Continuous-time Markov Decisions based on Partial Exploration

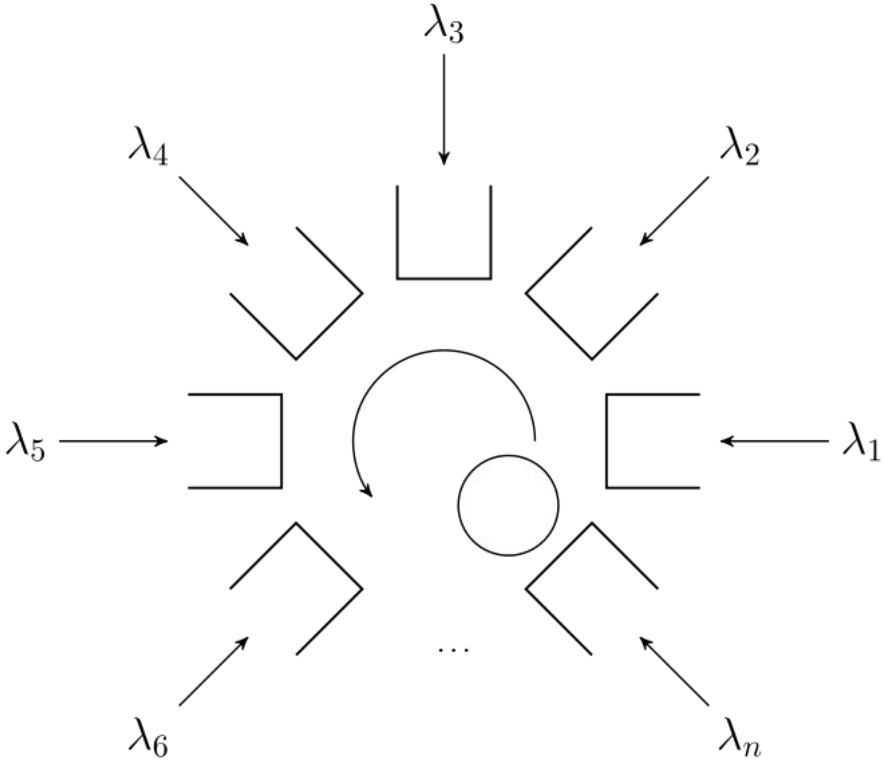
Pranav Ashok
Technical University of Munich

Highlights 2018, Berlin

Joint work with Yuliya Butkova[1], Holger Hermanns[1] and Jan Kretinsky[2]

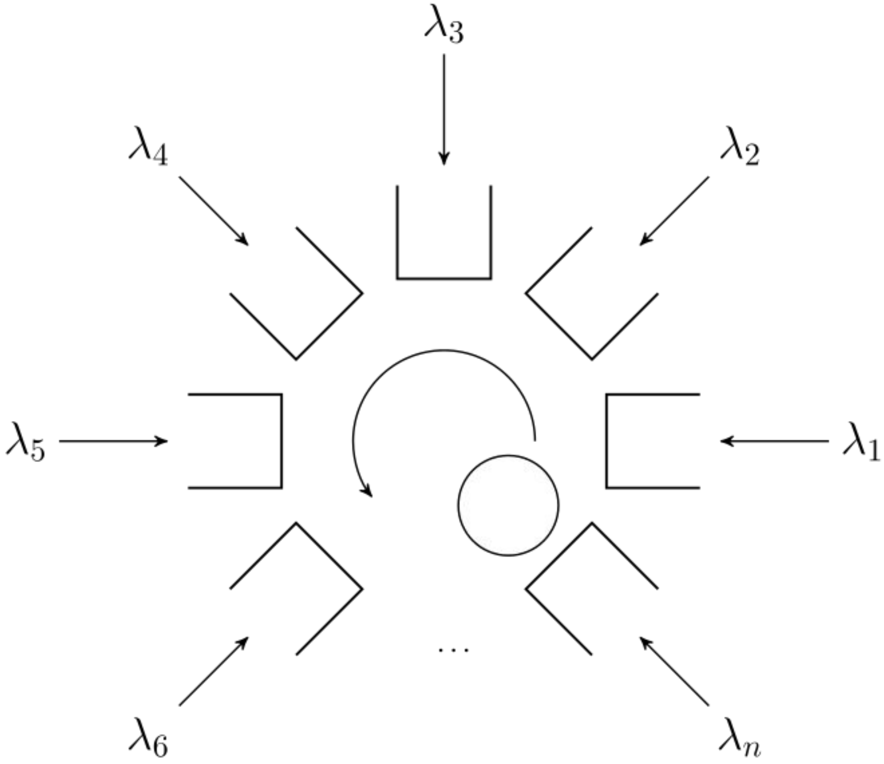
[1]*Saarland University, Germany*
[2]*Technical University of Munich, Germany*

Motivation



Motivation

- *n* students mail @ $\lambda_1, \lambda_2, ..., \lambda_n$ /day
- you pick a student's mail to process it
- **if** processed: remove from queue
- **else**: put it back into queue



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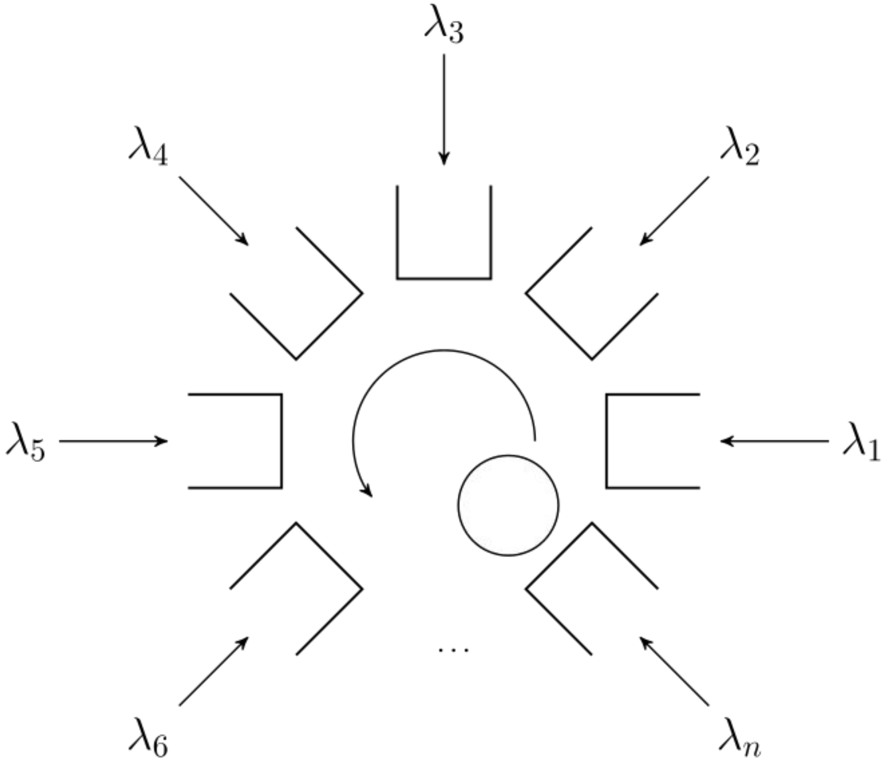
Q1: What is the max. prob. (over all strategies) that all queues are empty at the end of the week?



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- n students mail @ $\lambda_1, \lambda_2, \ldots, \lambda_n$ /day
- you pick a student's mail to process it
- **if** processed: remove from queue
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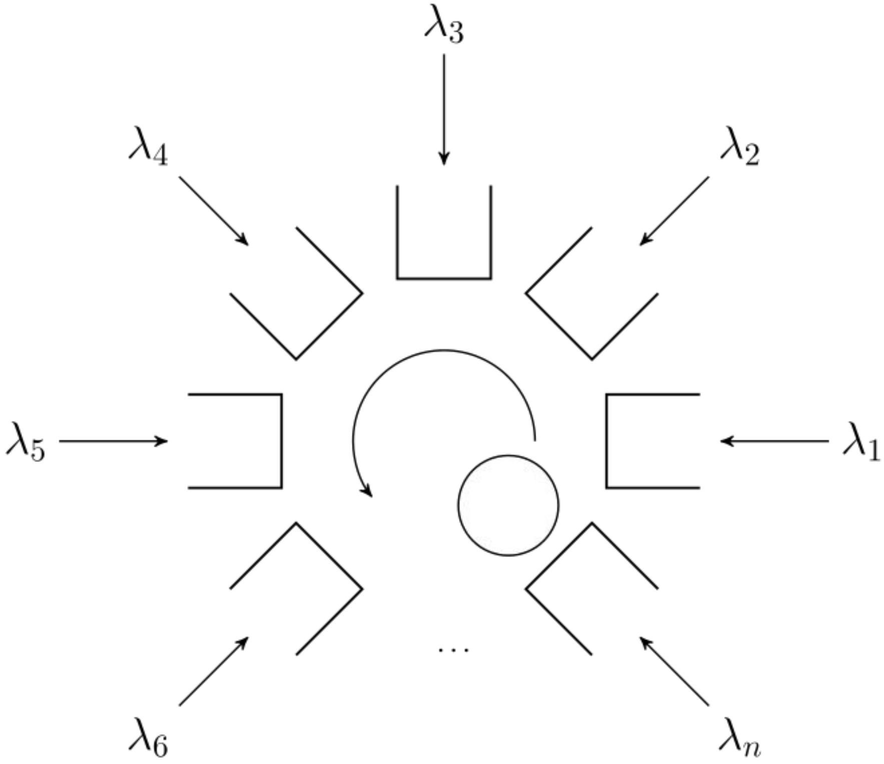
Q1: What is the max. prob. (over all strategies) that all queues are empty at the end of the week?

Q2: What is the min. prob. that student *X* quits your group after a semester?



Continuous-time Markov Decision Process (CTMDP)

Time-bounded Reachability

Maximal probability (over all strategies) of reaching some goal state within T time units

$$max_{\sigma}\ \mathbf{P}_{\sigma}(\Diamond^{\leq T} G)$$

Challenge

Existing reachability algorithms sometimes perform extremely bad in practice even though in **PTIME**

Can we improve them?

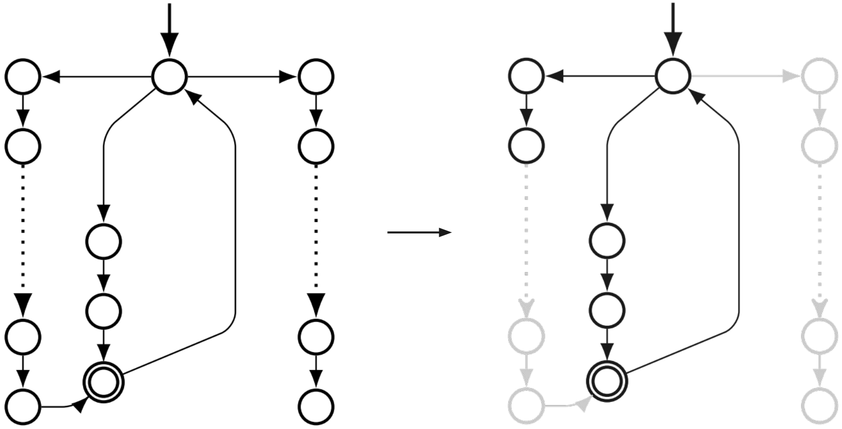
Contributions

- → Framework for time-bounded reachability (TBR) analysis
- → Use simulations to identify important parts of state-space
- → Instantiate with standard algorithms to show speed up

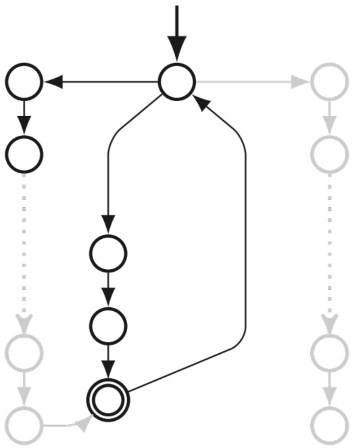
Key Idea

Partial Exploration Suffices

Not necessary to explore all states to get ε-optimal solution



What can we do with a partial model?



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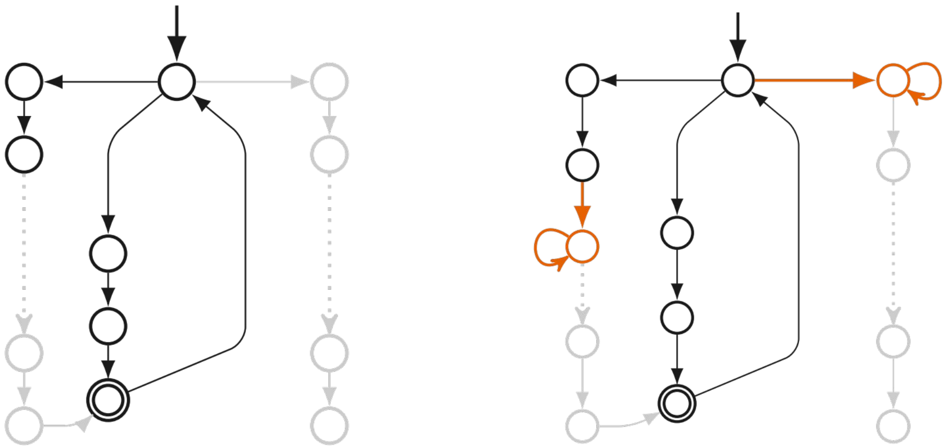

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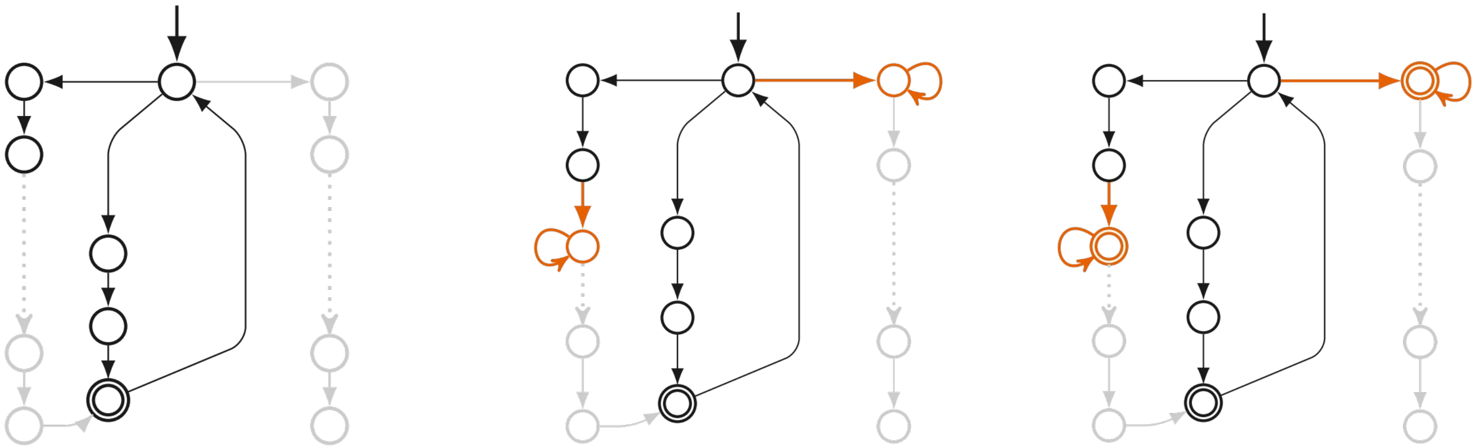

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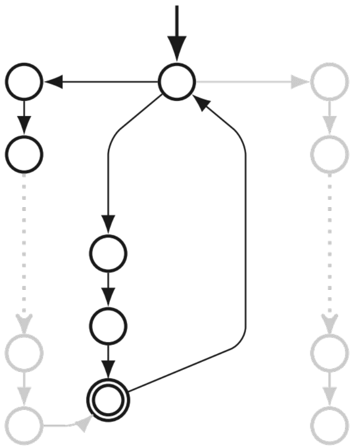
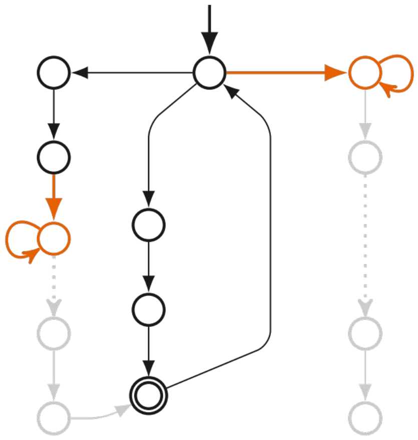
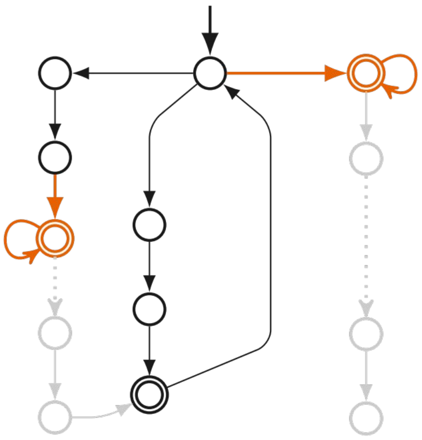


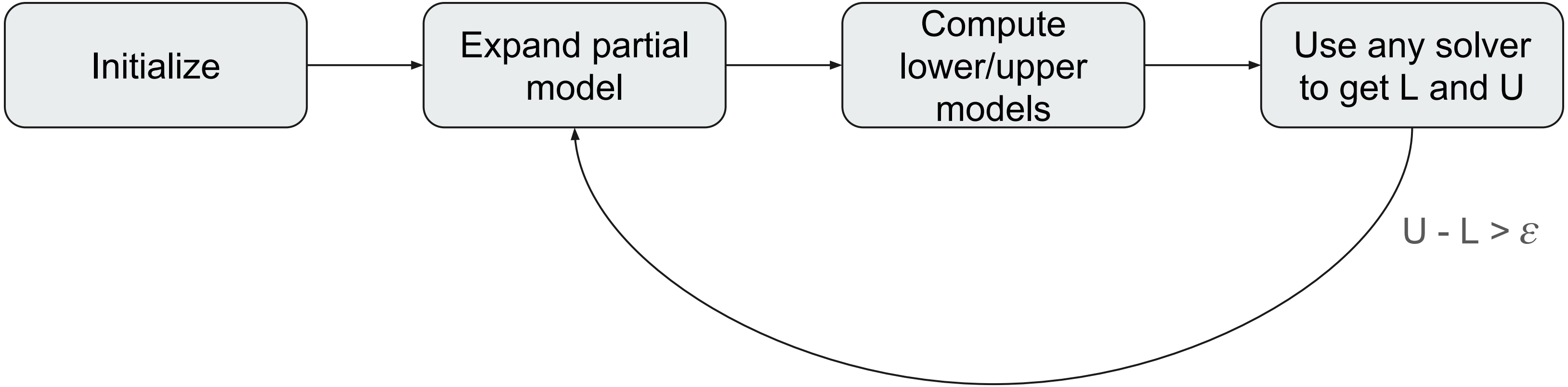


lower-bound model

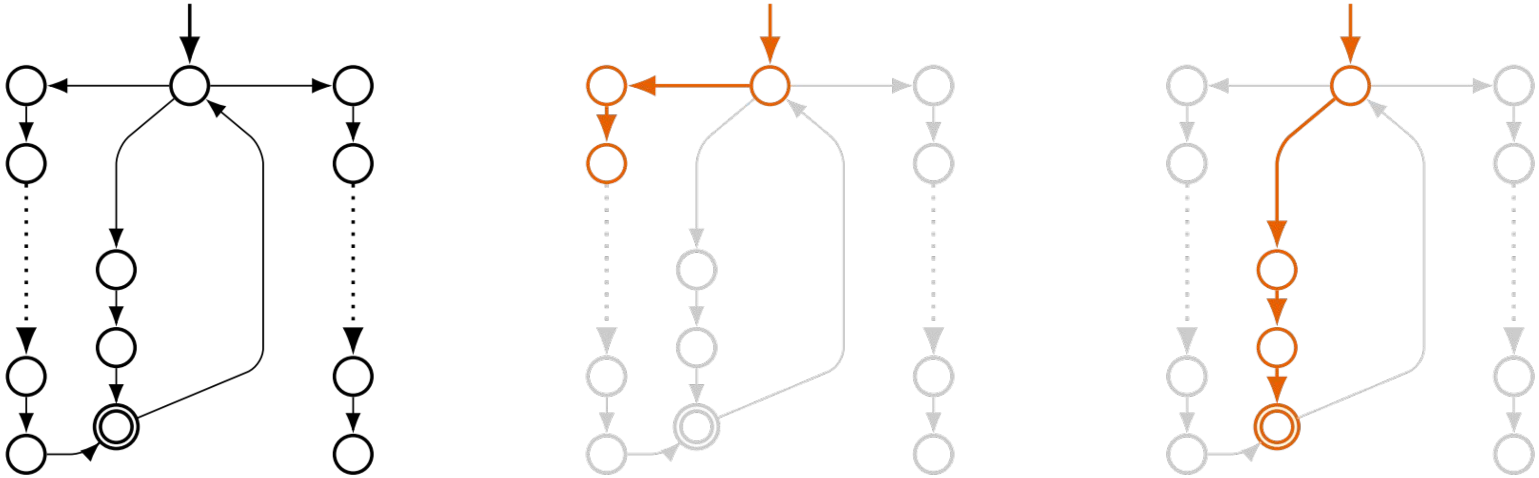


upper-bound model

The Framework



Partial model through simulations using σ_{sim}



Experiments I

Size of partial models

		Explored States	
Benchmark	States	by π_{sim}	%
`gfs-120`	1,479k	105	0.01
`ftwc-128`	597k	296	0.05
`erlang-10`^{6}`-10`	1,000k	559	0.06
`ps-4-24-one`	7,562k	23309	0.31
`sjs-2-7`	2k	2537	93.86
`ps-4-8-all`	119k	-	-

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Experiments II

Runtimes

Benchmark	States	Unif	Unif Improved	Adap	Adap Improved
erlang-10^6-10	1,000k	71	**1**	4	**1**
gfs-120	1,479k	-TO-	**2**	-TO-	**2**
ftwc-128	597k	251	**10**	114	**15**
ps-4-24-one	7,562k	507	-TO-	171	**105**
sjs-2-9	18k	6	99	2	-TO-
ps-4-8-all	119k	1475	-TO-	826	-TO-

TO $\longrightarrow$ > 1800s (30 min)

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Conclusion

- CTMDP TBR analysis **framework** based on partial exploration
- Partial model through simulations
- Usable with any TBR solver*
- Good on models with many *unimportant/improbable* states

*conditions apply, based on simulation strategy

Continuous-time Markov Decision Processes (CTMDP)

- **C** = (S, A, **R**, Goal)
- S: finite set of *states*; A: finite set of non-det choices
- Each choice ⟶ multiple *transitions*
- Each transition has a *rate* λ = **R**(s, a, s')
- Time *t* at which transition fired ⟵ *exp. dist* (λ)
- Next state chosen by a race between transitions