Exercise 1 (Ford-Fulkerson)

Consider the following flow network $G$ during the execution of Ford-Fulkerson. The edge labels, written $f/c$, denote the flow $f$ and capacity $c$ of the respective edge. Draw the residual graph $G_f$, find an augmenting path, and apply the Ford-Fulkerson operations. Continue the algorithm until you identified the maximum flow in $G$.

Solution:

The algorithm actually is finished now, with the maximum flow equal 3.

Exercise 2 (Marriage)

The maximum bipartite matching problem is defined as follows. Given a bipartite graph $((U, V), E)$, i.e. a graph where edges are exclusively between $U$ and $V$, find a largest set of edges such that no two edges share a vertex. Informally, this can be interpreted as marriage problem: The vertex sets represent the males and
females of a particular population, edges represent mutual interest. Now, we want to identify the maximum amount of marriages.

Think about how to apply Ford-Fulkerson to solve this problem.

Solution:
Introduce a super-source $s$, connected to all nodes in $U$, and a super-sink $t$, connected to all nodes in $V$. Now, applying Ford-Fulkerson to this graph immediately yields the solution.