Equivalence of Deterministic Tree-to-String Transducers Is Decidable

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Joint work with:

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Overview

Part 1: The General Setting

Part 2: Tree-to-Int Transducers

Part 3: Affine Spaces

Part 4: Polynomial Ideals

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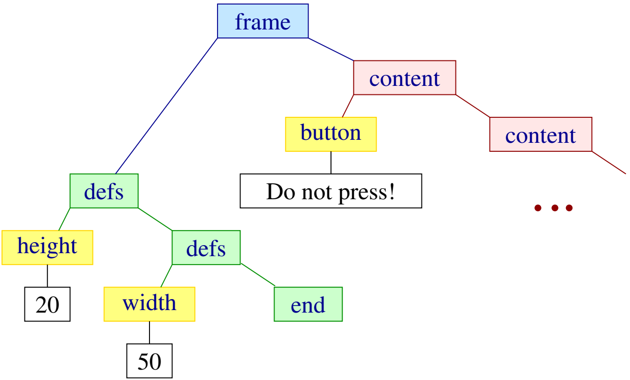
Tree-to-String Translation

Input



Tree-to-String Translation

Output

```
<frame height=20 width=50>
    <button>Do not press!</button>
    ...
</frame>
```

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Realized by:

$$
\begin{array}{lcl}
q(\text{frame}(x_1, x_2)) & \rightarrow & \texttt{<frame } q_1(x_1)q(x_2)\texttt{</frame>} \\
q_1(\text{end}) & \rightarrow & \texttt{>} \\
q_1(\text{defs}(x_1, x_2)) & \rightarrow & q_2(x_1)q_1(x_2) \\
q_2(\text{height}(x_1)) & \rightarrow & \texttt{height = } q_3(x_1) \\
 & \ldots &
\end{array}
$$

Tree-to-String Translation (cont.)

Output

```
<frame height=20 width=50>
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Or realized by:

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q_1(\text{end}) & \to & \epsilon \\
q_1(\text{defs}(x_1, x_2)) & \to & q_2(x_1)q_1(x_2) \\
q_2(\text{height}(x_1)) & \to & \texttt{height = } q_3(x_1) \\
 & \dots &
\end{array}
$$

Question

Are these two translations equivalent ?

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Unstructured output can be generated in surprisingly different ways ...

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\begin{array}{lcl}
q(f(x_1, x_2, x_3)) & \rightarrow & q(x_3)aq_1(x_2)bq(x_2) \\
q_1(f(x_1, x_2, x_3)) & \rightarrow & q_1(x_3)q_1(x_2)q_1(x_2)ba \\
q_1(e) & \rightarrow & ba \\
q(e) & \rightarrow & ab
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\end{array}
$$

versus

$$
\begin{array}{lcl}
q'(f(x_1, x_2, x_3)) & \to & ab\, q'(x_2)q'(x_2)q'(x_3) \\
q'(e) & \to & ab
\end{array}
$$

Notation

yDT — det. topdown tree-to-string transducer

$$q(f(x_1,\ldots,x_k)) \;\to\; T$$

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yDT — det. topdown tree-to-string transducer

$$q(f(x_1, \ldots, x_k)) \;\rightarrow\; T$$

yMDT — det. topdown macro tree-to-string transducer

$$q(f(x_1, \ldots, x_k), y_1, \ldots, y_l) \;\rightarrow\; T$$

// initially $y_i = \epsilon$

Related Work

problem statement	Engelfriet, 1980
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affine program invariants	MMO, S., 2004
polynomial program invariants	Letichevsky, Lvov, 1996 MMO, S., 2004

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From Arbitrary to Unary Output

Unary Output

$$q(f(x_1, x_2)) \ \rightarrow \ dd\, q_1(x_1)\, d\, q_1(x_1)\, q_2(x_2)$$

Succinct representation

$$q(f(x_1, x_2)) \ \rightarrow \ 3 + 2 \cdot q_1(x_1) + q_2(x_2)$$

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Encoding

letters $a, b, c, \ldots$	$\hat{=}$	digits $1, \ldots, h-1$
string $aabc$	$\hat{=}$	$1 + h \cdot (1 + h \cdot (2 + h \cdot 3))$

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Wanted

Transformation yDT $\Longrightarrow$ tree-to-int transducer

Transformation

The yDT rule

$$q(f(x_1, \ldots, x_k)) \;\to\; T$$

is simulated with the yMDT rule:

$$q(f(x_1, \ldots, x_k), y) \;\to\; [T]$$

where

$$\begin{array}{lcl} [\epsilon] & = & y \\ [a\, T] & = & a + h \cdot [T] \\ [q(x_i)\; T] & = & q(x_i, [T]) \end{array}$$

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Unary Transducers

Simplification

- A single transducer with states $Q = \{1, \ldots, n\}$.
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  $\mathcal{L}(B) = \text{dom}(p_0)$ for initial state p_0.

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  $\mathcal{L}(B) = \text{dom}(p_0)$ for initial state p_0.

Question

For states q, q' of the transducer, does it hold that

$$[\![q]\!](t) = [\![q']\!](t) \qquad (t \in \mathcal{L}(B))$$

Unary Transducers

Idea

- The semantics of a tree t can be seen as

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- For state p of B, let $V_p = \{[\![t]\!] \mid t \in \text{dom}(p)\}$.
- Consider $H(\mathbf{v}) = \mathbf{v}_q - \mathbf{v}_{q'}$.
- The following statements are equivalent:

  1. q, q' agree on inputs from $\mathcal{L}(B)$
  2. $H(\mathbf{v}) = 0 \qquad (\mathbf{v} \in V_{p_0})$

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  2. $H(\mathbf{v}) = 0 \qquad (\mathbf{v} \in V_{p_0})$
  3. $H(\mathbf{v}) = 0 \qquad (\mathbf{v} \in \text{Aff}(V_{p_0}))$

     // affine closure

Computing Affine Closures

Define

$$\begin{array}{rcl} [\![f]\!](\mathbf{x}_1, \ldots, \mathbf{x}_k) & = & ([\![T_1]\!](\mathbf{x}), \ldots, [\![T_n]\!](\mathbf{x})) \qquad \text{where} \\ & & q(f(x_1, \ldots, x_k)) \;\to\; T_q \end{array}$$

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Observation

$[\![f]\!] : \mathbb{Q}^n \times \ldots \times \mathbb{Q}^n \to \mathbb{Q}^n$ is affine.

Computing Affine Closures (cont.)

Consequence

$V'_p = \text{Aff}(V_p)$ is the least solution of:

$$V'_p \supseteq [\![f]\!](V'_{p_1}, \ldots, V'_{p_k})$$

$((p, f) \mapsto p_1 \ldots p_k$ transition of B) over the complete lattice of affine sub-spaces of $\mathbb{Q}^n$!

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Theorem

- Equivalence of total unary yDTs relative to some B is decidable in polynomial time.

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Theorem

- Equivalence of total unary yDTs relative to some B is decidable in polynomial time.
- In-Equivalence of linear yDTs is decidable in randomized polynomial time.

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Unary Transducers with Parameters

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\begin{array}{lcl}
q(f(x_1, x_2), y) & \rightarrow & q_1(x_1, q_1(x_2, 1)) \\
q_1(a(x_1), y) & \rightarrow & y + q_1(x_1, y) \\
q_1(e, y) & \rightarrow & 0 \\
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The semantics of a tree t is now a vector

$$[\![t]\!] : (\mathbb{Q} {\to} \mathbb{Q})^n$$

of affine functions in the parameters.

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of affine functions in the parameters.

$\Longrightarrow$

It can be represented by a matrix $([\![t]\!]_{qi})$.

The Semantics of Constructors

$$[\![f]\!] : (\mathbb{Q}^{n\times(l+1)} \times \ldots \times \mathbb{Q}^{n\times(l+1)}) \to \mathbb{Q}^{n\times(l+1)}$$

thus is of the form:

$$([\![f]\!](\mathbf{x}))_{qj} = \text{polynomial in the } \mathbf{x}_{iq'j'}$$

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thus is of the form:

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$\Longrightarrow$ The affine closure trick fails :-(

In the Example

$$\begin{array}{lcl} q(f(x_1, x_2), y) & \rightarrow & q_1(x_1, q_1(x_2, 1)) \\ q_1(a(x_1), y) & \rightarrow & y + q_1(x_1, y) \\ q_1(e, y) & \rightarrow & 0 \\ q'(f(x_1, x_2), y) & \rightarrow & q_1(x_2, q_1(x_1, 1)) \end{array}$$

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$$\begin{array}{lcl} ([\![f]\!](\mathbf{x}))_{q0} & = & \mathbf{x}_{1q_1 0} + \mathbf{x}_{1q_1 1} \cdot (\mathbf{x}_{2q_1 0} + \mathbf{x}_{2q_1 1} \cdot 1) \\ ([\![f]\!](\mathbf{x}))_{q'0} & = & \mathbf{x}_{2q_1 0} + \mathbf{x}_{2q_1 1} \cdot (\mathbf{x}_{1q_1 0} + \mathbf{x}_{1q_1 1} \cdot 1) \end{array}$$

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$$([\![f]\!](\mathbf{x}))_{q1} \quad = \quad 0 \quad = \quad ([\![f]\!](\mathbf{x}))_{q'1}$$

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Polynomial Invariants

Idea

- polynomial equality:

$$\mathbf{z}_{q1} \cdot \mathbf{z}_{q'1} \cdot \mathbf{z}_{q'0} - 2 \cdot \mathbf{z}_{q0} + 3 \doteq 0$$

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- $q_1 \doteq 0 \wedge \ldots \wedge q_r \doteq 0$ invariant at p iff

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can be described by polynomial ideals ...

Polynomial Ideals: A Primer

R ring. $I \subseteq R$ ideal, if

- $a + b \in I$ whenever $a, b \in I$;
- $r \cdot a \in I$ whenever $a \in I$ and $r \in R$.

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Hilbert (1890):

Every ideal of $\mathbb{Q}[\mathbf{z}]$ is finitely generated !

Consequences

Vanishing Ideal:

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- Invariants can be represented by polynomial ideals!
- Finite conjunctions suffice!
- There are effective algorithms for
  - membership
  - inclusion
  - equality

Inductive Invariants

Notation: $q_{qj}^{(f)} = (\llbracket f \rrbracket(\mathbf{x}))_{qj}$

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$p \mapsto I_p$ is inductive if for every transition $(p, f) \mapsto p_1 \ldots p_k$ of the automaton B,

$$I_p \subseteq \{q \in \mathbb{Q}[\mathbf{z}] \mid q[q^{(f)}/\mathbf{z}] \in \langle I_{p_1}(\mathbf{x}_1)\rangle_{\mathbb{Q}[\mathbf{x}]} \oplus \ldots \oplus \langle I_{p_k}(\mathbf{x}_k)\rangle_{\mathbb{Q}[\mathbf{x}]}\}$$

holds.

Main Result

Theorem

- Assume $p \mapsto I_p$ is inductive. Then for every $q \in I_p$,

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Corollary

Let $H(\mathbf{z}) = \mathbf{z}_{q0} - \mathbf{z}_{q'0}$. Then q, q' are equivalent (relative to $\mathcal{L}(B)$) iff

$$H \in \overline{I}_{p_0}$$

Discussion

- Inductive $p \mapsto I_p$ with $H \in I_{p_0}$ proves that H holds.

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- If H holds, it can be proven (somehow).
- $p \mapsto \bar{I}_p$ is a greatest fixpoint.

  Greatest fixpoint iteration may not terminate ...

- All inductive invariants, though, can be recursively enumerated!
- All potential counter examples can also be enumerated!!

Wrap-up

Theorem

- Equivalence of unary yMDTs is decidable.

Wrap-up

Theorem

- Equivalence of unary yMDTs is decidable.
- Equvalence of general yDTs is decidable.

Summary

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yDT		affine
nsn yMDT	linear yDT	affine
yMDT	yDT	polynomial

Thank you!