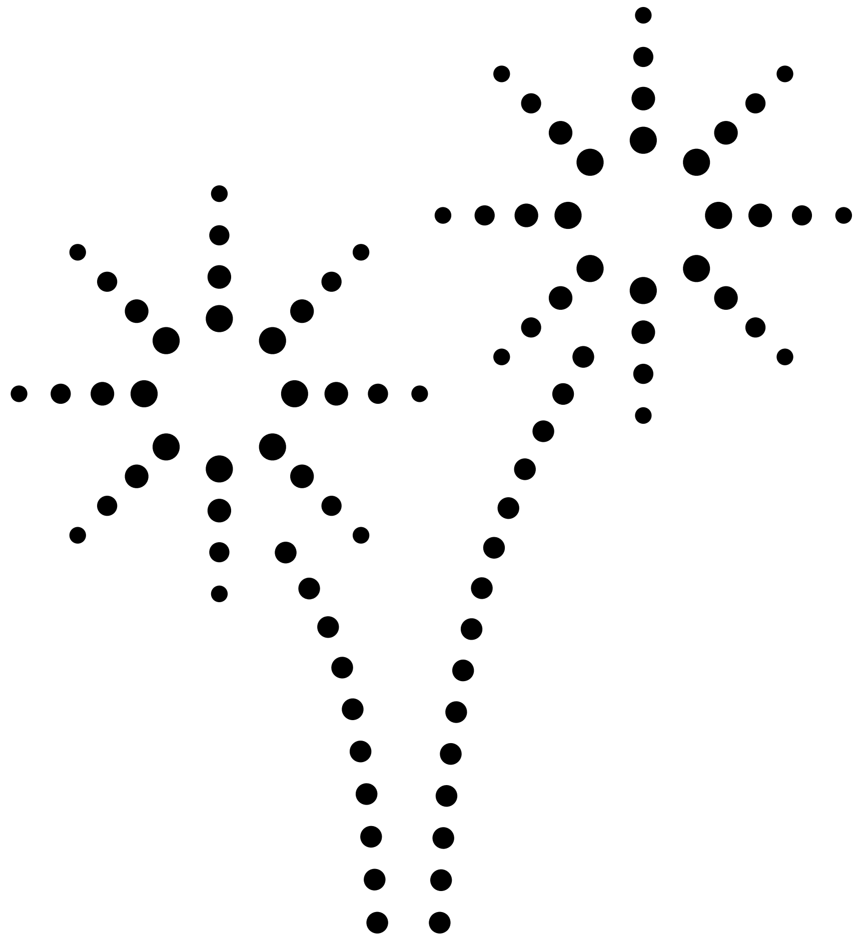
On the Home-Space Problem for Petri Nets



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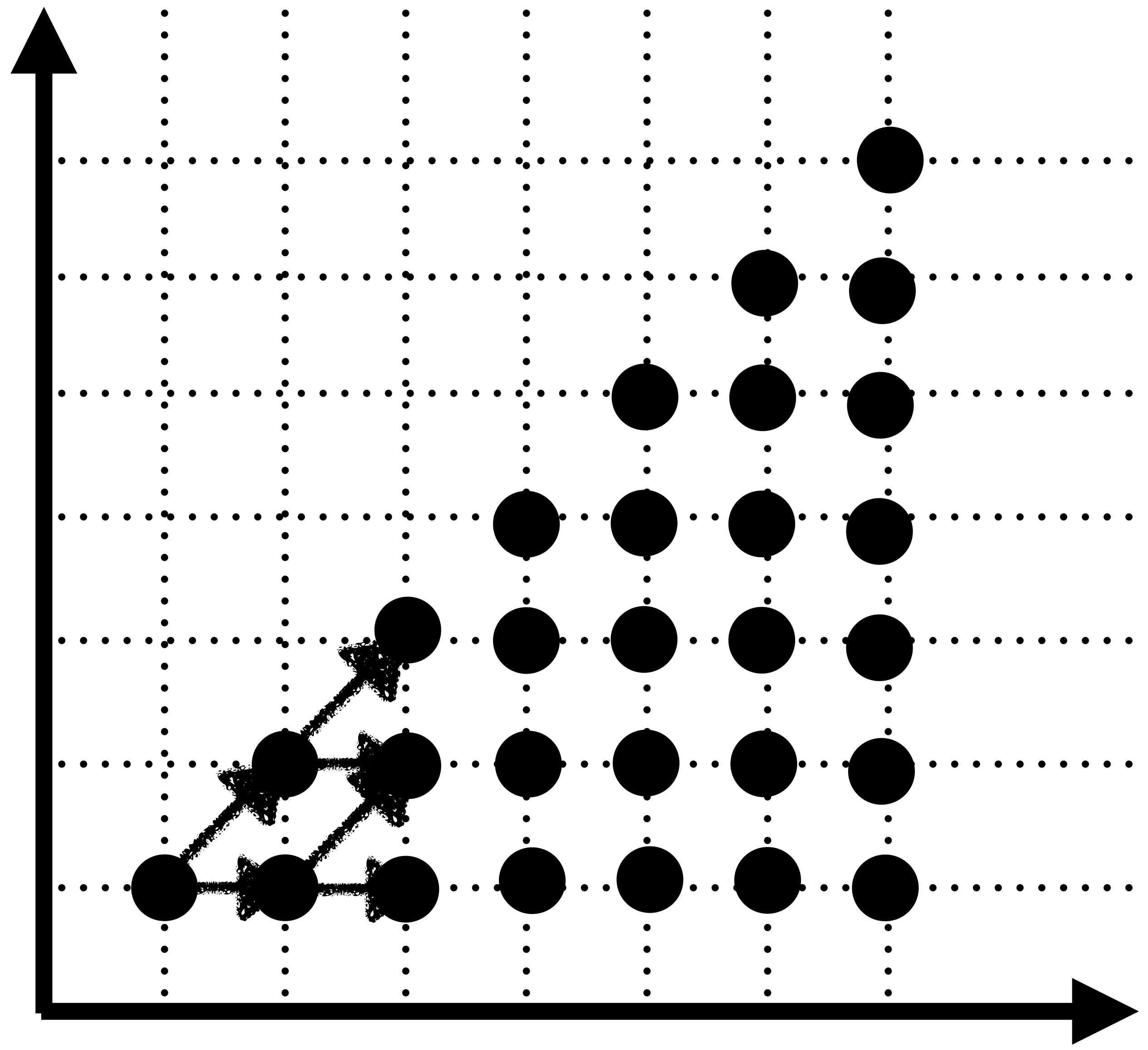
Joint work with Petr Jančar (Dept of Comp. Sci., Faculty of Science, Palacky Univ. Olomouc, Czechia)

Semilinear Sets

Linear Sets

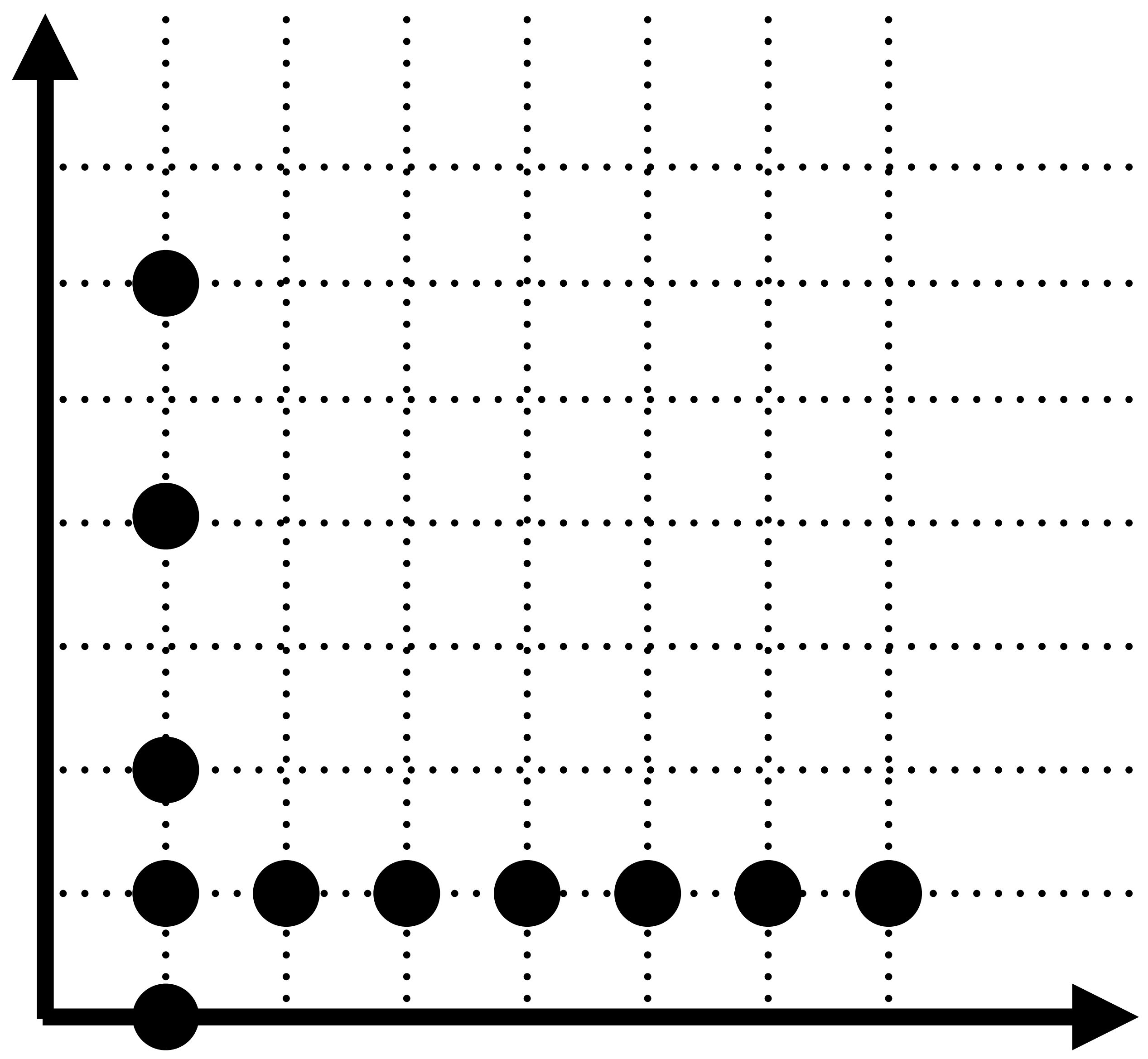
A linear set of $\mathbb{N}^d$ is a set of the form $b + \mathbb{N}p_1 + \cdots + \mathbb{N}p_k$ where $b, p_1, \ldots, p_k \in \mathbb{N}^d$.



$$(1,1) + \mathbb{N}(1,1) + \mathbb{N}(1,0)$$

Semilinear Sets

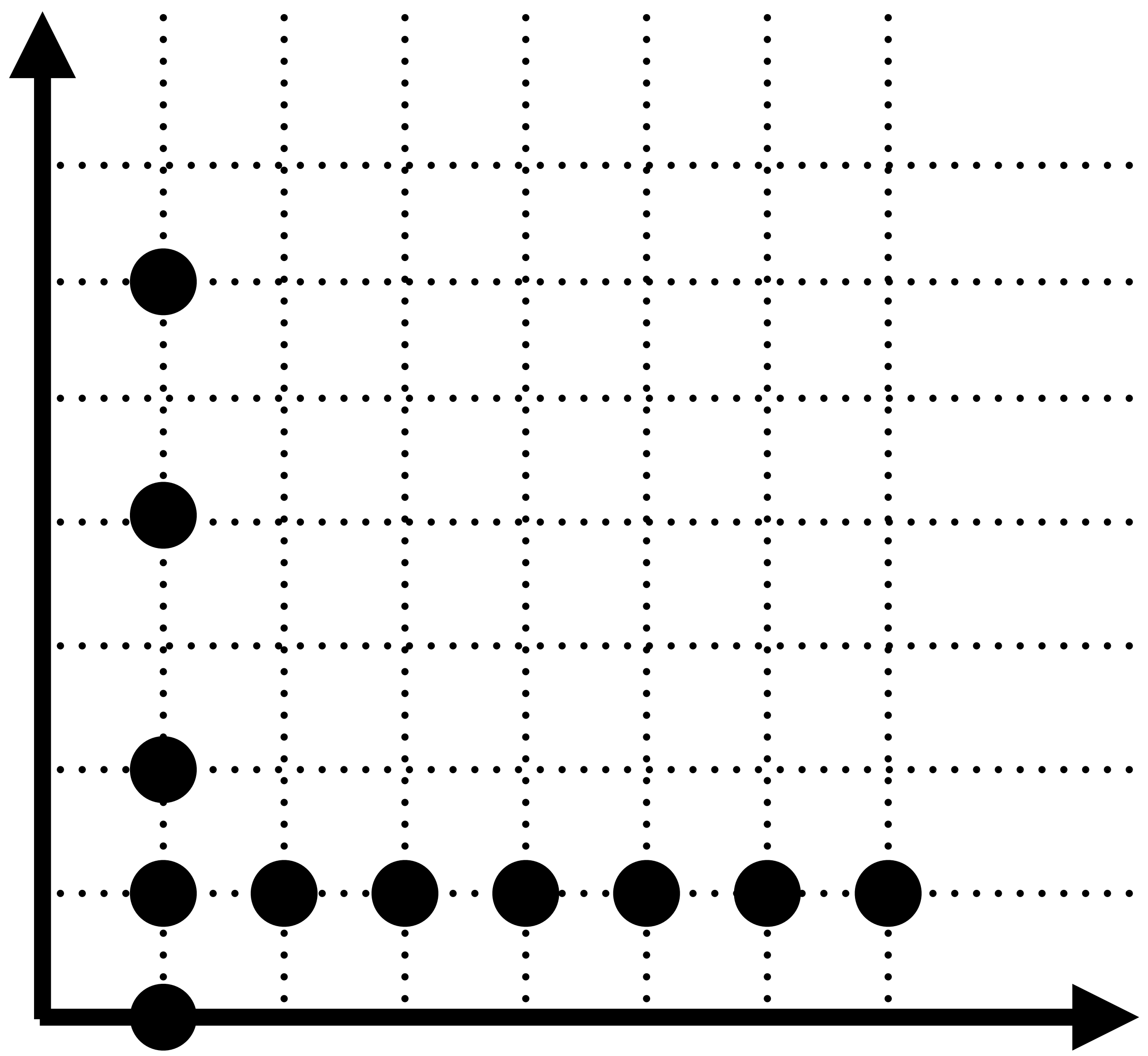
A semilinear set is a finite union of linear sets.



$$(1,1) + \mathbb{N}(1,0) \quad \cup \quad (1,0) + \mathbb{N}(0,2)$$

Presburger Arithmetic

1966, Ginsburg and Spanier : Sets definable in the Presburger arithmetic $FO(\mathbb{N}, +, 0, 1, \leq, (\equiv_k)_{k \in \mathbb{N}})$ are precisely the semilinear sets.



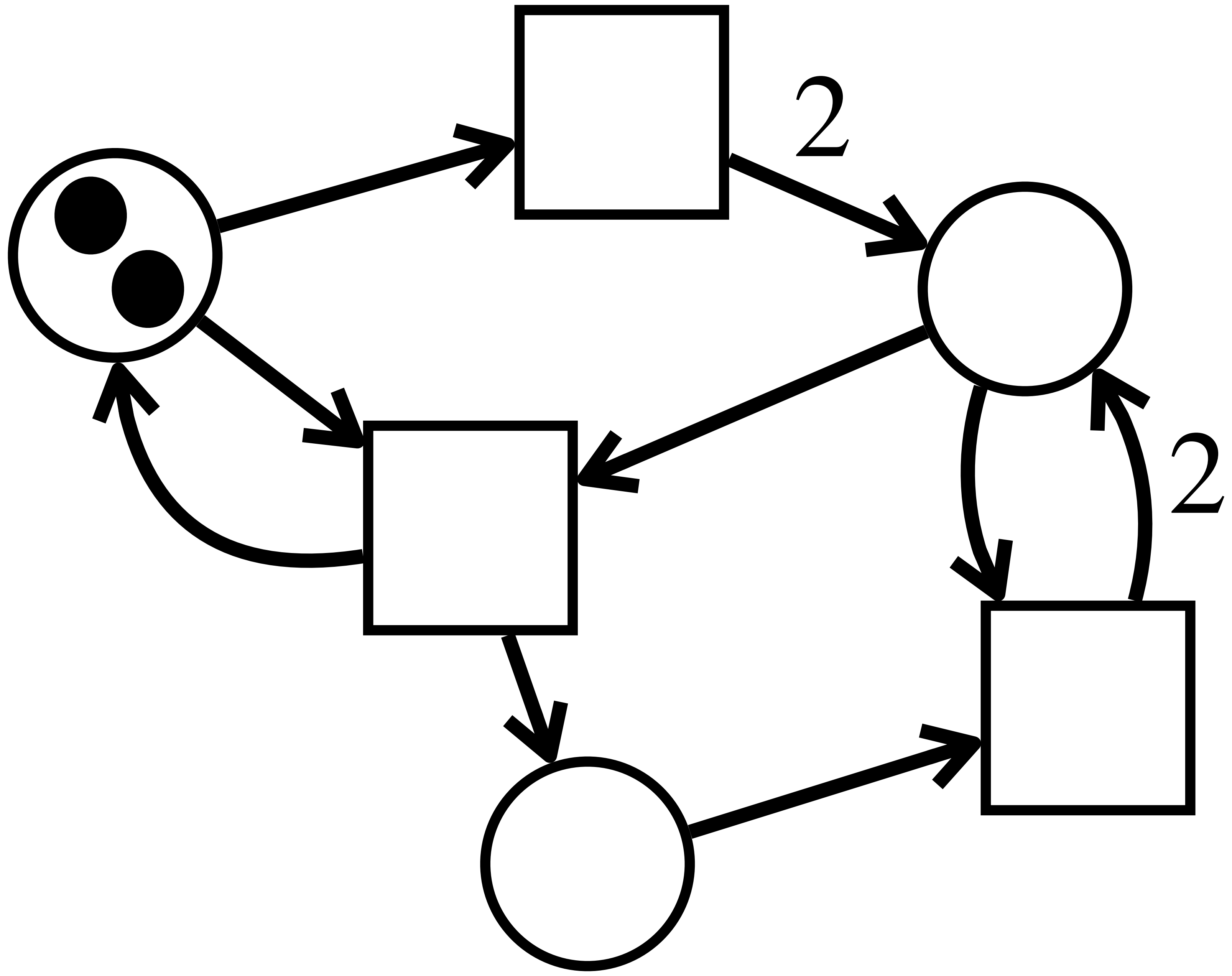
$$(1,1) + \mathbb{N}(1,0) \;\cup\; (1,0) + \mathbb{N}(0,2)$$

$$=$$

$$\{(x, y) \in \mathbb{N}^2 \mid (x \geq 1 \wedge y = 1) \vee (x = 1 \wedge y \equiv_2 0)\}$$

Petri Nets

Petri Nets



Petri Nets

Model of concurrency with extensive applications:

- hardware and software,
- database systems,
- chemical, biological and business processes.

Reachability Problem

INPUT:

- A Petri net and two configurations x, y.

OUTPUT:

- Decide whether $x \xrightarrow{*} y$.

Central Problem

Many problems reduce to the Petri net reachability problem:

- Formal languages.
- Logic.
- Concurrent systems.
- Process calculi.
- Population protocols. 🥷

Short History

- 1976, Lipton : Expspace lower-bound.
- 1981, Mayr : Decidable.
- 1982, Kosaraju and in 1992 Lambert clarified the algorithm.
- 2015, L. and Schmitz : Cubic-Ackermann upper-bound.
- 2019, L. and Schmitz : Ackermann upper-bound.
- 2019, Czerwinski, Lasota, Lazic, L., and Mazowiecki : Tower lower-bound.
- 2021, L., and Czerwinski and Orlikowski : Ackermann lower-bound.

$\Rightarrow$ The Petri net reachability problem is Ackermann-complete.

Semilinear Reachability Problem

INPUT:

- A Petri net and two semilinear sets X, Y of configurations.

OUTPUT:

- Decide whether $X \xrightarrow{*} Y$.

i.e. $\exists x \in X \ \exists y \in Y \ \ x \xrightarrow{*} y$

Home-Space 🏠

Home-Space

Given a Petri net and two sets of configurations X, H.

H is called an home-space for X if:

$$\forall y \quad X \xrightarrow{*} y \quad \Rightarrow \quad y \xrightarrow{*} H$$

Semilinear Home-Space Problem

INPUT:

- A Petri net and two semilinear sets of configurations X, H.

OUTPUT:

- Decide whether H is an home-space for X.

Semilinear Non-Reachability Problem

$\sim$

Semilinear Home-Space Problem

$\neg(X \xrightarrow{*} Y)$ iff $\mathrm{post}^*(X) \subseteq \overline{\mathrm{pre}^*(Y)}$

H home-space for X iff $\mathrm{post}^*(X) \subseteq \mathrm{pre}^*(H)$

History

- 1973, Henry G. Baker Jr : $\mathrm{post}^*_A(x) \subseteq \mathrm{pre}^*_B(y)$ undecidable if $A \neq B$
- 1989, Davide de Frutos Escrig and Colette Johnen : Home-space problem is decidable for hybrid linear sets H.

Our main contributions :

- The semilinear home-space problem is decidable.
- Moreover it is Ackermann-complete.

Inductive Non-Reachability Cores

C is called an inductive non-reachability core for H if:

- $\text{post}^*(C) = C$
- $C \cap H = \emptyset$
- $\forall x \quad x \xrightarrow{*} (H \cup C)$

In that case H is an home-space for X iff $\neg(X \xrightarrow{*} C)$

Stability Property

Theorem

If C_i is an inductive non-reachability core for H_i for every $1 \leq i \leq n$ then $C_1 \cap \ldots \cap C_n$ is an inductive non-reachability core for $H_1 \cup \ldots \cup H_n$.

1st Result

- Semilinear inductive non-reachability cores for semilinear sets are effectively computable.

2nd Result

A semilinear set H is an home-space for a semilinear set X

$$\Longleftrightarrow$$

$$\text{post}^*(X) \subseteq \text{pre}^*(H)$$

Semilinear set

Refined 2nd Result

A semilinear set H is an home-space for a semilinear set X

$$\iff$$

there exists a semilinear inductive invariant I and words $w_1, \ldots, w_k$ such that:

$$X \subseteq I \subseteq \mathrm{pre}(H, w_1^* \ldots w_k^*)$$

Future Work & Open Problems

Variants

- 2016, Eike Best and Javier Esparza : The existential variant is decidable i.e. : does there exist a configuration h such that $\{h\}$ is an home-space.

Open variant :

Given a Petri net and a semilinear set X of initial configurations, does there exists a semilinear set $H \subseteq \mathrm{post}^*(X)$ that is an home-space for X.

Inductive Non-reachability Core Problem

INPUT:

- A Petri net and two semilinear sets C, H of configurations.

OUTPUT:

- Decide whether C is an inductive non-reachability core for H.

Universal Reachability Problem

INPUT:

- A Petri net and a semilinear set X of configurations.

OUTPUT:

- Decide whether $\mathrm{post}^*(X)$ is the full set of configurations.

2018, Petr Jančar, L., Grégoire Sutre: Expspace-complete for singleton X and decidable in general.

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THANKS !

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