Stochastic Timed Automata

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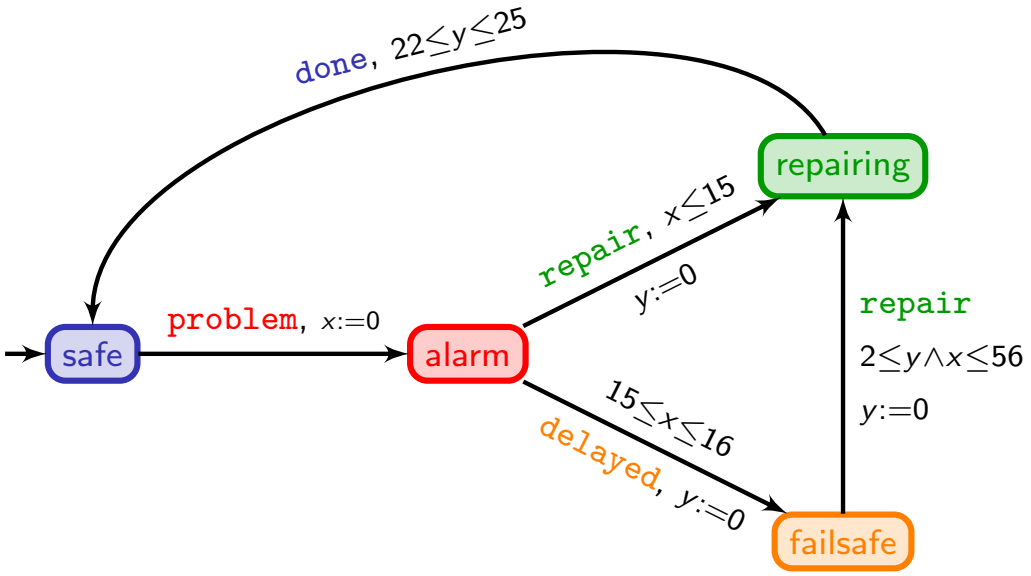
Based on joint works with Nathalie Bertrand, Thomas Brihaye, Pierre Carlier, Quentin Menet, Christel Baier, ...

The model of timed automata



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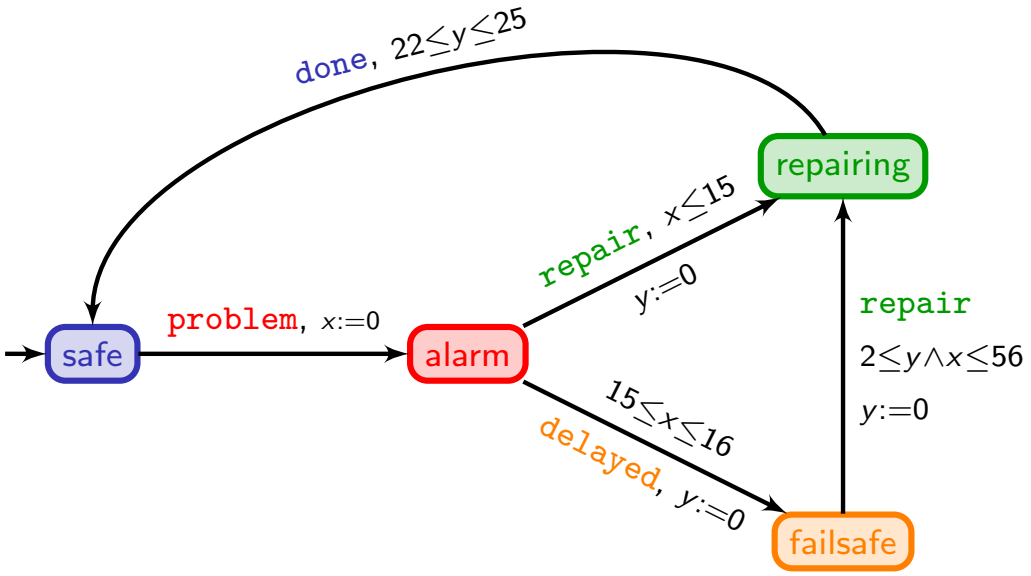
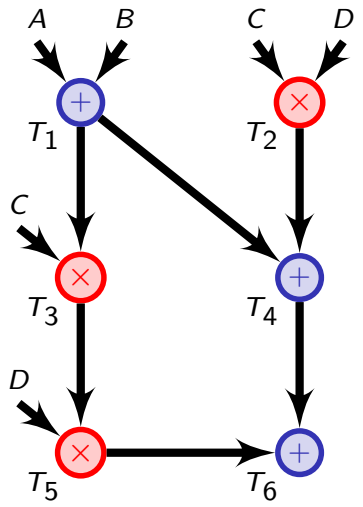


	safe	$\xrightarrow{23}$	safe	$\xrightarrow{\texttt{problem}}$	alarm	$\xrightarrow{15.6}$	alarm	$\xrightarrow{\texttt{delayed}}$	failsafe	
x	0		23		0		15.6		15.6	$\cdots$
y	0		23		23		38.6		0	

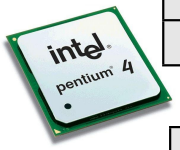
	failsafe	$\xrightarrow{2.3}$	failsafe	$\xrightarrow{\texttt{repair}}$	repairing	$\xrightarrow{22.1}$	repairing	$\xrightarrow{\texttt{done}}$	safe
$\cdots$	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

An example: The task graph scheduling problem

Compute $D \times (C \times (A+B)) + (A+B) + (C \times D)$ using two processors:



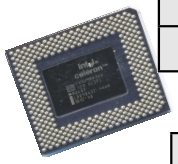
P_1 (fast):



time	
+	2 picoseconds
×	3 picoseconds

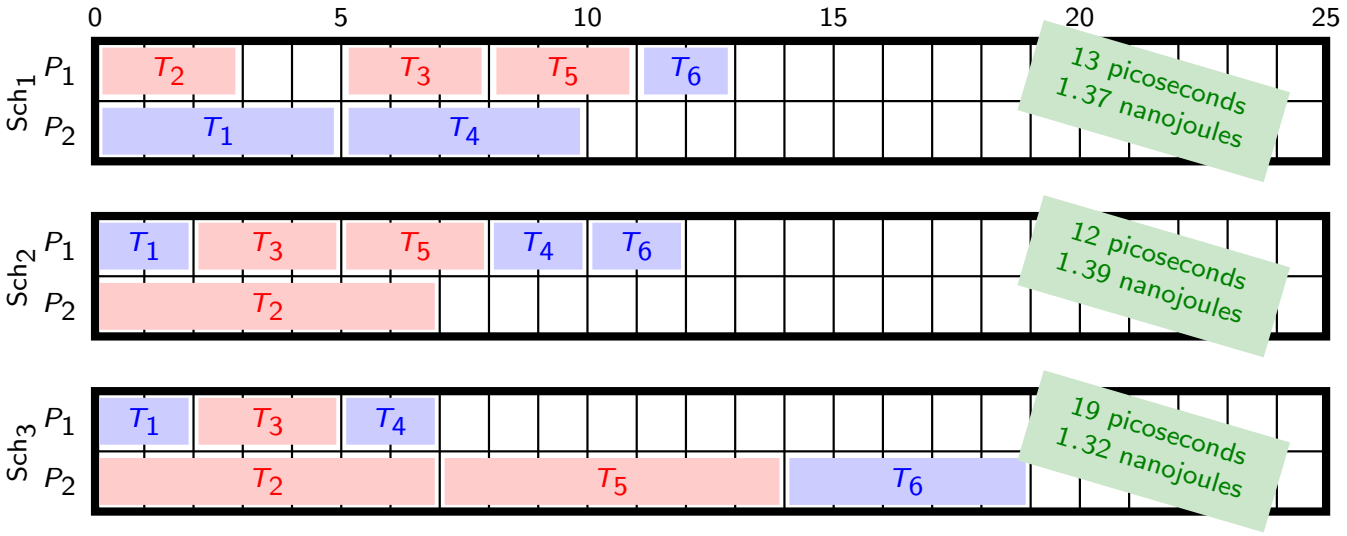
energy	
idle	10 Watt
in use	90 Watts

P_2 (slow):



time	
+	5 picoseconds
×	7 picoseconds

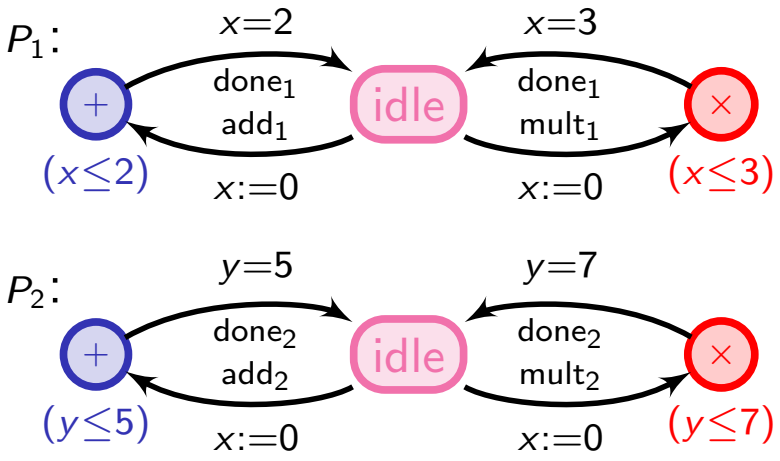
energy	
idle	20 Watts
in use	30 Watts



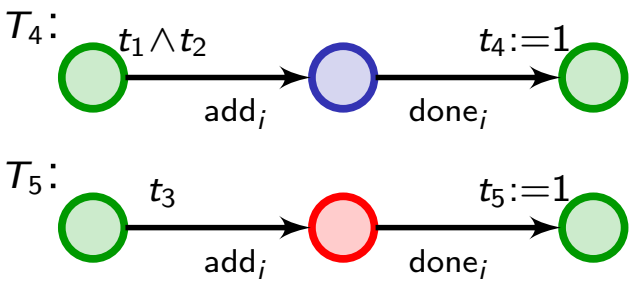
[BFLM10] Bouyer, Fahrenberg, Larsen, Markey. Quantitative Analysis of Real-Time Systems using Priced Timed Automata *(CACM)*.

Modelling the task graph scheduling problem

- Processors

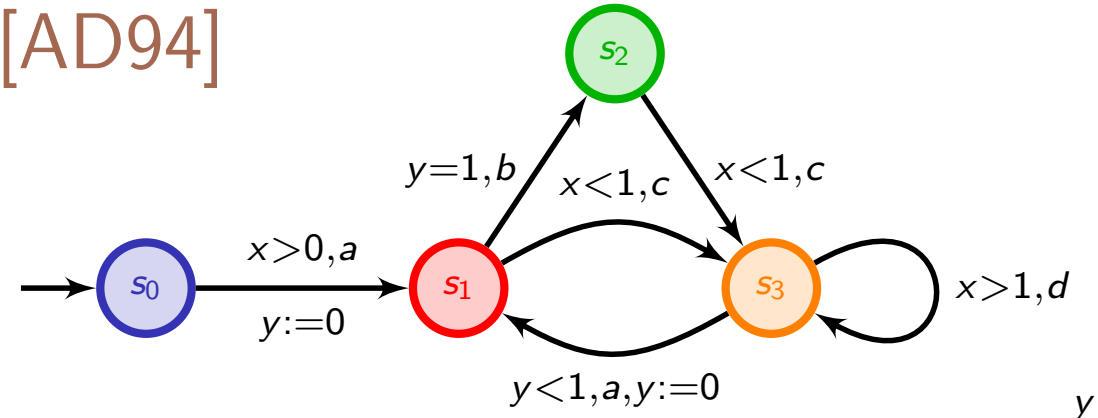


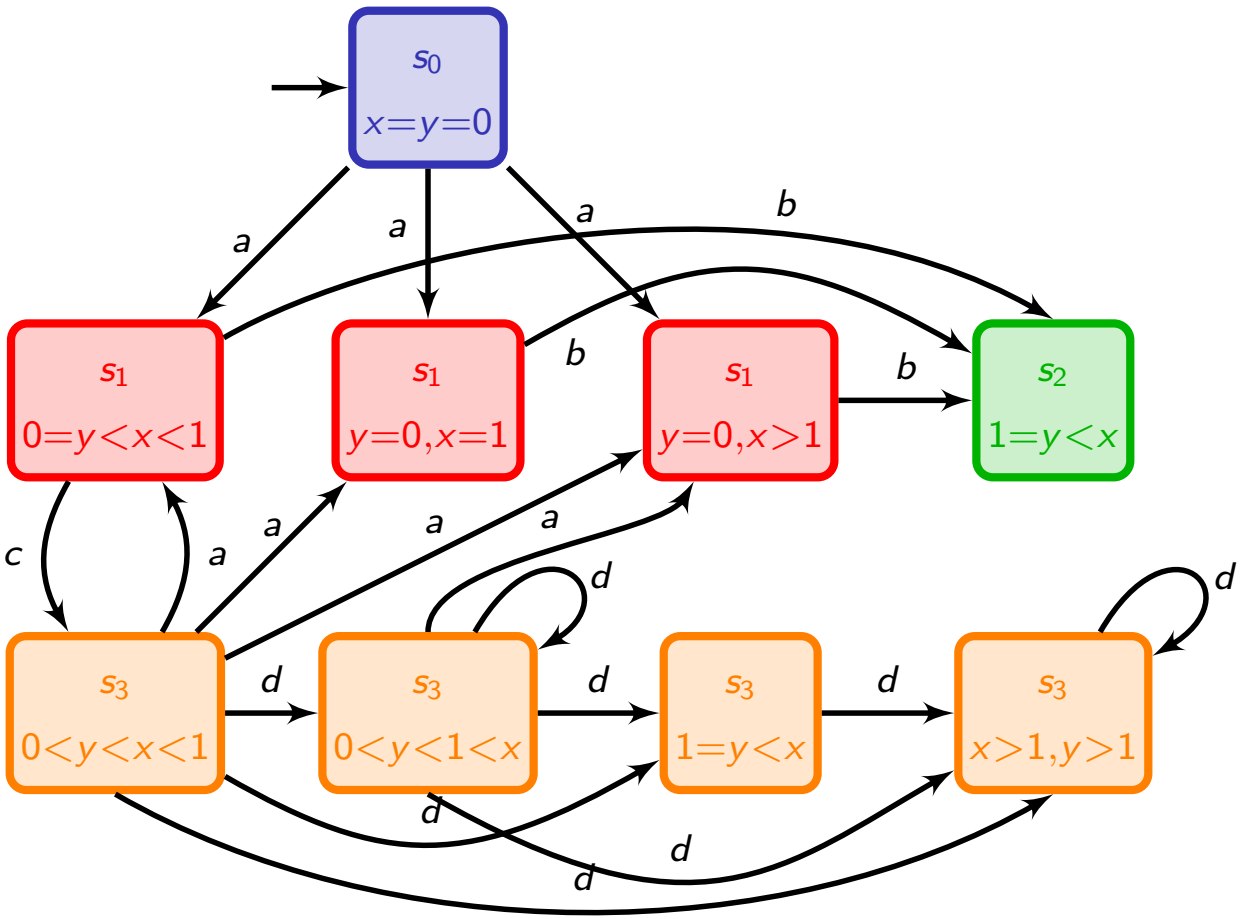
- Tasks

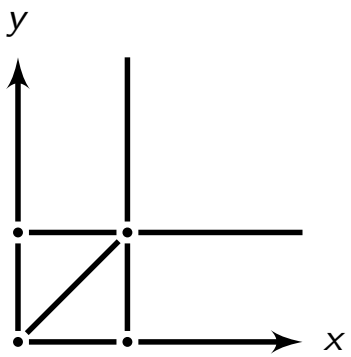


A schedule is a path in the product automaton

An example [AD94]

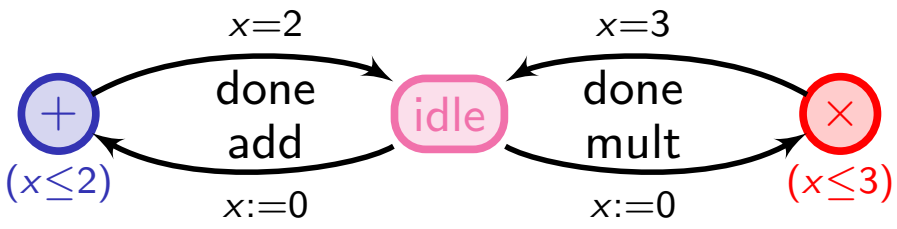






How to model uncertainty over delays?

- Using timed games



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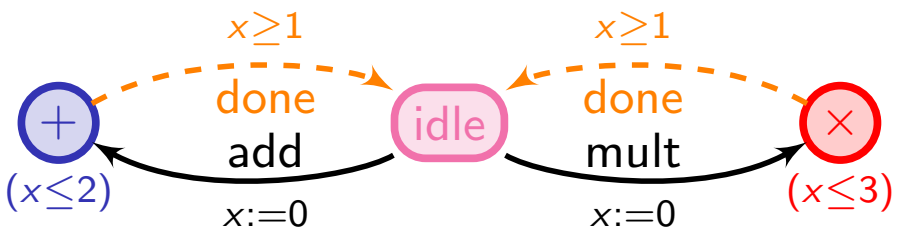

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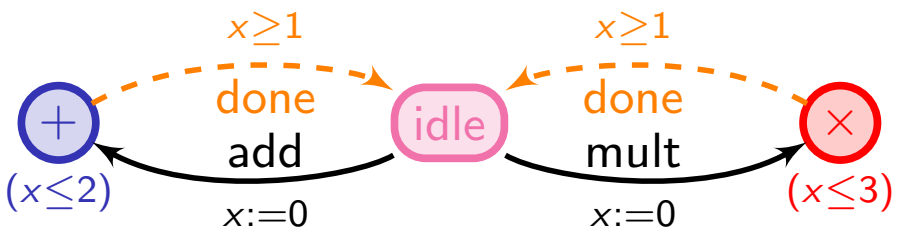

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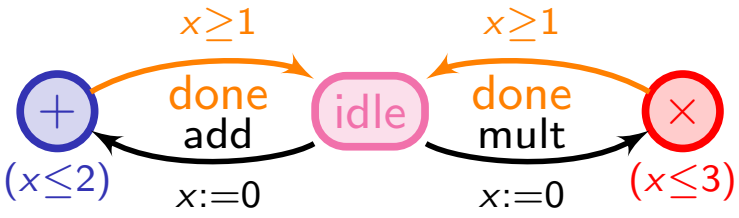

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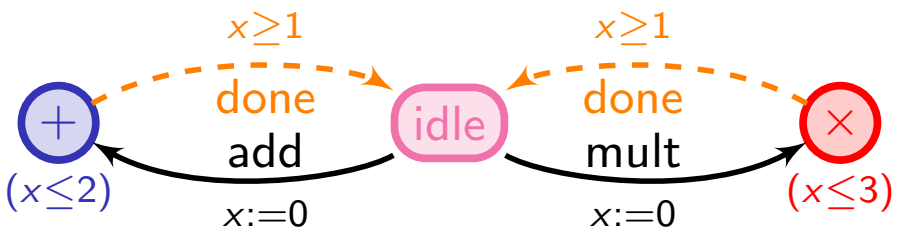

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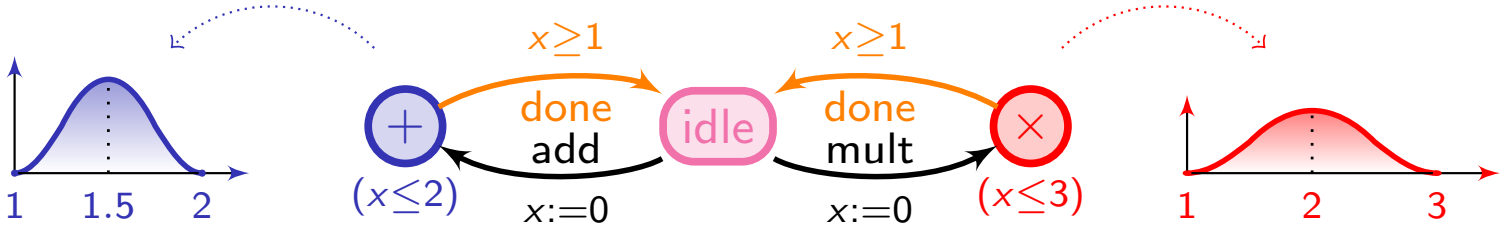


Existing models?

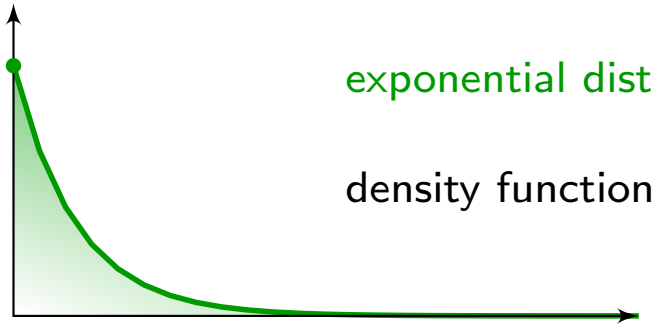
Models based on timed automata

- Probabilistic timed automata [KNSS99]
  $\rightsquigarrow$ only discrete probabilities over edges
- Continuous probabilistic timed automata [KNSS00]
  $\rightsquigarrow$ resets of clocks are randomized, but only few results

[KNSS99] Kwiatkowska, Norman, Segala, Sproston. Automatic verification of real-time systems with discrete probability distributions *(ARTS'99)*.
[KNSS00] Kwiatkowska, Norman, Segala, Sproston. Verifying quantitative properties of continuous probabilistic timed automata *(CONCUR'00)*.

How can we attach probabilities to delays?

- The example of continuous-time Markov chains



exponential distribution

density function $t \mapsto \begin{cases} \lambda \cdot \exp(-\lambda t) & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

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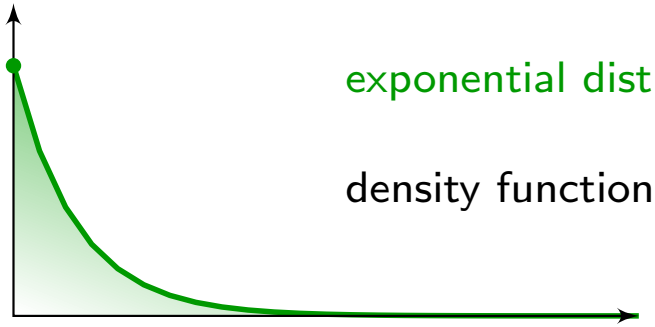

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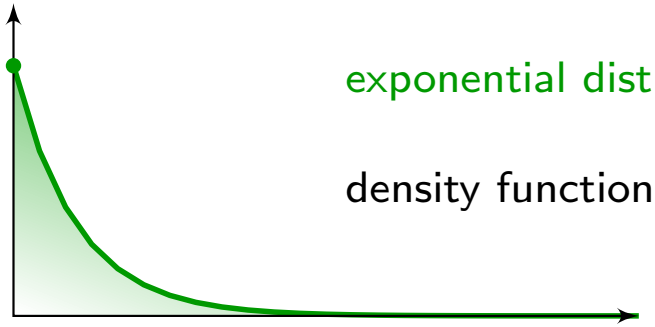

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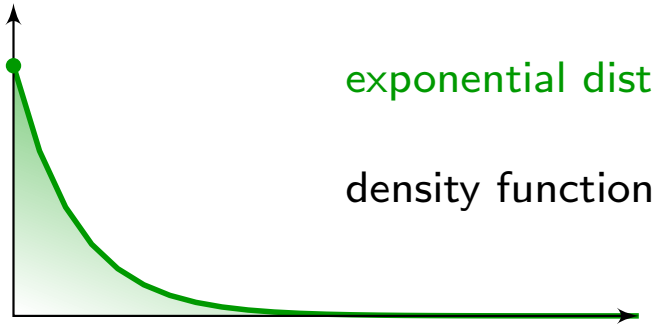

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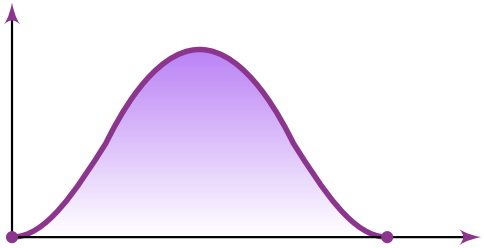

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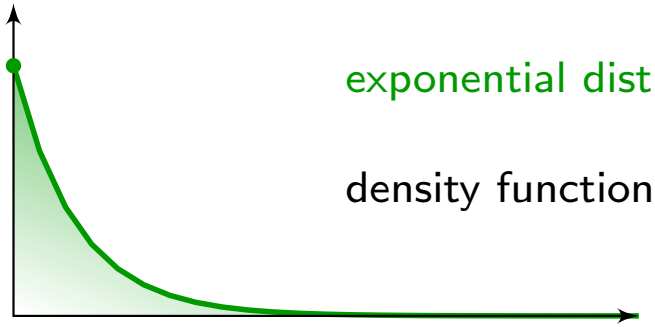

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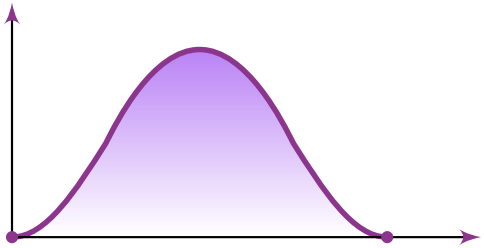

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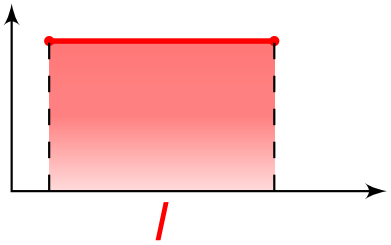
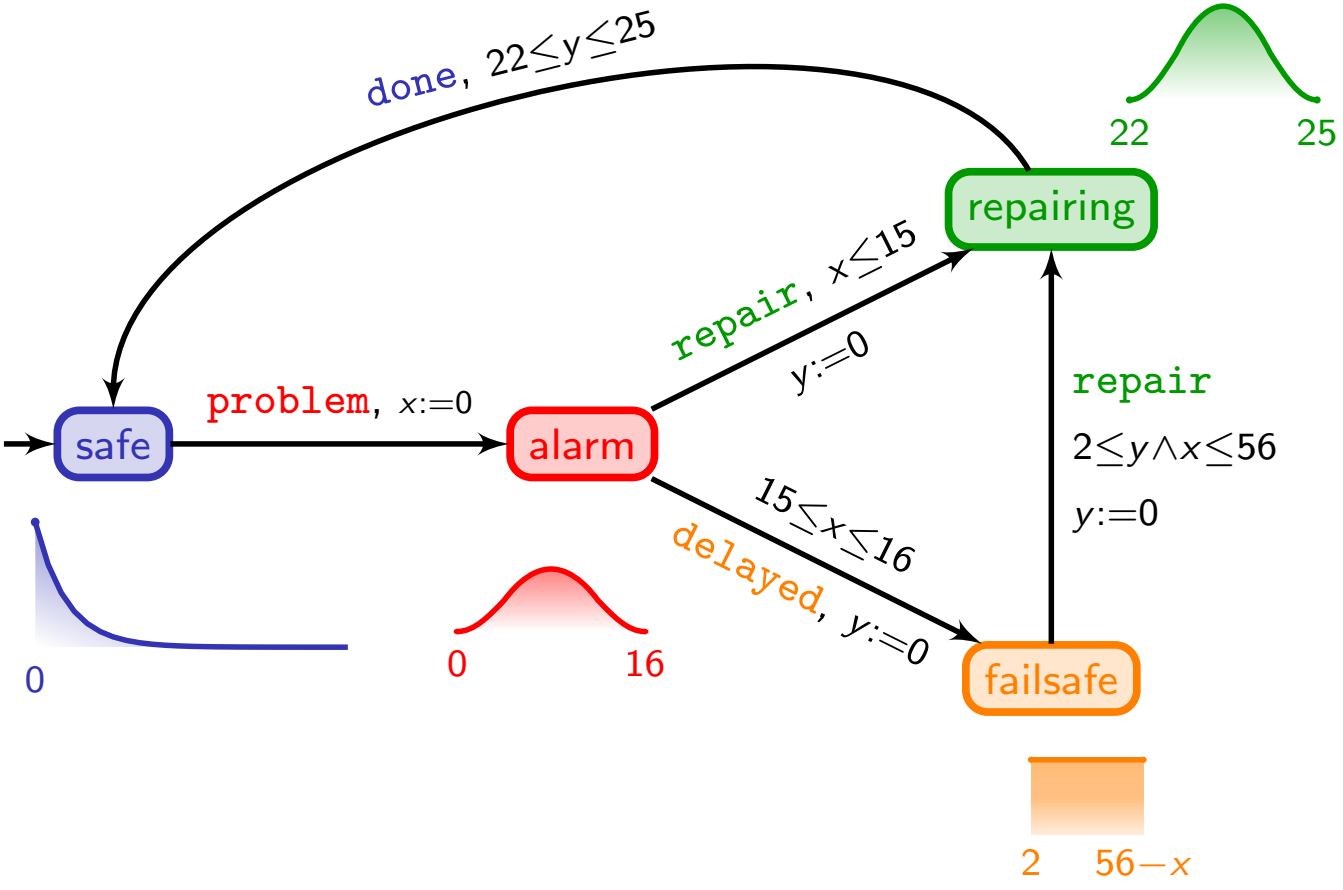


uniform distribution
density function $t \mapsto \begin{cases} \frac{1}{|I|} & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

How does a STA look like?



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- Finite-state generalized semi-Markov processes (residual-lifetime semantics) are STAs (if no fixed-delay events)
- Allows to express richer timing constraints

Almost-sure model-checking

We are interested in (automatic) model-checking algorithms!

- Qualitative model-checking: decide whether

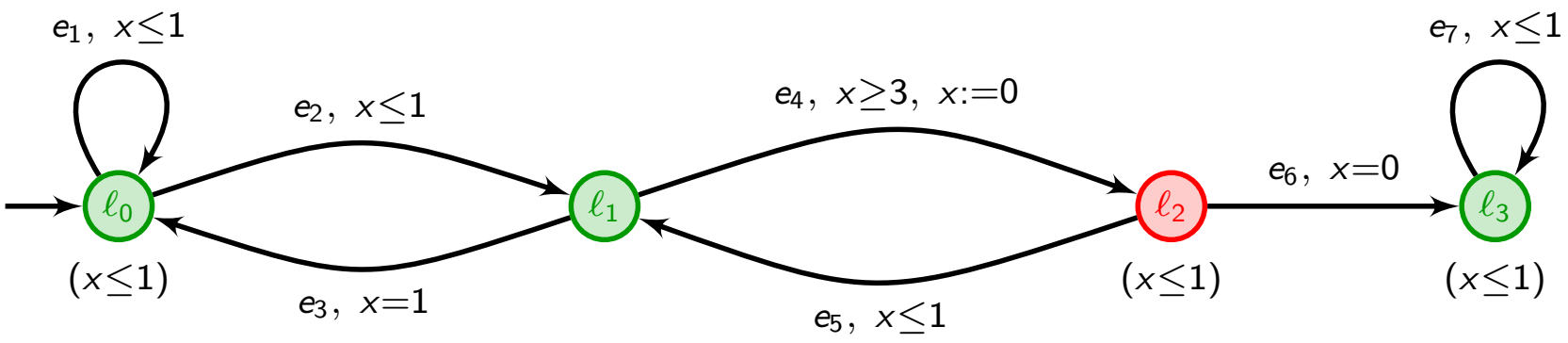
$$\mathbb{P}\big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\big) = 1$$

  We write $s \models\!\!\approx \varphi$ whenever it is the case.
  This is the almost-sure model-checking problem.

- Quantitative model-checking: compute (or approximate) the value

$$\mathbb{P}\big(\{\varrho \in \mathsf{Runs}(s) \mid \varrho \models \varphi\}\big)$$

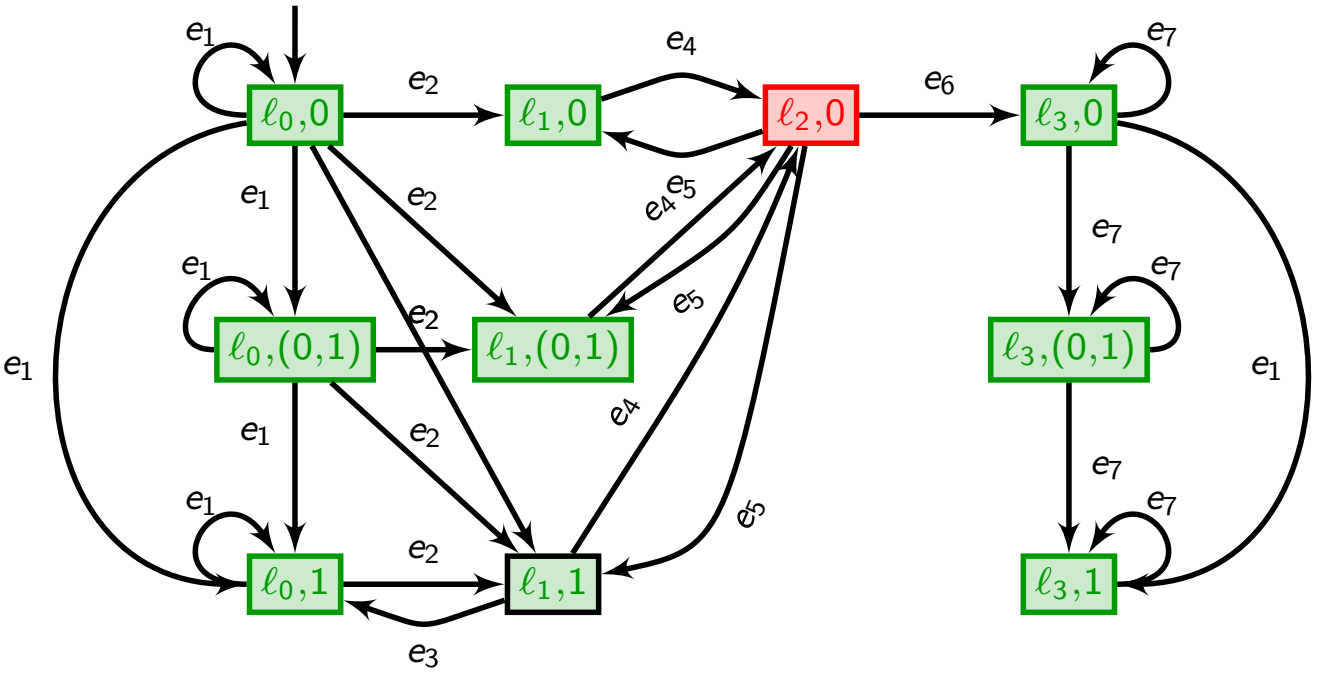
An example



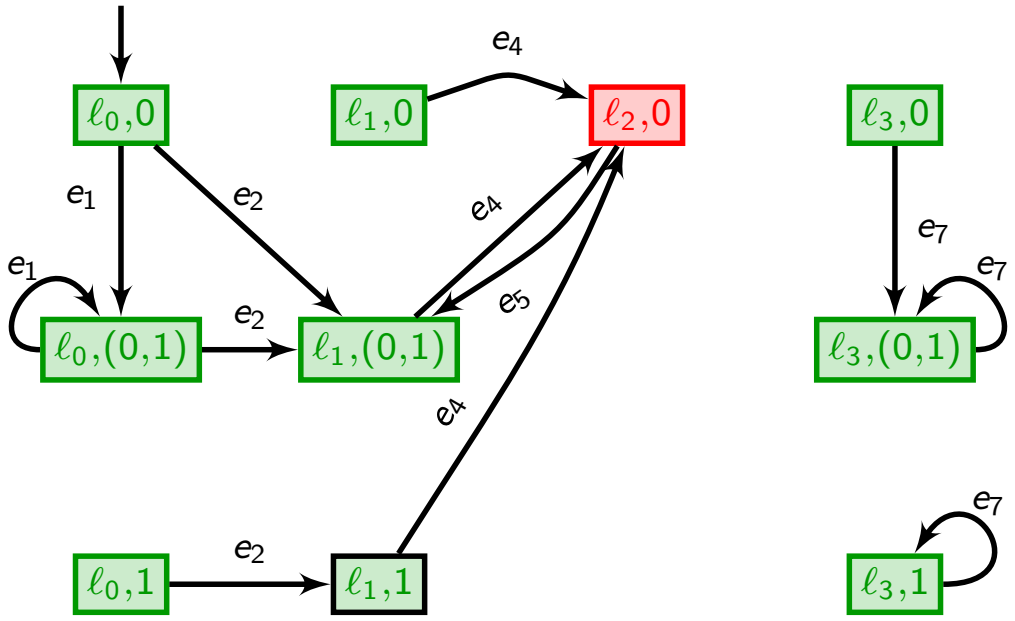
$$\mathcal{A} \not\models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\ \text{red}) \qquad \text{but} \qquad \mathbb{P}\big(\mathcal{A} \models \mathbf{G}(\text{green} \Rightarrow \mathbf{F}\ \text{red})\big) = 1$$

Indeed, almost surely, paths are of the form $e_1^* e_2 \big(e_4 e_5\big)^\omega$

The classical region automaton



The pruned region automaton



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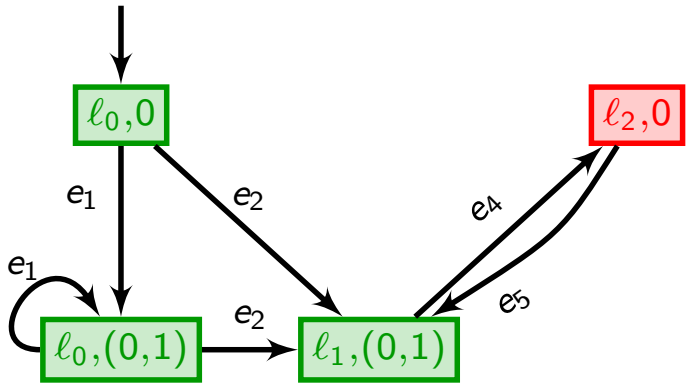

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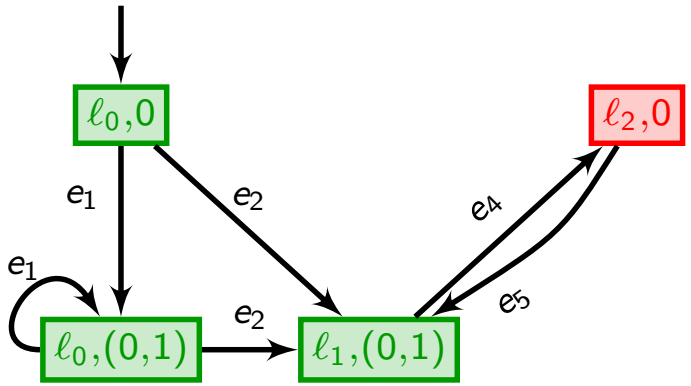

... viewed as a finite Markov chain $MC(\mathcal{A})$

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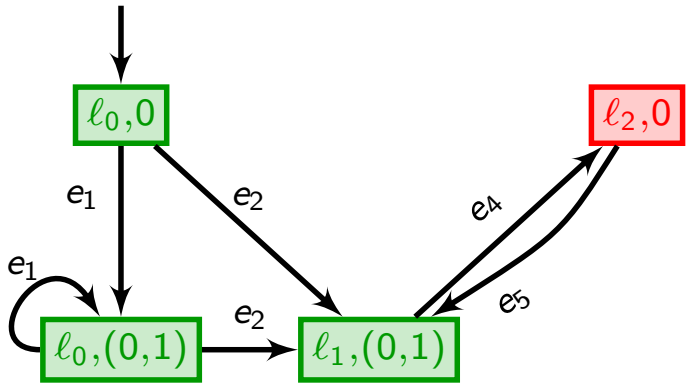

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When is that the case that

$$\mathbb{P}\big(\mathcal{A} \models \varphi\big) = 1 \quad \text{iff} \quad \mathbb{P}\big(MC(\mathcal{A}) \models \varphi\big) = 1\ ?$$