Quantitative Verification Session 7

December 15, 2016

1 Linear Temporal Logic

A Deterministic Rabin Automaton (DRA): $(Q, \Sigma, \delta, q_0, Acc)$ is a DFA with a different acceptance condition.

- $Acc = \{(E_i, F_i) \mid 1 \leq i \leq k\}$
- each accepting infinite path must visit for some i
  - all states of E_i at most finitely often and
  - some state of F_i infinitely often.

For each of the following LTL formulae, come up with a Deterministic Rabin Automaton which accepts the same words as the formula.

1. F a
2. F G a
3. G F a
4. a $\wedge$ X b
5. a U c
6. a $\rightarrow$ b
7. $\neg$ (a U b)
8. (a $\wedge$ X b) $\vee$ (a U c)

2 Markov Decision Processes (MDP)

2.1 Toy MDP and Understanding Strategies

Consider the following MDP

$[climb1]$ $s = 0 \rightarrow s' = 1;$

$[climb2]$ $s = 0 \rightarrow s' = 2;$

$[climb3]$ $s = 0 \rightarrow s' = 3;$

$[coffee]$ $s = 1 \rightarrow 0.1 : (s' = 4) + 0.9 : (s' = 5);$

$[hotchoc]$ $s = 1 \rightarrow s' = 6;$

$[coffee]$ $s = 2 \rightarrow 0.2 : (s' = 4) + 0.8 : (s' = 5);$

$[hotchoc]$ $s = 2 \rightarrow s' = 6;$

$[coffee]$ $s = 3 \rightarrow 0.5 : (s' = 4) + 0.5 : (s' = 5);$

$[hotchoc]$ $s = 3 \rightarrow s' = 6;$

State 4 represents being unhappy because the machine didn't give you coffee. State 5 represents being happy with the coffee you got. State 6 represents getting Hot Chocolate.

Q. List out a few strategies and draw the Markov Chain induced by the strategies.

Q. Find out the maximum and minimum probability of reaching State 5.

2.2 Another Toy Example

You are given two sets of two coins. Let us call the coins in the first set Ca, Cb and in the second set Da and Db. Ca has a probability of 0.4 of getting heads. Cb has a probability of 0.6 of getting heads. Both Da and Db are fair coins. You pick a set at random and then toss both the coins. Construct and analyse this MDP. What is the maximum probability of getting HH?

2.3 A Planning Example [1]

Assume that the initial fortune/fame is not known and design a Markov Decision Process which helps me planning for any initial condition (i.e. I start as Rich-Unknown/Rich-Famous/Poor-Unknown/Poor-Famous).

- I run a company
- I can choose to either save money or spend money on advertising
- If I advertise, I may become famous (50% prob.) but will spend money so I may become poor (50% prob)
- If I save money, I may become rich (50% prob.), but I may also become unknown because I don't advertise (50% prob)

Q. Examine what happens under the following strategies

- Always advertise
- Advertise when famous, save otherwise

References

[1] (Slides) Markov Decision Processes: Making Decision in the Presence of Uncertainty