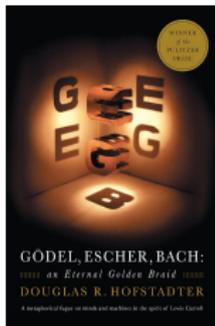
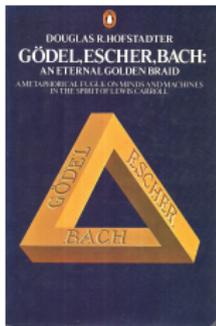


Perlen der Informatik I

Jan Křetínský

Technische Universität München
Winter 2021/2022

- ▶ language: English/German
- ▶ voluntary course
- ▶ lecture on Tuesday, in the slot 12 p.m. – 2 p.m.
- ▶ <https://www7.in.tum.de/~kretinsk/teaching/perlen.html>
- ▶ Gödel, Escher, Bach: an Eternal Golden Braid
by Douglas R. Hofstadter



- ▶ Frederick the Great
- ▶ Leonhard Euler, . . . , J.S. Bach
- ▶ improvised 6-part fugue
- ▶ canons

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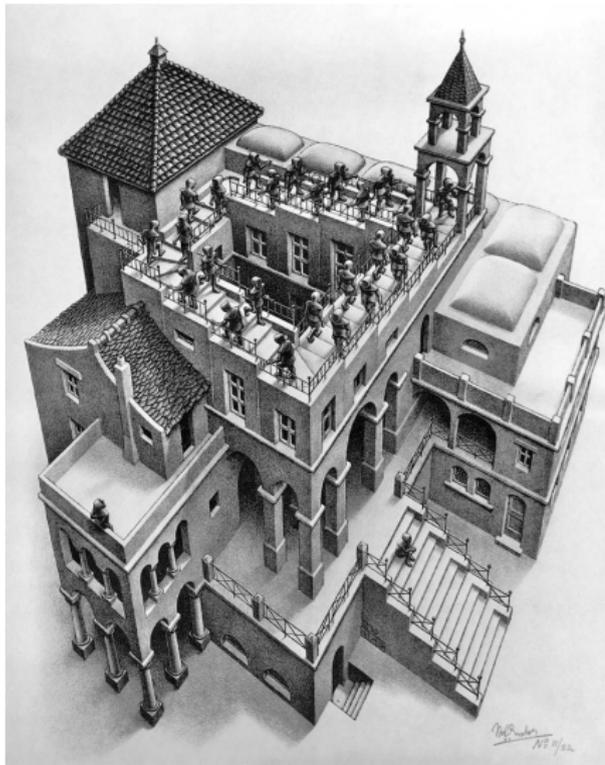


- ▶ Frederick the Great
- ▶ Leonhard Euler, . . . , J.S. Bach
- ▶ improvised 6-part fugue
- ▶ canons
 - ▶ copies differing in time, pitch, speed, direction (upside down, crab)
 - ▶ isomorphic
 - ▶ canon endlessly rising in 6 steps – “strange loop”



“Waterfall”

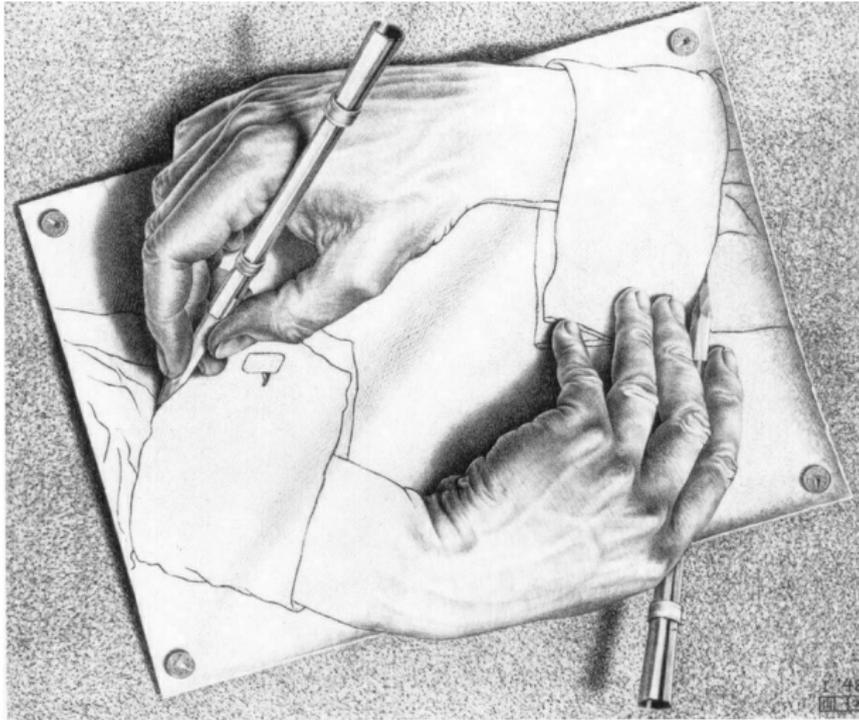
6-step endlessly falling loop



“Ascending and Descending”
illusion by Roger Penrose

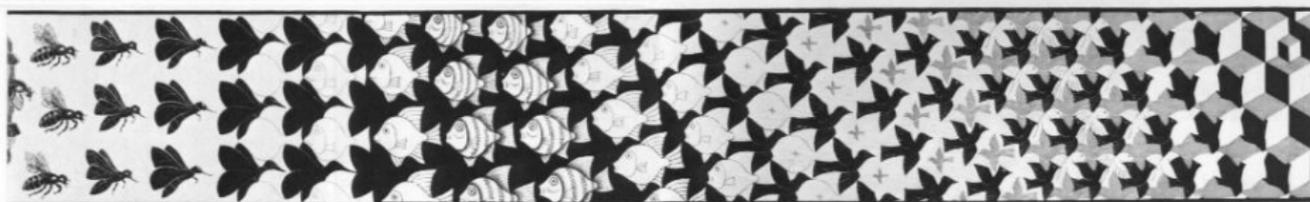
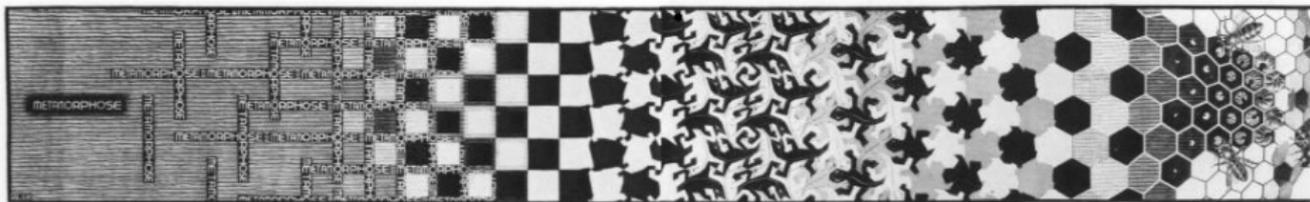


Penrose triangle
Faculty of Informatics, Brno



Dore

“Drawing hands”
his first strange loop



“Metamorphosis”
copies of one theme

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- ▶ Epimenides paradox: “All Cretans are liars”

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215473077557

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0011001000101011001100100011110100110101

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215473077557 is in binary
0011001000101011001100100011110100110101 read as ASCII
2+2=5
- ▶ **homework:**

34723379178930453204433293597543819411782291432109326918654063662

- ▶ different geometries, equally valid
- ▶ real world?
- ▶ proof?
- ▶ Russel's paradox
 - ▶ "ordinary" sets: $x \notin x$
 - ▶ "self-swallowing" sets: $x \in x$
 - ▶ R = set of all ordinary sets
- ▶ Grelling's paradox
 - ▶ self-descriptive adjectives ("pentasyllabic") vs non-self-descriptive
 - ▶ what about "non-self-descriptive"?
- ▶ self-reference
drawing hands
The following sentence is false. The preceding sentence is true.



- ▶ prohibition (Principia mathematica)
- ▶ types, metalanguage
- ▶ “In this lecture, I criticize the theory of types”
cannot discuss the type theory
- ▶ David Hilbert: consistency and completeness

- ▶ Babbage
The course through which I arrived at it was the most entangled and perplexed which probably ever occupied the human mind.
Ada Lovelace (daughter of Lord Byron)
Mechanized intelligence
“Eating its own tail” (altering own program)
- ▶ axiomatic reasoning, mechanical computation, psychology of intelligence
- ▶ Alan Turing ~ Gödel’s counterpart in computation theory
Halting problem is undecidable.
Can intelligent behaviour be programmed? Rules for inventing new rules...
Strange loops in the core of intelligence
- ▶ materialism, de la Metrie: L’homme machine