Rewriting Models of Boolean Programs

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Joint work with Ahmed Bouajjani

Automatic verification using model-checking

Initiated in the early 80s in USA and France.

- 25 Years of Model Checking Workshop on August 16

Exhaustive examination of the state space using (hopefully) clever techniques to avoid state explosion.

Very successful for hardware or "low-level" software:

- Applied to commercial microprocessors, telephone switches launching protocols, brake systems, or the dutch Delta works.
- Model-checking groups at all major hardware companies.
- ACM Software System Award 2001, Gödel Prize 2000, Kannellakis Awards 1998 and 2005.

Software model-checking

Big research challenge of the 00s: extension to ‘high-level’ software.

Three main research questions:

- Integration of the tools in the software development process.
  - Users trust their hardware but may not trust their software: “post-mortem” verification, “backstage” verification tools . . .
- Automatic extraction of models from code.
- Algorithms for infinite-state systems.
  - Software systems “more often” infinite-state.

The lazy approach to software verification

Construct a sequence of increasingly faithful models that under- or overapproximate the code.

Underapproximations: 32-bit integer $\longrightarrow$ 2-bit integer, 500MB heap $\longrightarrow$ 10B heap.

Overapproximations using predicate abstraction:

- Define a set of predicates over the dataspace (e.g. $x < y$, “list is empty”).
- Partition the dataspace into $2^{\text{number of predicates}}$ abstract values.
- Execute symbolically: one boolean variable per predicate.

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Both are boolean programs:

- Same control-flow structure as code + possibly nondeterminism.
- Only one datatype: booleans.
- Conceptually could also take any enumerated type but booleans are the bridge to SAT and BDD technology.

Rewriting models of boolean programs

Boolean programs are still pretty complicated objects:

- Procedures/methods and recursion.
- Concurrency and communication (threads, cobegin-coend sections).
- Object-orientation.

Must be “compiled” into simpler and formal models.

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Use rewriting to model boolean programs. In a nutshell:

- Model program states as terms.
- Model program instructions as term-rewriting rules.
- Model program executions as sequences of rewriting steps.

Fundamental analysis problems

Reachability

- But reachability between two states not enough for verification purposes.
- Safety properties often characterized by an infinite set of dangerous states.

Symbolic reachability: Find a finite symbolic representation of the (possibly infinite) set of states reachable or backward reachable from a given (possibly infinite) set of states.

- $pre^*(S)$ denotes the set of predecessors of S.
  (states backward reachable from states in S)
- $post^*(S)$ denotes the set of successors of S.
  (states forward reachable from states in S)

Program for the rest of the talk

Rewriting models for:

- Procedural sequential programs.
- Multithreaded while-programs.
- Multithreaded procedural programs.
- Procedural programs with cobegin-coend sections.

For each of those:

- Complexity of the reachability problem.
- Finite representations for symbolic reachability.

A rewriting model of procedural sequential programs

State of a procedural boolean program: $(\, g, \ell, n, (\ell_1, n_1) \ldots (\ell_k, n_k) \,)$, where

- g is a valuation of the global variables,
- ℓ is a valuation of local variables of the currently active procedure,
- n is the current value of the program pointer,
- l_i is a saved valuation of the local variables of the caller procedures, and
- n_i is a return address.

Modelled as a string $g \, \langle \ell, n \rangle \, \langle \ell_1, n_1 \rangle \, \ldots \, \langle \ell_k, n_k \rangle$

Instructions modelled as string-rewriting rules, e.g. $t \, \langle t, m_0 \rangle \longrightarrow f \langle f \, t \, f, p_0 \rangle \, \langle t, m_1 \rangle$

Prefix-rewriting policy:

$$\frac{u \longrightarrow w}{u \, v \xrightarrow{\; r \;} w \, v}$$

An example

bool function *foo*(ℓ)

f_0: **if** ℓ **then**

f_1: **return** $\mathbf{false}$

else

f_2: **return** $\mathbf{true}$

fi

procedure *main*$()$

global b

m_0: **while** b **do**

m_1: $b := foo(b)$

od;

m_2: **return**

$$
\begin{array}{rcl}
b\,\langle t, f_0\rangle & \longrightarrow & b\,\langle t, f_1\rangle \\
b\,\langle f, f_0\rangle & \longrightarrow & b\,\langle f, f_2\rangle \\
b\,\langle \ell, f_1\rangle & \longrightarrow & f \\
b\,\langle \ell, f_2\rangle & \longrightarrow & t
\end{array}
$$

$$
\begin{array}{rcl}
t\,m_0 & \longrightarrow & t\,m_1 \\
f\,m_0 & \longrightarrow & f\,m_2 \\
b\,m_1 & \longrightarrow & b\langle b, f_0\rangle\,m_0 \\
b\,m_2 & \longrightarrow & \epsilon
\end{array}
$$

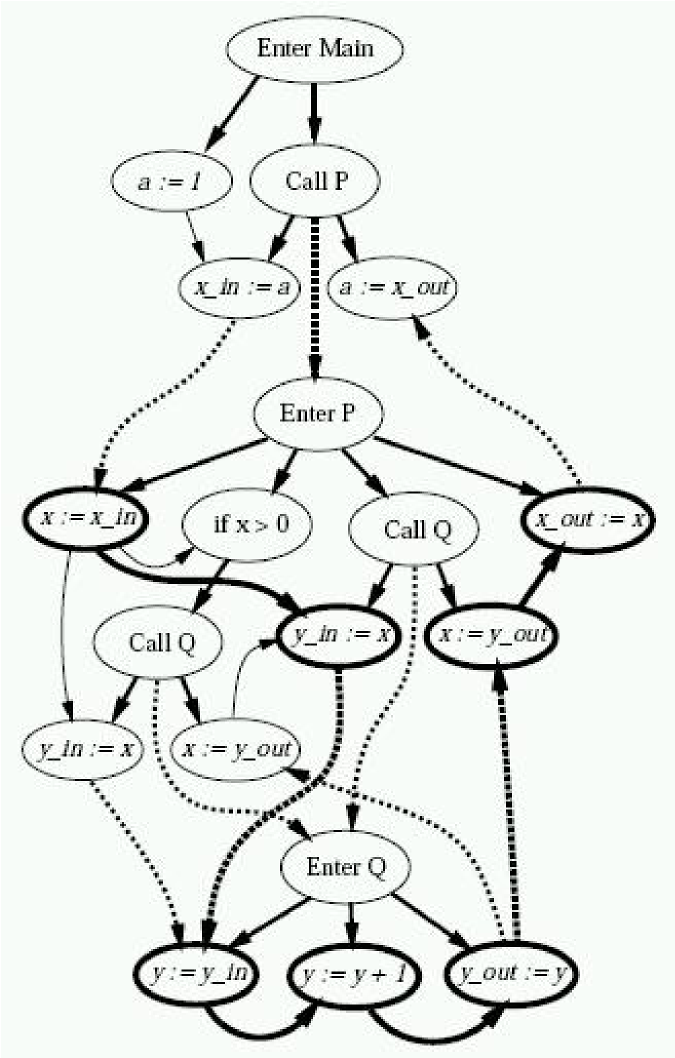
(b and ℓ stand for both t and f)

Comparison with Horwitz, Reps, and Binkley 90

procedure $Q(y)$
q_0: $y := y + 1$
q_1: **return**

procedure $P(x)$
p_0: **if** $x > 0$ **then**
p_1: **call** $Q(x)$
fi;
p_2: **call** $Q(x)$
p_3: **return**

procedure *Main*();
local a
m_0: $a := 1$
m_1: **call** $P(a)$
m_2: **return**



$$\langle y, q_0\rangle \rightarrow \langle y+1, q_1\rangle$$
$$\langle y, q_1\rangle \rightarrow \epsilon$$

$$\langle x^+, p_0\rangle \rightarrow \langle x^+, p_1\rangle$$
$$\langle x^-, p_0\rangle \rightarrow \langle x^+, p_2\rangle$$
$$\langle x, p_1\rangle \rightarrow \langle x, q_0\rangle\langle x, p_2\rangle$$
$$\langle x, p_2\rangle \rightarrow \langle x, q_0\rangle\langle x, p_3\rangle$$
$$\langle x, p_3\rangle \rightarrow \epsilon$$

$$\langle a, m_0\rangle \rightarrow \langle 1, m_1\rangle$$
$$\langle a, m_1\rangle \rightarrow \langle a, p_0\rangle\langle a, m_2\rangle$$
$$\langle a, m_2\rangle \rightarrow \epsilon$$

Prefix string rewriting. From theory . . .

First studied by Büchi in 64 under the name regular canonical systems as a variant of semi-Thue systems.

Theorem: Given an effectively regular (possibly infinite) set S of strings, the sets $\mathit{pre}^*(S)$ and $\mathit{post}^*(S)$ are also effectively regular.

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Polynomial algorithms by Bouajjani, E., Maler and Finkel, Willems, Wolper in 97.

- Saturation algorithms: the automata for $pre^*(S)$ and $post^*(S)$ are essentially obtained by adding transitions to the automaton for S.
  (Algorithms for similar models by Alur, Etessami, Yannakakis, and Benedikt, Godefroid, Reps and . . .)

. . . to applications

Efficient algorithms by E., Hansel, Rossmanith and Schwoon in 00.

Theorem (informal): Let Σ, R be the alphabet and set of rules of a 2-normalized prefix-rewriting system system and let A be a “small” NFA over Σ.

An NFA for $post^*(L(A))$ can be constructed in $O(|\Sigma||R|^2)$ time and space.

An NFA for $pre^*(L(A))$ can be constructed in $O(|\Sigma|^2|R|)$ time and $O(|\Sigma||R|)$ space.

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"Model Checking an Entire Linux Distribution for Security Violations"
by Schwarz et al. at ACSAC 05.

Büchi did it

Moshe Vardi, 25MC Workshop:

Büchi automata, introduced by Büchi in the early 60s to solve problems in second-order number theory, have been translated, unlikely as it may seem, into effective algorithms for model checking tools.

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This talk:

Regular canonical systems, introduced by Büchi in the early 60s because he liked them, have been translated, unlikely as it may seem, into effective algorithms for software model checking tools.

A rewriting model of multithreaded while-programs

Communication through global variables.

State determined by: $\{\, g, (\ell_0, n_0), (\ell_1, n_1) \ldots (\ell_k, n_k) \,\}$ where

- g is a valuation of the global variables,
- ℓ_i is a valuation of the local variables of the i-th thread, and
- n_i is the value of the program pointer of the i-th thread.

Modelled as a multiset

$$g \parallel \langle \ell_0, n_0 \rangle \parallel \langle \ell_1, n_1 \rangle \parallel \ldots \parallel \langle \ell_k, n_k \rangle$$

Instructions modelled as multiset-rewriting rules, e.g.

$$t\, f \parallel m_0 \rightarrow f\, f \parallel m_1 \parallel \langle f, p_0 \rangle$$

Multiset rewriting, or rewriting modulo assoc. and comm. of $\parallel$.

An example

thread $p()$
p_0: **if** ? **then**
p_1: $b := \mathrm{true}$
else
p_2: $b := \mathrm{false}$
fi;
p_3: **end**

$$
\begin{array}{lcl}
b \parallel p_0 & \rightarrow & b \parallel p_1 \\
b \parallel p_0 & \rightarrow & b \parallel p_2 \\
b \parallel p_1 & \rightarrow & t \parallel p_3 \\
b \parallel p_2 & \rightarrow & f \parallel p_3 \\
b \parallel p_3 & \rightarrow & \epsilon
\end{array}
$$

thread $main()$
global b
m_0: **while** b **do**
m_1: **fork** $p()$
od;
m_2: **end**

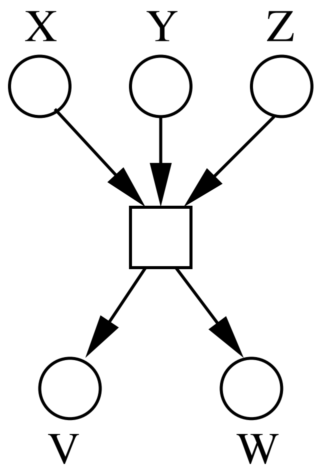
$$
\begin{array}{lcl}
t \parallel m_0 & \rightarrow & t \parallel m_1 \\
f \parallel m_0 & \rightarrow & f \parallel m_2 \\
b \parallel m_1 & \rightarrow & b \parallel m_0 \parallel p_0 \\
b \parallel m_2 & \rightarrow & \epsilon
\end{array}
$$

Multiset rewriting

Theorem [Mayr, Kosaraju, Lipton, 80s]: The reachability problem for multiset-rewriting is decidable but EXPSPACE-hard.

- Equivalent to the reachability problem for Petri nets.
- A place for each alphabet letter.
- A Petri net transition for each rewrite rule.

$$X \parallel Y \parallel Z \longrightarrow V \parallel W$$



Algorithms (not only proofs) quite complicated.

Negative results for $pre^*(\{s\})$ and $post^*(\{s\})$.

Symbolic reachability for *pre** and upward-closed sets

Upward-closed set: if some multiset t belongs to the set, then $t \parallel t'$ also belongs to the set for every t'.

Finitely representable e.g. by the its of minimal elements.

Upward-closed sets capture properties that can be decided by inspecting a bounded number of threads (e.g. mutual exclusion).

Theorem [Abdulla et al. 96]: Given a multiset-rewriting system and an upward-closed set of states S, the set $pre^*(S)$ is upward-closed and effectively constructible.

- Very simple algorithm: compute $pre(S), pre^2(S), pre^3(S) \ldots$.

Extensions applied to multithreaded Java [Delzanno, Raskin, Van Begin 04].

Monadic multiset-rewriting

Monadic rules $\equiv$ no global variables $\equiv$ no communication

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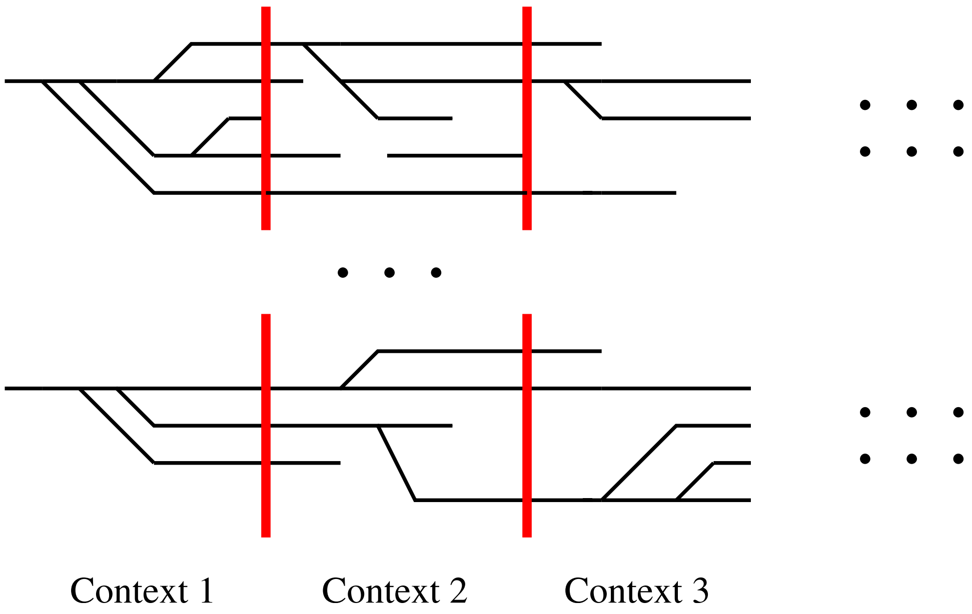
They are good for underapproximations [Qadeer and Rehof 05]



Reachability

Theorem [Huyhn 85, E.95]: The reachability problem for monadic multiset-rewrite systems is NP-complete.

- Membership in NP not completely trivial.
- Hardness very easy, reduction from SAT:

  A thread for each variable x_i that (a) nondeterministically chooses $l_i \in \{x_i, \overline{x}_i\}$ and (b) spawns a clause thread for each clause satisfied by l_i.

  The thread for a clause does nothing and terminates.

  Formula satisfiable iff there is state at which one thread per clause is active.

Symbolic reachability for semi-linear sets

Semi-linear sets usually defined as subsets of $\mathbb{N}^n$.

- Finite union of linear sets.
- $\{r + \lambda_1 p_1 + \ldots + \lambda_n p_n \mid \lambda_1, \ldots, \lambda_n \in \mathbb{N}\}$.

Language interpretation: "commutative closure" of the regular languages.

Similar properties to regular languages: closure under boolean operations, decidable (but no longer polynomial) membership problem, etc.

Theorem [E.95]: Given a monadic multiset-rewriting system and a semi-linear set of states S, the sets $post^*(S)$ and $pre^*(S)$ are semi-linear and effectively constructible.

Multithreaded procedural programs

Two-counter machines can be simulated by a program with two recursive threads communicating over two global (boolean) variables:

- Tops of the recursion stacks contains two copies of the machine's control point.
- Depths the two recursion stacks model the values of the counters.
- Calls and returns model increasing and decrementing the counters.
- One variable to ensure alternation of moves.
- One variable to keep the two copies of the control point "synchronized".

If communication takes place by rendezvous the two variables are no longer needed: programs without variables are still Turing powerful.

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Communication-free case: [Bouajjani, Müller-Olm and Touili 05]

Communication through nested locks: [Kahlon and Gupta 06]

A rewriting model for the communication-free case

State of a multithreaded procedural program without global variables: multiset $\{s_1, s_2 \ldots, s_k\}$ of states of procedural programs, where

$$s_i = (\ell_{i0}, n_{i0})\,(\ell_{i1}, n_{i1}) \ldots (\ell_{im}, n_{im})$$

Modelled as a string $\# w_k \# w_{k-1} \# \ldots \# w_1$ where

$$w_i = \langle \ell_{i0}, n_{i0} \rangle \, \langle \ell_{i1}, n_{i1} \rangle \, \ldots \, \langle \ell_{im}, n_{im} \rangle$$

Instructions modelled as string-rewriting rules. A new thread is inserted to the left of its creator, e.g.

$$\# \, \langle b, m_1 \rangle \longrightarrow \# \, p_0 \, \# \, \langle f, m_3 \rangle$$

Threads “in the middle” of the string should also be able to “move”: back to ordinary rewriting

$$\frac{u \longrightarrow w}{v_1 \, u \, v_2 \xrightarrow{r} v_1 \, w \, v_2}$$

An example

process $p()$;

p_0:	**if** (?) **then**		$\# p_0$	$\longrightarrow$	$\# p_1$
p_1:	**call** $p()$		$\# p_0$	$\longrightarrow$	$\# p_2$
	else		$\# p_1$	$\longrightarrow$	$\# p_0 \, p_3$
p_2:	**skip**		$\# p_2$	$\longrightarrow$	$\# p_3$
	fi;		$\# p_3$	$\longrightarrow$	$\#$
p_3:	**return**				

process $main()$

m_0:	**if** (?) **then**		$\# m_0$	$\longrightarrow$	$\# m_1$
m_1:	**fork** $p()$		$\# m_0$	$\longrightarrow$	$\# m_2$
	else		$\# m_1$	$\longrightarrow$	$\# p_0 \# m_3$
m_2:	**call** $main()$		$\# m_2$	$\longrightarrow$	$\# m_0 \, m_3$
	fi;		$\# m_3$	$\longrightarrow$	$\# \epsilon$
m_3:	**return**		$\# \#$	$\longrightarrow$	$\#$

Analysis

Theorem [BMOT05]: For every effectively regular set S of states, the set $pre^*(S)$ is regular and a finite-state automaton recognizing it can be effectively constructed in polynomial time.

- Similar to pre^* for monadic string-rewriting [Book and Otto 93].

Theorem [BMOT05]: For every effectively context-free set S of states, the set $post^*(S)$ is context-free and a pushdown automaton recognizing it can be effectively constructed in polynomial time.

Counterexample to regularity: P that spawns a copy of Q and calls itself.

The number of threads is equal to the depth of the recursion.

Reachability set: $\{(\#q)^n \#p^{(n+1)} \mid n \geq 0\}$.

Cobegin-coend sections

Difference with threads: implicit synchronization induced by the coend.

- “Threads have to wait for its siblings to terminate.”
- Corresponds to calling procedures in parallel.

Rewriting model only works well for the communication-free (monadic) case.

States modelled as terms with both $\|$ and $\cdot$ as infix operators e.g

$$(\langle t, p_1 \rangle \parallel q_2) \cdot \langle t\, f, m_1 \rangle$$

Rewriting modulo assoc. of $\cdot$ and assoc. and comm. of $\|$.

This model is called monadic process rewrite systems (monadic PRS) [Mayr 00].

Analysis

Symbolic reachability with commutative hedge automata (CHA) [Lugiez 03].

Theorem [Bouajjani and Touili 05]: Given a monadic PRS, for every CHA-definable set of terms T, the sets $post^*(T)$ and $pre^*(T)$ are CHA-definable and effectively constructible.

Weaker approach: construct not the sets $post^*(T)$ or $pre^*(T)$ themselves, but representatives w.r.t. the equational theory.

Sufficient for control reachability problems.

Theorem [Lugiez and Schnoebelen 98, E. and Podelski 00]:
Let R be a monadic PRS and let A be a bottom-up tree automaton.
One can construct in $O(|R| \cdot |A|)$ time bottom-up tree automata recognizing a set of representatives of $post^*(L(A))$ and $pre^*(L(A))$.

Conclusions

Rewriting concepts can be used to give elegant semantics to programming languages.

- String/multiset rewriting correspond to sequential/parallel computation.
- Monadic/non-monadic rewriting correspond to absence or presence of communication.
- Rewriting modulo useful for combining concurrency and procedures.

Symbolic reachability is the key problem to solve.

Comparison with process algebras:

- Process algebras have a notion of hiding or encapsulation.
- Rewriting much closer to automata theory $\longrightarrow$ algorithms.