Verification of Population Protocols

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Joint work with Pierre Ganty, Jérôme Leroux,
and Rupak Majumdar

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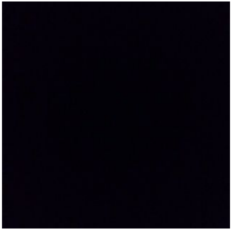

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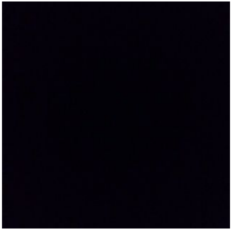

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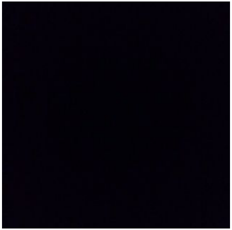

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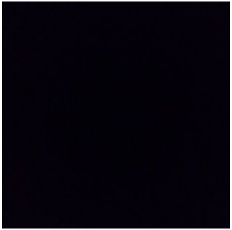

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Random bumps guarantee eventual consensus

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Syntax

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Intuition:

if	$(q_1, q_2) \mapsto (q_1', q_2') \in \Delta$ **and**
	two agents in states q_1 and q_2 "meet",
then	the agents can interact and
	change their states to q_1', q_2'.

Assumption: at least one interaction for each (q_1, q_2)

Semantics

Configuration: mapping $C\colon Q \to \mathbb{N}$, where $C(q)$ is the current number of agents in state q.

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Fair Execution: **if** C appears infinitely often and $C \to C'$
then C' appears infinitely often.

(Fairness constraint approximating random scheduler)

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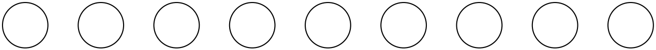
- A PP-scheme (Q, Δ)

Q: 

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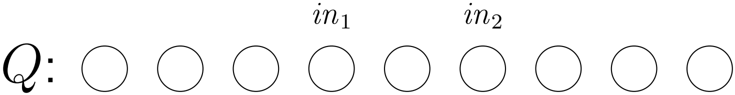

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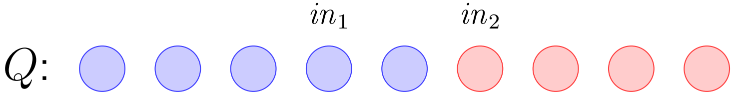

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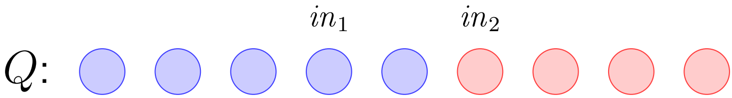
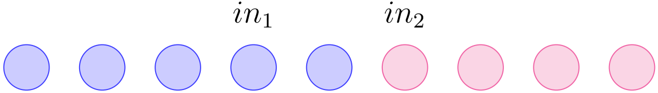


A fair execution stabilizes to $b \in \{\mathrm{true}, \mathrm{false}\}$ if from some point on every agent stays within the b-states. ("All agents agree on b").

Computing with PPs

A PP computes the value b for input $(n_1, \ldots, n_k)$ if every fair execution starting at the configuration



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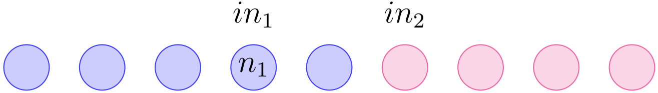

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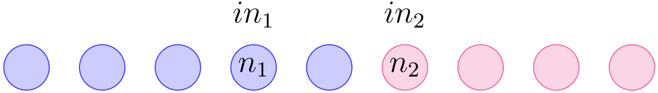

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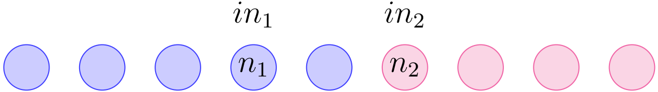

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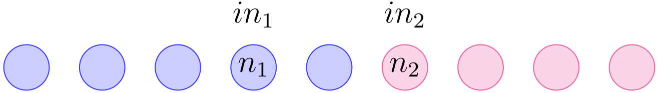

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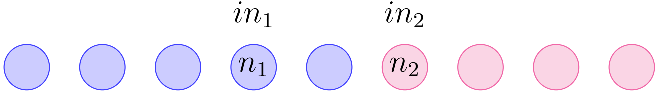

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A PP computes $P \colon \mathbb{N}^n \to \{\mathrm{true}, \mathrm{false}\}$ if it computes $P(n_1, \ldots, n_k)$ for every input $(n_1, \ldots, n_k)$

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- Probabilistic PPs (Angluin *et al.* 2004-2006, Chatzigiannakis and Spirakis, 2008)
- Fault-tolerant PPs (Delporte-Gallet *et al.* 2006)
- Private computation in PPs (Delporte-Gallet *et al.* 2007)
- PPs with identifiers (Guerraoui *et al.* 2007)
- PPs with a leader (Angluin *et al.* 2008)
- Mediated PPs (Michail *et al.*, 2011)
- Trustful PPs (Bournez *et al.*, 2013)

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Well-specification problem: Given a protocol, decide if it is well-specified.

Correctness problem: Given a protocol and a Presburger predicate, decide if the protocol is well-specified and computes the predicate.

Verifying population protocols: Previous work

- Use model-checkers (SPIN, PRISM , . . .) to verify correctness for some inputs
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Theorem: The well-specification and correctness problems can be reduced to the reachability problem for Petri nets, and are thus decidable.

Theorem: The reachability problem for Petri nets can be reduced to the well-specification and correctness problems for PPs with leader.

From PPs to Petri nets

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PP	Net $+$ infinite family of initial markings

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Key Theorem: The set of bottom configurations of a PP is effectively Presburger.

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