Exercise 1. There exist a family of stationary distributions which are equal to the initial distribution \( \pi^* = \pi_0 = [0, x, y] \) where \( x + y = 1 \).

Exercise 2. This problem is a small modification of the worked out example in [1, Sec. 8.10].

Exercise 3. This is Exercise 8.6 of [1, ch. 8]. It can be worked out in a similar manner as the previous question. Contact me in case you need any clarifications.

Exercise 4. This is a straightforward application of what you had seen in the lecture slides. The answer works out to 1.

Exercise 5. It is not practical to manually perform VI for this Markov chain. But we shall illustrate the first few steps.

We apply the following equations repeatedly on the initial vector \( v_0 = [0, 0, 0, 1] \) representing the values of states \([s_0, s_1, s_2, s_3]\). Since \( s_3 \) is a target state, its value is fixed at 1. Other values are initialized with 0.

\[
\begin{align*}
v_n(s_0) &= 0.5v_{n-1}(s_1) + 0.5v_{n-1}(s_2) \\
v_n(s_1) &= 0.55v_{n-1}(s_1) + 0.25v_{n-1}(s_2) + 0.25 \\
v_n(s_2) &= v_{n-1}(s_2) \\
v_n(s_3) &= 1
\end{align*}
\]

Step 1: \( v_1 = [0, 0.25, 0, 1] \)

Step 2: \( v_2 = [0.125, 0.375, 0, 1] \)

Step 3: \( v_3 = [0.1875, 0.4375, 0, 1] \)

Step 4: \( v_4 = [0.21875, 0.46875, 0, 1] \)

...  

Step 15: \( v_{15} = [0.249984..., 0.499984..., 0, 1] \)

and so on...

References