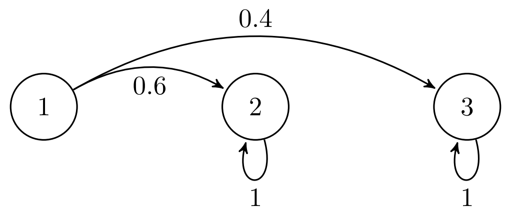
Quantitative Verification Session 6

November 30, 2017

Stationary Distributions

Exercise 1. Consider the following Markov Chain. How many stationary distributions does it have?



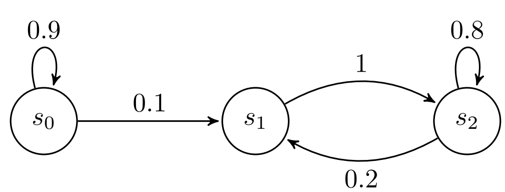
Exercise 2. [1, ch. 8] Imagine jobs arriving at a server with an unbounded queue. The server works on a job at the head of the queue and when finished, moves on to the next job. The server never drops a job, but just allows them to queue up. Suppose at every time step, with probability p = 1/50 one job arrives, and independently, with probability q = 1/30 one job departs. Note that during a time step, we might have both an arrival and a transmission, or neither. Draw the Markov chain modeling this server. What is the average number of jobs in the system?

Exercise 3. [1, ch. 8] We define a threshold queue with parameter T as follows: When the number of jobs is $<$ T , then the number of jobs decreases by 1 with probability 0.4 and increases by 1 with probability 0.6 at each time step. However, when the number of jobs increases to $>$ T, then the reverse is true, and the number of jobs increases by 1 with probability 0.4 and decreases by 1 with probability 0.6 at each time step.

- Assume that the limiting probabilities exist. Use the stationary equations to derive the limiting probability distribution as a function of T, for arbitrary threshold T.
- Compute the mean number of jobs, $\mathbf{E}[\mathrm{N}]$, in a threshold queue as a function of T.

Reachability

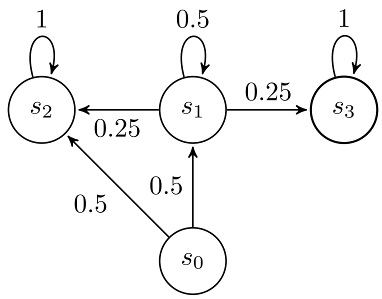
Exercise 4. Use the matrix form of the reachability equation, $x = Ax + b$ to see whether s_0 almost surely reaches s_2 in the following Markov chain



Exercise 5. Let $x(s)$ denote $P_s(\Diamond B)$ where $\Diamond B = \{s_0 s_1 \cdots \mid \exists i : s_i \in B\}$. Then

- For all $s \in B : x(s) = 1$
- For all $s \in S \setminus B : x(s) = \sum_{t \in S \setminus B} P(s,t)x(t) + \sum_{u \in B} P(s,u)$

Use value iteration (step-by-step) on the following Markov chain to compute the probability of s_0 eventually reaching $B = \{s_3\}$.



References

[1] Mor Harchol-Balter, Performance Modeling and Design of Computer Systems.