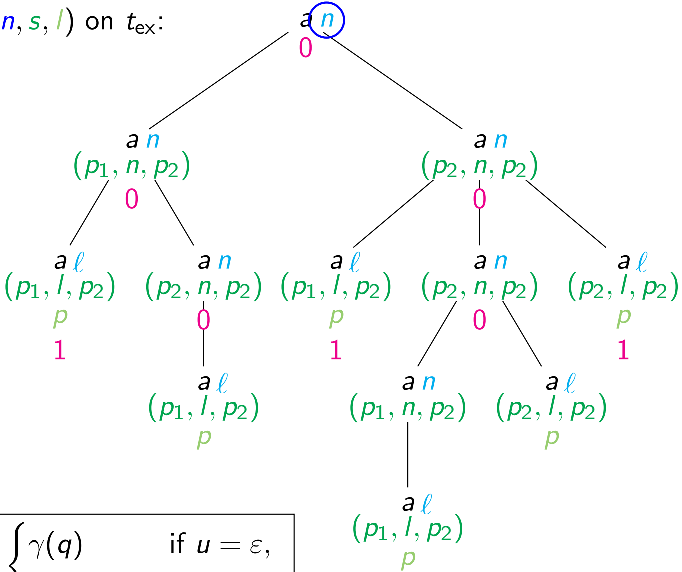

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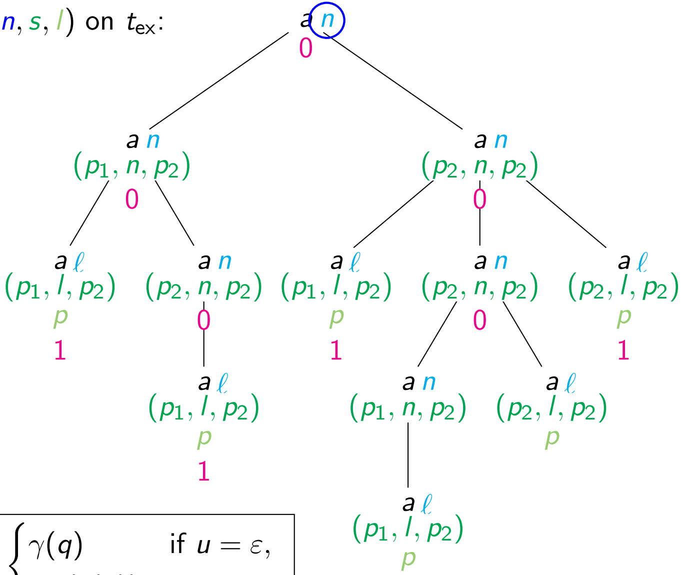

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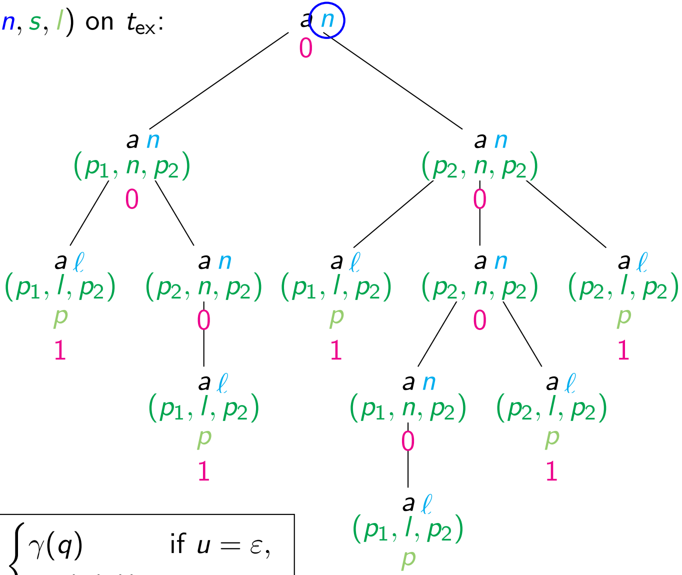

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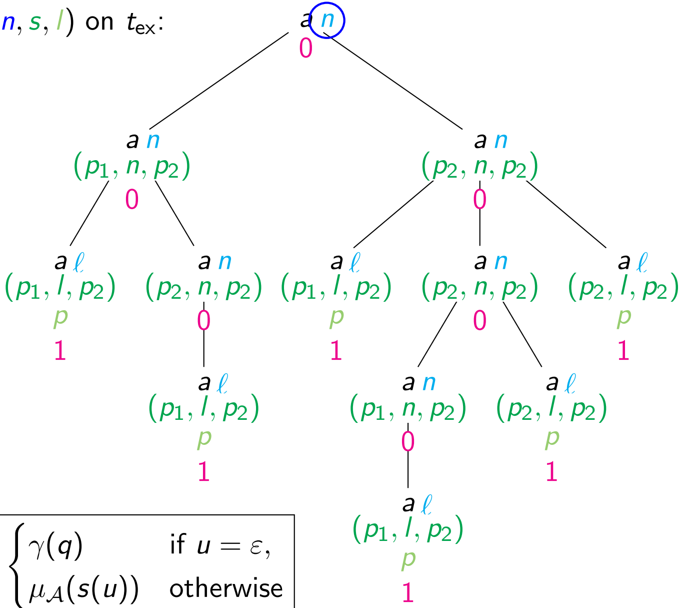

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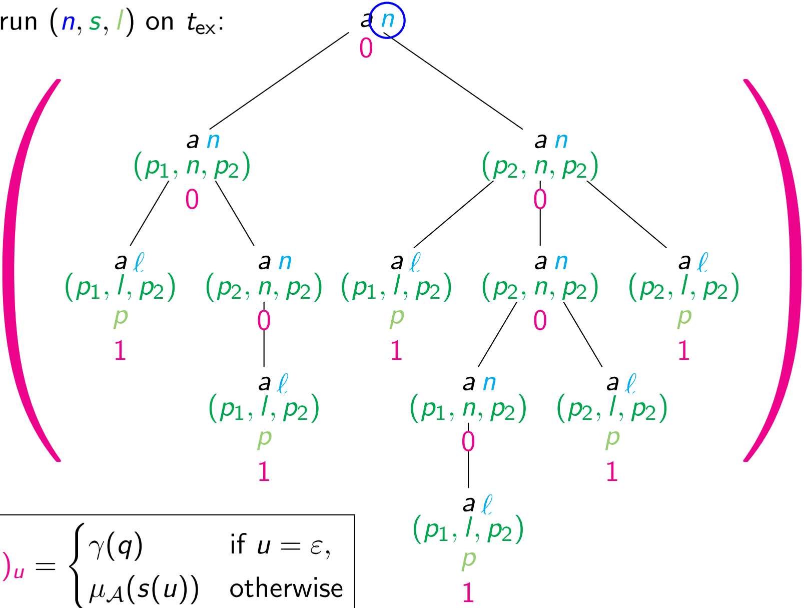
avg



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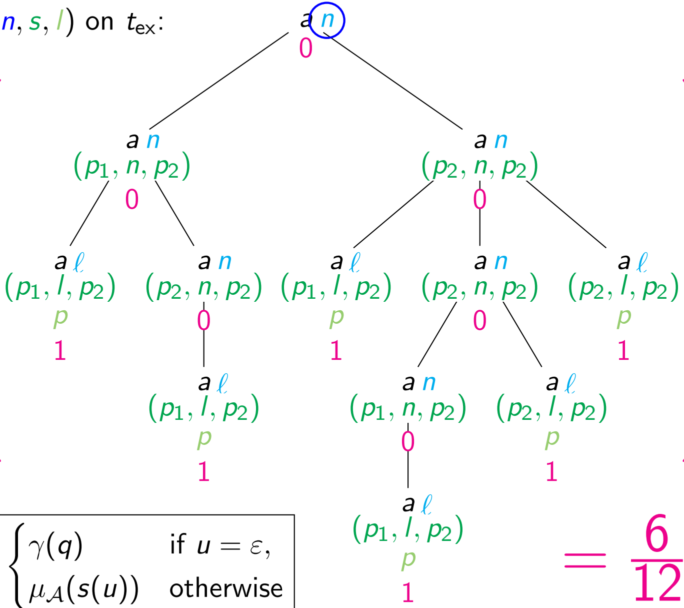

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  - has initial, final weights

Lemma

$\mathbb{K}$ *commutative semiring.*

$\{$*s-wuta over* $\mathbb{K}\} = \{$*s-wuta with initial, final weights in* $\{\mathbb{0}, \mathbb{1}\}\}$

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  - multiplies weights of a run

$\rightarrow$ tv-monoid $\mathbb{K}_{\text{tv}} = (K, +, \text{Val}, \mathbb{0})$ with $\text{Val}(t) = \prod_{u \in \text{pos}(t)} t(u)$

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$\{$*s-wuta over* $\mathbb{K}\} = \{$*s-wuta with initial, final weights in* $\{\mathbb{0}, \mathbb{1}\}\}$

  - multiplies weights of a run

→ tv-monoid $\mathbb{K}_{\text{tv}} = (K, +, \text{Val}, \mathbb{0})$ with $\text{Val}(t) = \prod_{u \in \text{pos}(t)} t(u)$

- $\|\mathcal{M}\|_{\mathbb{K}} = \|\mathcal{M}\|_{\mathbb{K}_{\text{tv}}}$

Remark

Wuta over tv-monoids subsume wuta over commutative semirings

- wuta $\mathcal{M} = (Q, \mathcal{A}, \gamma)$ over commutative semiring $\mathbb{K} = (K, +, \circ, \mathbb{0}, \mathbb{1})$:
  - has initial, final weights

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Remark

wuta restricted to semirings are strict subclass of s-wuta

Weighted MSO Logic for Unranked Trees over tv-Monoids

Definition (Syntax of wMSO over $\mathbb{D}$)

- boolean formulas:
  $\beta ::= \mathrm{label}_a(x) \mid \mathrm{desc}(x,y) \mid x \leq y \mid x \sqsubseteq y \mid x \in X \mid \neg\beta \mid \beta \wedge \beta \mid \forall x \beta \mid \forall X \beta$
- weighted MSO formulas:
  $\varphi ::= d \mid \beta \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid \exists x \varphi \mid \forall x \varphi \mid \exists X \varphi$

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Semantics of φ: maps every (tree t, assignment σ) to a $d \in \mathbb{D}$

- defined inductively:

true	false	d	$\vee$	$\exists$	$\forall$	$\wedge$
$\mathbb{1}$	$\mathbb{0}$	d	$+$	$\sum$	Val	$\diamond$

example wMSO formula over $\mathbb{Q}_{\text{avg}}$ and $\Sigma = \{a\}$

- wMSO formula $\varphi_{ex}(x)$ with $[\![\varphi_{ex}(x)]\!](t, \sigma) = \begin{cases} 1 & \text{if } \sigma(x) \text{ is leaf} \\ 0 & \text{otherwise} \end{cases}$

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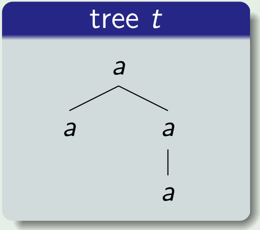

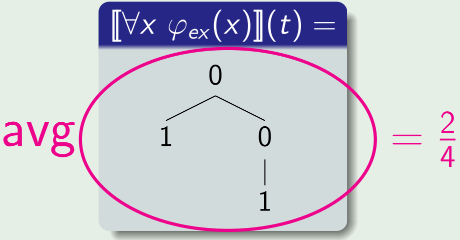

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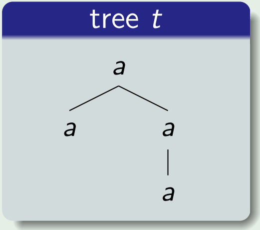

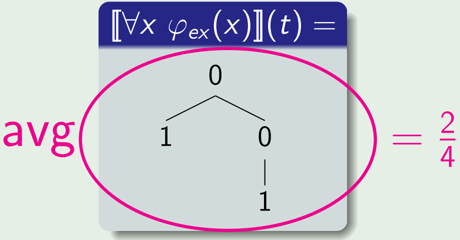

$$[\![\forall x\ \varphi_{ex}(x)]\!](t) = \|\mathcal{M}_{ex}\|(t)$$

Weighted Tree Automata and Weighted MSO Logic

words: restriction of $\forall$-quantifier and conjunction
$\curvearrowright$ restrictions on the underlying ptv-monoid

conditionally commutative tv-semirings or cctv-semirings

- $\diamond$ associative
- $\diamond$ is distributive over $+$,Val
- $\forall\ t, t' \in U_{\mathbb{D}}$:

$$\mathrm{pos}(t) = \mathrm{pos}(t') \ \wedge \ \mathrm{im}(t), \mathrm{im}(t') \text{ commute}$$

$$\to \mathrm{Val}(t) \diamond \mathrm{Val}(t') = \mathrm{Val}((t_u \diamond t'_u)_{u \in \mathrm{pos}(t)})$$

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Example

$\mathbb{Q}_{\mathrm{avg}+} = (\mathbb{Q} \cup \{-\infty, \infty\}, \max, \mathrm{avg}, \min, -\infty, \infty)$ is a cctv-semiring

Theorem

$\mathbb{D}$ *cctv-semiring,* $S\colon U_\Sigma \to \mathbb{D}$ *tree series. TFAE:*

1. $S = \|\mathcal{M}\|$ *for a wuta* $\mathcal{M}$
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Satisfiability Problem

$\exists$ tree t with $[\![\varphi]\!](t) \neq 0$ for given formula φ?

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$$\Leftrightarrow$$

Problem

Is $supp(\mathcal{A}) \neq \emptyset$ for a given wuta $\mathcal{A}$?

Definition (Support)

$\mathcal{A}$ weighted automaton.

$$supp(\mathcal{A}) = \{t \in U_\Sigma \mid \|\mathcal{A}\|(t) \neq 0\}$$

- since the 80's: support of probabilistic automata has been studied e.g. in books by Paz, Bukharaev

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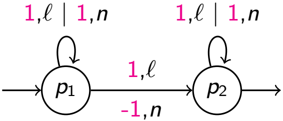

in semiring $(\mathbb{Z}, +, \cdot, 0, 1)$

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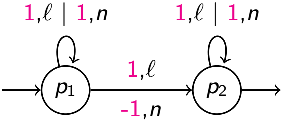

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- effective construction of support automata for weighted automata over commutative, zero-sum free semirings (Kirsten)

Construction of Support Automaton M_s for wuta $\mathcal{M}$

Theorem

$\mathcal{M}$ wuta over zero-sum free, commutative semiring.
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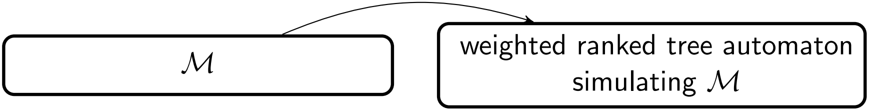

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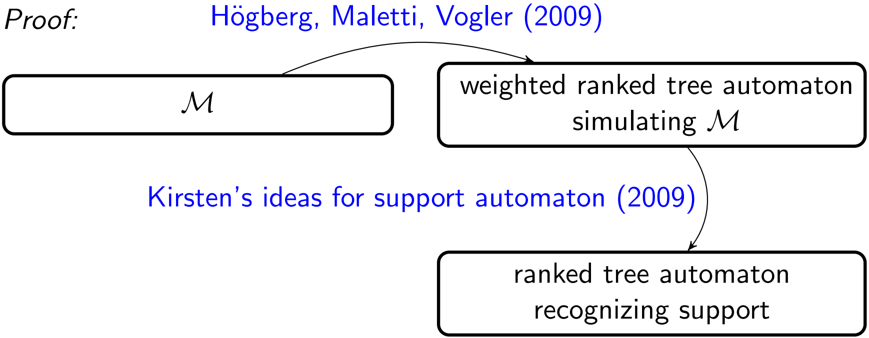

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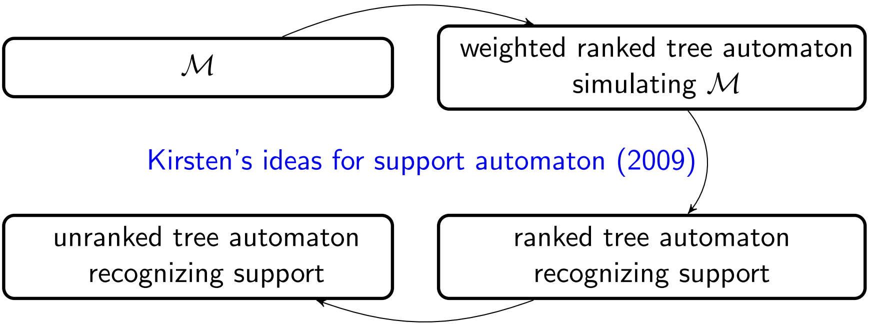

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Lemma (Droste, Heusel)

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Corollary

Non-emptyness problem decidable
for nested weighted automata over zero-sum free, commutative semiring

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$\rightarrow$ generalizes of respective results for:

- s-wuta and wMSO over commutative semirings (Droste, Vogler)
- wta and wMSO for ranked trees over tv-monoids (Droste et al.)

- found model for unranked trees for average, discounting, semirings
- four further equivalent results for other versions of the logic

Theorem

$\mathcal{M}$ wuta over zero-sum free, commutative semiring.
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$\rightarrow$ partially solves satisfiability problem of Bollig et al.