Lemma 1. Let \((N, M_0)\) be a T-system, and let 
\[ M_0 \xrightarrow{\sigma_1 \sigma_2} \] 
such that 
- \( t \in A(r_1) \) (\( A(r_i) \) is the set of tokens that occur in \( r_i \))
- \( A(r_1) \subseteq A(r_2) \)

Then \( M_0 \xrightarrow{\sigma_1 \sigma_2} \)

Proof. By induction on the length of \( r_2 \)

Base: \( |r_2| = 0 \)

Step: \( r_2 = \sigma_2^{-1} u \)

We have to show \( M_0 \xrightarrow{\sigma_1 \sigma_2} \) \( \equiv M_0 \xrightarrow{\sigma_1 \sigma_2} u \)

6. We show \( M_0 \xrightarrow{\sigma_1 \sigma_2} u \)

Two cases:
1) \( u \cap \sigma_1 \sigma_2 = \emptyset \)
   - If \( t \) is neither \( u \) in \( \sigma_1 \sigma_2 \), then it was already in \( \sigma_1 \) before
2) \( u \cap \sigma_1 \sigma_2 \neq \emptyset \)
   - Since \( u \cap A(r_1) \) and \( \sigma_1 \sigma_2 \subseteq A(r_1) \), we have \( u \cap A(r_1) \)
   - So after \( \sigma_1 \sigma_2 u \) there are at least 2 tokens in any place "between \( u \) and \( \sigma_1 \sigma_2 u \)" (because \( t \in A(r_1) \))
   - So \( u \) was already included before \( \sigma_1 \sigma_2 u \), and so \( \sigma_1 \sigma_2 u \) is forbidden.

6. We show \( M_0 \xrightarrow{\sigma_1 \sigma_2} u \)

By (a) we have \( M_0 \xrightarrow{\sigma_1 \sigma_2} u \)

By (b) we have \( |r_1| < |r_2| \) we get 
\[ M_0 \xrightarrow{\sigma_1 \sigma_2} u \]
Lemma 2. Let \((N, M_0)\) be a \(1\)-bounded T-system and let \(M_0 \xrightarrow{a} M, \ 1 \leq i \leq 3\). Then there is \(\sigma_1, \sigma_2\) with that:

(a) \(M_0 \xrightarrow{\sigma_1, \sigma_2} M\)

(b) no transition occurs more than once in \(\sigma_1\)

(c) \(A(\sigma_2) \subseteq A(\sigma_1)\)

**Proof.** For we prove the result with \(\leq\) instead of \(\in\).

By induction on \(|\sigma|\)

Base: \(|\sigma| = 1\). Take \(\sigma_1 = \sigma\) and \(\sigma_2 = \sigma\).

Step: \(\sigma = e + t, \ t \neq e\)

By IH: \(M_0 \xrightarrow{\sigma_1, \sigma_2} M\) where

- no transition occurs more than once in \(\sigma_1\)
- \(A(\sigma_2) \subseteq A(\sigma_1)\)

- If \(t \in A(\sigma_1)\) then take \(\sigma_1 = \sigma, \sigma_2 = \sigma + t\)

- If \(t \notin A(\sigma_1)\) then by Lemma 1

\[M_0 \xrightarrow{t, \sigma} M, \text{ and take } \sigma_1 = \sigma + t, \sigma_2 = \sigma\]

- Now assume \(A(\sigma_2) = \sigma(\sigma_2)\)

- If \(A(\sigma_1)\) contains every transition then replace \(\sigma_1\) by \(\sigma\) ![because \(N_0 \xrightarrow{\sigma_1}, \sigma_2)\)]

- If \(A(\sigma_1)\) does not contain any transition then the net is not T-reducible.

For: T-system \(N_0 \xrightarrow{\sigma_1}\)

- \(A(\sigma_1)\) does not contain any transition
- \(A(\sigma_2) = A(\sigma_1)\)
Theorem Let \((N, A)\) be a \(1\)-safe T-system and let \(M\) be reducible from \(N\).

Then \(M \rightarrow \Sigma \) with \(151 \leq \frac{n(n-1)}{2}\) where \(n\) is the number of transitions.

Proof By repeated application of Lemma 2 there is

\[ \Sigma_1 \Sigma_2 \cdots \Sigma_k \rightarrow \Sigma \]

- no transition occurs more than once in

\[ \Sigma_1 \] for any \(1 \leq i \leq k \)

- \( \Sigma_i \leq \Sigma_{i+1} \) for any \(1 \leq i \leq k-1 \)

It follows

\[ 151 \leq n \]
\[ 152 \leq n-1 \]
\[ 153 \leq n-2 \]

so

\[ |\Sigma_1 - \Sigma| \leq \sum_{i=1}^{n} (n-i) = \sum_{i=1}^{n} \frac{n(n-1)}{2} \]

If \((N, A)\) is bare, then

\[ |\Sigma_1 - \Sigma| \leq \sum_{i=1}^{n} \frac{n(n-1)}{2} \]