Fundamental Algorithms 8

Exercise 1 (Parallel Scalar)
Write a parallel program that computes the scalar product of two vectors (stored in two arrays). Discuss the runtime complexity on the EREW PRAM model. How many processors can be used?

Exercise 2 (Parallel Vector)
Extend the program of exercise 1 to compute a matrix-vector product. Again, discuss the runtime complexity on the EREW PRAM and state the number of processors that are used.

Exercise 3 (Parallel Optimization)
Given the following parallel algorithm PrefixPRAM for prefix multiplication (with EREW-PRAM). First, argue why the algorithm is correct. Then, assume that the $j$-loop is changed to a sequential loop. State why the resulting algorithm now no longer is correct and suggest how to change the $j$-loop to obtain a correct sequential implementation.

Algorithm 1: PrefixPRAM

\begin{algorithm}
\textbf{Input:} $A$: Array[1..$2^k$]  
\hfill $\quad \text{tmp} \leftarrow \text{Array}[1..2^k]$;  
\hfill $\quad \text{for } l = 0 \text{ to } k - 1 \text{ do}$  
\hfill $\quad \quad \text{for } j = 2^l + 1 \text{ to } n \text{ in parallel do}$  
\hfill $\quad \quad \quad \text{tmp}[j] \leftarrow A[j - 2^l]$;  
\hfill $\quad \quad \quad A[j] \leftarrow \text{tmp}[j] \cdot A[j]$;  
\hfill $\quad \quad \text{end}$  
\hfill $\quad \text{end}$
\end{algorithm}