Fundamental Algorithms 7

Exercise 1 (Hash Function)
Let $n = 1000$. Compute the values of the hash function $h(k) = \lfloor n(a k - \lfloor a k \rfloor) \rfloor$ for the keys $k \in \{61, 62, 63, 64, 65\}$, using $a = \frac{\sqrt{5} - 1}{2}$. What do you observe?

Exercise 2 (Hash Table)
Let $T$ be a hash-table of size 9 with the hash function $h : U \rightarrow \{0, 1, \ldots, 8\}, k \mapsto k \mod 9$. Write down the entries of $T$ after the keys 5, 28, 19, 15, 20, 33, 12, 17, and 10 have been inserted. Use chaining to resolve collisions.

Exercise 3 (Open Hash Tables)
Now, let $T$ be a hash table of size 11, using open addressing with the following hash functions

1. $h(k, i) := (k + i) \mod 11$
2. $h(k, i) := (k \mod 11 + 2i + i^2) \mod 11$
3. $h(k, i) := (k \mod 11 + i \cdot (k \mod 7)) \mod 11$

Insert the keys 5, 19, 27, 15, 30, 34, 26, 12, and 21 (in that order) and state which keys require the longest probe sequence in the resulting tables.

Exercise 3
Consider a universe $U$ of keys, where $|U| > mn$, and a hash function $h : U \rightarrow \{0, 1, \ldots, n - 1\}$. Show that there are at least $m$ elements of $U$ which are mapped to the same hash value, i.e. there is a subset $A$ of $U$ with $|A| = m$ and $h(a_1) = h(a_2)$ for all $a_1, a_2 \in A$. 