Quantitative Verification Session 10

January 18, 2018

CTMCs

Exercise 1. Given two independent random variables $X_1 \sim exp(\lambda_1)$ and $X_2 \sim exp(\lambda_2)$, how is the random variable $Z = \min(X_1, X_2)$ distributed?

Exercise 2. There are two ways of looking at a CTMC which are summarized by this figure. These two views are equivalent. For now, prove that the second implies the first.

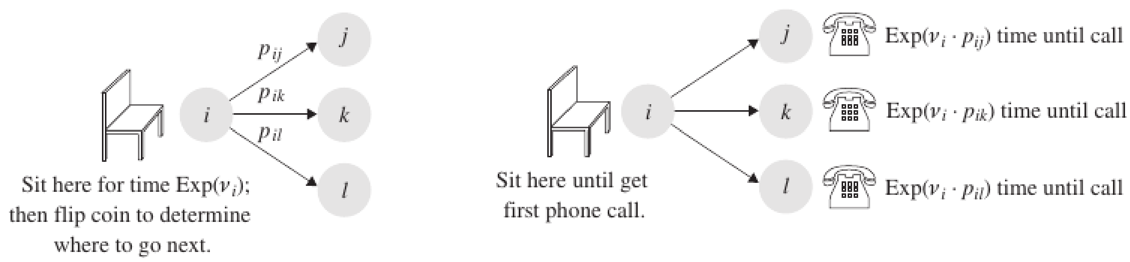

Figure 1: Two views of CTMC (from [1, Ch. 12])

Exercise 3. Prove that adding a self-loop in state i with rate $v_i p_{ii}$ does not change the eventual behaviour at the state. More specifically (1) prove that the time at which the state i is left for good is distributed identically as the time at which state i is left when there is no self-loop; (2) prove that the probability of eventually going to a state $j (\neq i)$ in the case with self-loop is equal to the probability of going to the same state when there was no self-loop.

Exercise 4. With the help of Ex. 2 and Ex. 3, suggest how to convert an arbitrary CTMC to an equivalent CTMC in which at *every state* the sojourn time is distributed according to $exp(\gamma)$.

Exercise 5. Understand the simple BitTorrent like file-sharing protocol of (`peer2peer/peer2peer4_5.sm`) in `prism-examples` [5]. Using Prism, plot the graph (probability vs. time) of the probability that all clients have received all blocks in time t.

Note: You can observe Fox-Glynn bound computations in the log!

Exercise 6. There are two routers placed in tandem (the *out* of one of them is directly connected to the *in* of the other). Both routers can handle only one packet at a time, i.e. they don't have a queue. If a new packet arrives at the first router while it is already processing another packet, then it is dropped. On the other hand, if the first router has finished processing its packet while the second is busy, then the packet waits in the first router. Packets arrive at the first router according to $Poisson(\lambda)$, they are serviced by the first router $\sim exp(\mu_1)$ and by the second router $\sim exp(\mu_2)$.

- What proportion of packets enter the system?
- What are the expected number of jobs in the system at any given time?

This is an adaptation of the example in [1, Sec. 16.1.1]. Refer the book for the solution.

References

[1] Mor Harchol-Balter, Performance Modeling and Design of Computer Systems.

[2] `http://people.ee.duke.edu/~kst/markovpapers/numerical.pdf`

[3] `https://books.google.at/books?id=rIvh-QQrYAoC&pg=PA10`

[4] `https://pms.cs.ru.nl/iris-diglib/src/getContent.php?id=2011-Jansen-UnderstandingFoxGlynn`

[5] `http://www.prismmodelchecker.org/casestudies/peer2peer.php`