A False History of True Concurrency

Javier Esparza

Software Reliability and Security Group
Institute for Formal Methods in Computer Science
University of Stuttgart
The early 60s
Abstract Models of Computation in the early 60s

Lambda calculus (Church 35)

Turing machines (Turing 36)

Finite automata (Kleene 56, Moore 56, Mealy 56, Scott and Rabin 59)

Pushdown automata (Oettinger 61, Chomsky 62, Evey 63, Schutzenberger 63)
Semantics: executions

**States:** current configurations of the machine

One or more initial states

Possibly some distinguished final states

**Transitions:** moves between configurations

- Lambda calculus: $$(\lambda x.xx)(\lambda y.y) \rightarrow (\lambda y.y)(\lambda z.z)$$
- Turing machine: $$0010q_1011 \rightarrow 001q_201011$$
- Finite automaton: $$q_1 \xrightarrow{a} q_2$$
- Pushdown automaton: $$(q_1, XYYZ) \xrightarrow{a} (q_2, XXYXYZ)$$

**Executions:** alternating sequences of states and transitions
Abstract machines are implemented as physical systems.
Abstract machines are implemented as
  can simulate physical systems

SIMULA project (Nygaard and Dahl) started in 1962
Abstract machines are implemented as physical systems can simulate physical systems

SIMULA project (Nygaard and Dahl) started in 1962

A plane (physical system)
Abstract machines are implemented as physical systems can simulate.

SIMULA project (Nygaard and Dahl) started in 1962.

A plane (physical system) can be simulated by a plane simulator (abstract machine).
Abstract machines are implemented as physical systems can simulate physical systems

SIMULA project (Nygaard and Dahl) started in 1962

A plane (physical system)

can be simulated by a plane simulator (abstract machine)

which can be implemented in a video console (physical system)
Abstract machines are implemented as physical systems can simulate physical systems

SIMULA project (Nygaard and Dahl) started in 1962

A plane (physical system) can be simulated by a plane simulator (abstract machine) which can be implemented in a video console (physical system) which can be simulated by a hardware simulator (abstract machine)
Abstract machines are implemented as physical systems. SIMULA project (Nygaard and Dahl) started in 1962.

A plane (physical system) can be simulated by a plane simulator (abstract machine) which can be implemented in a video console (physical system) which can be simulated by a hardware simulator (abstract machine) which is implemented in a PC (physical system) . . .
C.A. Petri points out a discrepancy between how *Theoretical Physics* and *Theoretical Computer Science* described systems in 1962:

*Theoretical Physics* describes systems as a collection of interacting particles (subsystems), without a notion of global clock or simultaneity.

*Theoretical Computer Science* describes systems as sequential virtual machines going through a temporally ordered sequence of global states.

Petri’s question:

Which kind of abstract machine should be used to describe the physical implementation of a Turing machine?
A graphical representation of interacting finite automata:
Petri Nets

A graphical representation of interacting finite automata:
Petri Nets

A graphical representation of interacting finite automata:
A graphical representation of interacting finite automata:
Petri Nets

A graphical representation of interacting finite automata:
Petri Nets

A graphical representation of interacting finite automata:
Petri Nets

A graphical representation of interacting finite automata:
A graphical representation of interacting finite automata:
The interleaving semantics of Petri nets

An execution semantics

State: marking (distribution of tokens)

Transitions: $M \xrightarrow{a} M'$

Executions: $M_0 \xrightarrow{a_0} M_1 \xrightarrow{a_1} M_2 \ldots$
\[
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]
\[
\begin{bmatrix}
s \\ r \\ q
\end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]
\[
\begin{align*}
    s &= \begin{bmatrix} 0 \\ 0 \\
    r &= \begin{bmatrix} 0 \\ 0 \\
    q &= \begin{bmatrix} 0 \\
\end{align*}
\]
The diagram consists of nodes labeled as $s_0$, $r_0$, $q_0$, $s_1$, $r_1$, $q_1$, and $b$. The edges between these nodes are as follows:

- $s_0$ to $s_1$
- $s_0$ to $b$
- $r_0$ to $c$
- $r_0$ to $d$
- $q_0$ to $e$
- $q_0$ to $d$
- $r_1$ to $d$
- $r_1$ to $q_1$
- $s_1$ to $a$
- $b$ to $c$
- $b$ to $d$

The transitions between states are defined by the following matrices:

- $s$:
  - $0$
- $r$:
  - $0$
- $q$:
  - $0$

The transition $c$ is from state $b$ to state $c$ with the following matrix:

- $c$:
  - $0$ to $1$
  - $0$ to $0$
\[
\begin{align*}
    s &: \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
    r &: \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
    q &: \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} & \xrightarrow{c} \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} & \begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix} & \xrightarrow{b} \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
\begin{pmatrix} s \\ r \\ q \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\
\begin{pmatrix} s \\ r \\ q \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\end{align*}
\]
\[
\begin{align*}
\mathbf{s} & \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
\mathbf{r} & \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\end{align*}
\]
s
r
q

\[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\]  \[\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\]  \[\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\]  \[\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\]  \[\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}\]
\[
\begin{align*}
\begin{array}{cccc}
 s & 0 & 0 & 0 \\
r & 0 & 1 & 0 \\
q & 0 & 0 & 1 \\
\end{array}
\begin{array}{cccc}
 0 & 0 & 1 & 0 \\
 1 & 0 & 0 & 1 \\
 0 & 0 & 1 & 0 \\
\end{array}
\end{align*}
\]
\[
\begin{align*}
\begin{bmatrix} s \\ r \\ q \end{bmatrix} & \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}
\end{align*}
\]
\[
\begin{align*}
&\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
\end{align*}
\]
\begin{array}{c}
s\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{d} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{e} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{a} \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\end{array}
\[
\begin{align*}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\overset{c}{\rightarrow}
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\overset{b}{\rightarrow}
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\overset{e}{\rightarrow}
\begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}
\overset{d}{\rightarrow}
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\overset{e}{\rightarrow}
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}
\overset{a}{\rightarrow}
\begin{bmatrix}
0 \\
1 \\
1
\end{bmatrix}
\end{align*}
\]
\[ \begin{align*}
  s & \xrightarrow{0} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & c & \xrightarrow{1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & b & \xrightarrow{1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & e & \xrightarrow{1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & d & \xrightarrow{1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & e & \xrightarrow{1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & a & \xrightarrow{0} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & b & \xrightarrow{1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\end{align*} \]
The true concurrency semantics of Petri nets

\[
\begin{array}{cccccc}
& a & b & c & d & e \\
& s_0 & & r_0 & & q_0 \\
& & s_1 & & r_1 & & q_1 \\
\end{array}
\]
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets

![Petri net diagram]

- States: $s_0$, $s_1$, $r_0$, $r_1$, $q_0$, $q_1$
- Transitions: $a$, $b$, $c$, $d$, $e$

1.0
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets

Diagram: A Petri net model with places and transitions labeled with letters a, b, c, d, e, s₀, s₁, r₀, r₁, q₀, q₁.
The true concurrency semantics of Petri nets

```
s_0
  \arrow{a}
  \arrow{b}
  \arrow{c}
  \arrow{d}
  \arrow{e}

s_1
  \arrow{r_0}
  \arrow{r_1}
  \arrow{q_0}
  \arrow{q_1}
```
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The true concurrency semantics of Petri nets
The interleaving thesis:

The total order assumption is a reasonable abstraction, adequate for practical purposes, and leading to nice mathematics.

The true concurrency thesis:

The total order assumption does not correspond to physical reality and leads to awkward representations of simple phenomena.
The standard example

In interleaving semantics, a system composed of $n$ independent components

![Diagram](image)

has $n!$ different executions

The automaton accepting them has $2^n$ states

In true concurrency semantics, it has only one nonsequential execution
30 years of concurrency theory in one slide

Interleaving semantics

- Petri nets/vector addition systems (Hack, Kosaraju, Mayr, ... 70s–80s)
- Process algebras (Milner 80)
- Temporal logic (Pnueli 77)
- Model checking (Clarke, Emerson, Queille, Sifakis 81)

True concurrency

- Axiomatic concurrency theory (Best, Fernandez, Petri ... 70s–80s)
- Trace theory (formal languages, Mazurkiewicz 77)
- Event structures (domain theory, Winskel 80)
- True concurrency semantics of process algebras (Montanari, Winskel ... 80s)
- Partial order model checking (E., Godefroid, Peled, Wolper 90s)
- Temporal logics for true concurrency (90s–00s)
Temporal Logics for True Concurrency
LTL: a temporal logic for sequential runs

Syntax: \( \varphi ::= \text{true} \mid \neg \varphi \mid \varphi \lor \psi \mid \langle a \rangle \varphi \mid F \varphi \mid G \varphi \mid \varphi \mid \varphi U \psi \)

where \( a \) belongs to a finite set \( Act \) of actions

Formulas interpreted on \text{runs} over \( Act \) : elements of \( Act^\omega \)

Semantics:
\( \rho \models \langle a \rangle \varphi \)  if  \( \rho = a \rho' \) and  \( \rho' \models \varphi \)
\( \rho \models F \varphi \)  if  \( \rho' \models \varphi \) for some suffix \( \rho' \) of \( \rho \)
\( \rho \models G \varphi \)  if  \( \rho' \models \varphi \) for all suffixes \( \rho' \) of \( \rho \)
\( \rho \models \varphi U \psi \)  if  \( \rho' \models \psi \) for some suffix \( \rho' \) of \( \rho \) and
\( \rho'' \models \phi \) for all suffixes \( \rho'' \) between \( \rho \) and \( \rho' \)
Examples

Invariants: \( G \varphi \)

\[
G(\langle a_1 \rangle \text{true} \lor \ldots \lor \langle a_n \rangle \text{true})
\]

deadlock freedom

Response, recurrence: \( G(\varphi \Rightarrow F \psi) \)

\[
G(\langle \text{request} \rangle \text{true} \Rightarrow F \langle \text{taken} \rangle \text{true})
\]

eventual access to a resource

\[
G F \langle \text{active} \rangle \text{true}
\]

process remains active

Reactivity: \( G F \varphi \Rightarrow G F \psi \)

\[
G F(\langle \text{request}_1 \rangle \text{true} \land \neg \langle \text{taken}_2 \rangle \text{true}) \Rightarrow
\]

strong fairness

\[
G F \langle \text{taken}_1 \rangle \text{true}
\]
Fix a system $S$ with action alphabet $Act$

We use LTL over $Act$ to specify properties of $S$

$S$ satisfies a formula $\varphi$, denoted $S \models_{\text{LTL}} \varphi$, if all its executions satisfy $\varphi$

The model checking problem: given $S$ and $\varphi$, decide if $T \models_{\text{LTL}} \varphi$
Kamp’s theorem: LTL has the same expressivity as the first-order theory of runs

$$FO(Act) ::= R_a(x) \mid x \leq y \mid \neg \varphi \mid \phi \lor \psi \mid \exists x. \varphi$$

The satisfiability and model-checking problems are PSPACE-complete

Construct a Büchi automaton of size $2^{O(|\varphi|)}$ accepting the runs satisfying $\varphi$

(Intersect it with an automaton accepting all executions of $S$)

Check for emptiness
Fix a distributed alphabet $Act = (Act_1, \ldots, Act_n)$ of actions

$a \in Act_i \cap Act_j$ means that $a$ is a joint action of the $i$-th and the $j$-th agents

Denote by $NS(Act)$ the set of nonsequential runs over $Act$

- The line of the $i$-th component only contains actions of $Act_i$
- Joint actions ‘synchronize’ the lines of its agents

Example: $Act_1 = \{a, b\}$, $Act_2 = \{a, d\}$, $Act_3 = \{c, d\}$
LTL now interpreted on $\text{NS}(\text{Act})$

Same semantics as LTL, but with a new notion of suffix

Prefixes of a nonsequential run:

- All minimal places belong to the prefix
- If a transition belongs to the prefix, so do its output places

Suffixes: ‘complements’ of prefixes
Semantics:

\[ \nu \models \langle a \rangle \varphi \quad \text{if} \quad \nu \text{ can be extended by an } a\text{-labelled event } e \]
\[ \text{such that } \nu \cup \{e\} \models \varphi \]

\[ \nu \models F \varphi \quad \text{if} \quad \nu \models \varphi \text{ for some suffix } \nu' \text{ of } \nu \]

\[ \nu \models G \varphi \quad \text{if} \quad \nu \models \varphi \text{ for all suffixes } \nu' \text{ of } \nu \]

\[ \nu \models \varphi \ U \psi \quad \text{if} \quad \nu' \models \psi \text{ for some suffix } \nu' \text{ of } \rho \text{ and} \]
\[ \nu'' \models \phi \text{ for all suffixes } \nu'' \text{ between } \nu \text{ and } \nu' \]
\[ \langle a \rangle \text{true} \land \langle b \rangle \text{true} \text{ unsatisfiable in LTL, satisfiable in LTrL} \]

Example satisfies \( GF(\langle a \rangle \langle e \rangle \text{true}) \) as a formula of LTrL, but not as a formula of LTL

Better specification of ‘reset states’
Fix a system $S = (S_1, \ldots, S_n)$ with distributed alphabet $Act$,

We use LTL over $Act$ to specify properties of $S$.

$S$ satisfies a formula $\varphi$ if all its nonsequential executions satisfy $\varphi$.

The model checking problem: given $S$ and $\varphi$, decide if $T \models_{\text{LTL}} \varphi$. 

Results on LTrL

Expressively complete for the first order theory of nonsequential runs

\[ FO(Act) ::= R_a(x) \mid x \leq y \mid \neg \phi \mid \phi \lor \psi \mid \exists x. \phi \]

\[ \leq \text{ interpreted on partial orders over } Act \text{ that respect the distribution} \]

Thiagarajan and Walukiewicz, LICS '97: LTrL + P_a modalities

Diekert and Gastin, CSL '99: LTrL + X_A^* modalities

Diekert and Gastin, ICALP '00

Non-elementary satisfiability and model checking problems (Walukiewicz ICALP'98)

LTL allows to specify \(2^n\)-counters with formulas of length \(O(n)\)

LTrL allows to specify \(\text{Tower}(2, n)\)-counters with formulas of length \(O(n)\)
Local state of a component: ‘a position in its time line’

Identify a local state (place) with the prefix determined by all its predecessors

A component always has complete information about its causal past

Components exchange full information when they synchronize

Example: \( \text{Act}_1 = \{a, b\} \), \( \text{Act}_2 = \{a, d\} \), \( \text{Act}_3 = \{c, d\} \)

\[
\begin{array}{c}
\text{1} & \text{2} & \text{3} \\
\quad & \quad & \quad \\
da & d & c \\
\quad & \quad & \quad \\
\quad & \quad & \quad \\
b & \quad & \quad \\
\quad & \quad & \quad \\
\end{array}
\]
Syntax: $\varphi ::= \textbf{true} \mid \neg \varphi \mid \varphi \land \psi \mid \langle a \rangle^i \varphi \mid \mathbf{F}^i \varphi \mid \mathbf{G}^i \varphi \mid \varphi \mathbf{U}^i \psi$

where $a$ is an action and $1 \leq i \leq n$

Semantics:

$\langle a \rangle^i \varphi$ means $\varphi$ holds at $i$’s next local state

$\mathbf{F}^i \varphi$ means $\varphi$ holds eventually at $i$’s timeline

$\mathbf{G}^i \varphi$ means $\varphi$ holds always along $i$’s timeline

$\varphi \mathbf{U}^i \psi$ means $\varphi$ holds until $\psi$ holds along $i$’s timeline

Gets interesting when formulas use several indices: $\mathbf{F}^1 \mathbf{G}^2 \langle a \rangle^1 \textbf{true}$
Results on Local LTL

PSPACE-complete satisfiability and model checking problems (Thiagarajan, LICS ’94)

Generalization of the Büchi automaton construction

Technical problem: to keep track of the latest gossip

Expressiveness still unclear

Completeness results for logics with similar flavours (Gastin, Mukund, Kumar MFCS ’03)

Difficult to specify with
Outlook

Interleaving semantics ‘default’ semantics in practice

True concurrency brought in when interleaving ‘fails’

Challenge: automatic synthesis of distributed systems

Interleaving logics insensitive to distribution requirements

Maybe the ‘killer application’ for true concurrency?