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Learning Workflow Petri Nets*

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Abstract. Workflow mining is the task of automatically producing a workflow model from a set of event logs recording sequences of workflow events; each sequence corresponds to a use case or workflow instance. Formal approaches to workflow mining assume that the event log is complete (contains enough information to infer the workflow) which is often not the case. We present a learning approach that relaxes this assumption: if the event log is incomplete, our learning algorithm automatically derives queries about the executability of some event sequences. If a teacher answers these queries, the algorithm is guaranteed to terminate with a correct model. We provide matching upper and lower bounds on the number of queries required by the algorithm, and report on the application of an implementation to some examples.

1. Introduction

Modern workflow management systems offer modelling capabilities to support business processes (see for example [1]). However, constructing a formal or semi-formal workflow model of an existing business process is a non-trivial task, and for this reason *workflow mining* has been extensively studied (see [4] for a survey). In this approach, information about the business processes is gathered in form of logs recording sequences of workflow events, where each sequence corresponds to a use case. The logs are then used to extract a formal model. Workflow mining techniques have been implemented in several systems, most prominently in the ProM tool [3], and successfully applied.

Most approaches to process mining use a combination of heuristics and formal techniques, like machine learning or neural networks, and do not offer any kind of guarantee about the relationship between

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the business process and the mined model. Formal approaches have been studied using *workflow graphs* [5] and *workflow nets* [2, 11] as formalisms. These approaches assume that the logs provide enough information to infer the model, i.e., that there is one single model compatible with them. In this case we call the logs *complete*. This is a strong assumption, which often fails to hold, for two reasons: first, the number of use cases may grow exponentially in the number of tasks of the process, and so may the size of a complete set of logs. Second, many processes have "corner cases": unusual process instances that rarely happen. A complete set of logs must contain at least one instance of each corner case.

In this paper we propose a learning technique to relax the completeness assumption on the set of logs. In this approach the model is produced by a *Learner* that may ask questions to a *Teacher*. The Learner can have initial knowledge in the form of an initial set of logs; if the log contains enough information to infer the model, the Learner produces it. If not, it iteratively produces *membership queries* of the form: Does the business process have an instance (a use case) starting with a given sequence of tasks? For instance, in the standard example of complaint processing (see Figure 1 and [2]), a membership query could have the form "Is there a use case in which first the complaint is registered and then immediately rejected?" The Teacher would answer no, because a decision on acceptance or rejection is made only after the customer has been sent a questionnaire.

Notice that the Learner does not guess the queries, they are automatically constructed by the learning algorithm. Under the assumption that the Teacher provides correct answers, the learning process is guaranteed to terminate with a correct model: a model whose executions coincide with the possible event sequences of the business process. In other words, we provide a formal framework with a correctness and completeness guarantee which only assumes the existence of the Teacher.

It could be objected that if a Teacher exists, then a workflow model must already exist, and there is no need to produce it. To see the flaw in this argument, observe that Teachers can be employees, databases of client records, etc., that have knowledge about the process, but usually lack the modelling expertise required to produce a formal model. Our learning algorithm only requires from the Teacher the low-level ability to recognize a given sequence of process actions as the initial sequence of process actions of some use case. To draw a partial analogy, witnesses of a crime can usually answer questions about the physical appearance of the criminal, but they are very rarely able to draw the criminal's portrait: this requires interaction with a police expert. This interaction can be seen as a learning process: the Teacher is the witness, and the Learner is the police expert. The teacher has knowledge about the criminal, but is unable to express it in the form of a portrait. The Learner has the expertise required to produce a portrait, but needs input from the Teacher.

Like [2, 16, 11, 17], we use *workflow nets*, introduced by van der Aalst, as formal model of business processes. Loosely speaking, a workflow net is a Petri net with a distinguished initial and final marking. Van der Aalst convincingly argues that well-formed business processes (an informal notion) correspond to *sound* workflow nets (a formal concept). A workflow net is *sound* [2] if it is live and bounded. In this paper we follow van der Aalst's ideas. Given a Teacher, we wish to learn a sound workflow net for the business process. It is easy to come up with a naive correct learning algorithm. However, a first naive complexity analysis yields that the number of queries necessary to learn a workflow net can be triple exponential in the number of tasks of the business process in the worst case. This seems to indicate that the approach is useless. However, we show how the special properties of sound workflow nets, together with a finer complexity analysis, lead to WNL, a new learning algorithm requiring a single exponential number of queries in the worst case. We also provide an exponential lower bound, showing that WNL is asymptotically optimal. Finally, in a number of experiments we show that despite the exponential worst-case complexity the algorithm is able to synthesize interesting workflows. Notice also that the complexity is analysed for the case in which no initial event log is provided, that is, the case in which all knowledge has to be extracted from the Teacher by asking membership queries.

Technically, the triple exponential complexity of the naive algorithm is a consequence of the following three facts:

(a) the size of a deterministic finite automaton (DFA) recognizing the language of a net with n transitions can be a priori double exponential in n;

(b) learning such a DFA using only membership queries requires exponentially many queries in the size of the DFA (follows from [7] and [19, 15]); and

(c) the algorithms of Darondeau et al. for synthesis of Petri nets from regular languages [8] are exponential in the size of the DFA.

In the paper we solve (a) by proving that the size of the DFA is only single exponential; we solve (b) by exhibiting a better learning algorithm for sound workflow nets requiring only polynomially many queries; finally, we solve (c) by showing that for sound workflow nets the algorithms for synthesis of Petri nets from regular languages can be replaced by the algorithms for synthesis of bounded nets from minimal DFA, which are of polynomial complexity. Notice that our solution very much profits from the restriction to sound workflow nets, but that this restriction is given by the application domain: that sound workflow nets are an adequate formalization of well-formed business processes has been proved by the large success of the model in both the workflow modelling and Petri net communities.

Outline In the next section, we fix the notation of automata, recall the notion of Petri nets and workflow nets, and cite results on synthesis of Petri nets from automata. Our learning algorithm WNL is elaborated in Section 3. In Section 4 we describe some optimizations and heuristics to improve the performance of our basic algorithm. Section 5 reports on our implementation and experimental results. Finally, we sum up our contribution in the conclusion.

2. Preliminaries

We assume that the reader is familiar with elementary notions of graphs, automata and net theory. In this section we fix some notations and define some less common notions.

2.1. Automata and Languages

A deterministic finite automaton (DFA) is a 5-tuple $A = (Q, \Sigma, \delta, q_0, F)$ where Q is a finite set of *states*, Σ is a finite *alphabet*, $q_0 \in Q$ is the *initial state*, $\delta\colon Q \times \Sigma \to Q$ is the (partial) *transition function* and $F \subseteq Q$ is the set of *final states*. We denote by $\widehat{\delta}$ the function $\widehat{\delta}\colon Q \times \Sigma^* \to Q$ inductively defined by $\widehat{\delta}(q, \epsilon) = q$ and $\widehat{\delta}(q, wa) = \delta(\widehat{\delta}(q, w), a)$. The language $\mathcal{L}(q)$ of some state $q \in Q$ is the set of words $w \in \Sigma^*$ such that $\widehat{\delta}(q, w) \in F$. The language recognized by a DFA A is defined as $\mathcal{L}(A) := \mathcal{L}(q_0)$. A language is *regular* if it is accepted by some DFA.

Myhill-Nerode theorem and minimal DFAs Given a language $L \subseteq \Sigma^*$, we say two words $w, w' \in \Sigma^*$ are *L-equivalent*, denoted by $w \sim_L w'$, if $wv \in L \Leftrightarrow w'v \in L$ for every $v \in \Sigma^*$. The language L is regular iff L-equivalence partitions Σ^* into a finite number of equivalence classes. Given a regular language L, there exists a unique DFA A up to isomorphism with a minimal number of states such that $\mathcal{L}(A) = L$; this automaton A is called the *minimal DFA* for L. The number of states of this automaton is equal to the number of equivalence classes.

Given a DFA $A = (Q, \Sigma, \delta, q_0, F)$, we say two states $q, q' \in Q$ are *A-equivalent* if $\mathcal{L}(q) = \mathcal{L}(q')$. We can quotient A with respect to this equivalence relation. The states of the quotient DFA are the equivalence classes of $\sim_A$. The transitions are defined by "lifting" the transitions of A: for every transition $q \xrightarrow{a} q'$, add $[q] \xrightarrow{a} [q']$ to the transitions of the quotient DFA, where $[q]$ and $[q']$ denote the equivalence classes of q and q'. The initial state is $[q_0]$, and the final states are $\{[q] \mid q \in F\}$. The quotient DFA recognizes the same language as A, and is isomorphic to the minimal DFA recognizing L.

It is easy to see that the minimal automaton for a prefix-closed regular language has a unique non-final state (a trap state). For simplicity, we sometimes identify this automaton with the one obtained by removing the trap state together with its ingoing and outgoing transitions.

2.2. Petri Nets

A (marked) Petri net is a 5-tuple $N = (P, T, F, W, m_0)$ where P is a set of *places*, T is a set of *transitions* with $P \cap T = \emptyset$, $F \subseteq (P \times T) \cup (T \times P)$ is a *flow relation*, $W : (P \times T) \cup (T \times P) \to \mathbb{N}$ is a *weight function* satisfying $W(x, y) > 0$ iff $(x, y) \in F$, and $m_0 : P \to \mathbb{N}$ is a mapping called the *initial marking*.

For each transition or place x we call the set ${}^\bullet x := \{y \in P \cup T : \ (y, x) \in F\}$ the *preset* of x. Analogously we call $x^\bullet := \{y \in P \cup T : \ (x, y) \in F\}$ the *postset* of x. A net is *pure* if no transition belongs to both the pre- and postsets of some place.

Given an arbitrary but fixed numbering of P and T, the *incidence matrix* of N is the $|P| \times |T|$-matrix C given by: $C(p_i, t_j) = W(t_j, p_i) - W(p_i, t_j)$.

A transition $t \in T$ is *enabled* at a marking m, if $\forall p \in {}^\bullet t : m(p) \geq W(p, t)$. If a transition t is enabled it can *fire* to produce the new marking m', written as $m \xrightarrow{t} m'$.

$$m'(p) := m(p) + \sum_{p' \in P} C(p', t)$$

Given $w = t_1 \cdots t_n \in T^*$ (i.e. $t_i \in T$), we write $m_0 \xrightarrow{w} m$ if $W = \varepsilon$ and $m = m_0$ or there exist markings $m_1, \ldots, m_n$ such that $m_n = m$ and $m_0 \xrightarrow{t_1} m_1 \xrightarrow{t_2} m_2 \ldots m_{n-1} \xrightarrow{t_n} m_n$. Then, we say that m is *reachable*. The set of reachable markings of N is denoted by $\mathcal{M}(N)$ and defined by $\mathcal{M}(N) = \{m : \ \exists w \in T^*.\ m_0 \xrightarrow{w} m\}$. It is well-known that if $m_0 \xrightarrow{w} m$, then $m = m_0 + C \cdot P(w)$, where $P(w)$, the *Parikh vector* of w, is the vector of dimension $|T|$ having as i-th component the number of times that t_i occurs in w. We call this equality the *marking equation*.

A net N is *k-bounded* if $m(p) \leq k$ for every reachable marking m and every place p of N, and *bounded* if it is k-bounded for some $k \geq 0$. A 1-bounded net is also called *safe*. A net is *reversible* if for every firing sequence $m_0 \xrightarrow{w} m$ there is a sequence v_w leading back to the initial state, i.e. $m \xrightarrow{v_w} m_0$. N is *live* if every transition can fire eventually at every marking, i.e. $\forall m \in \mathcal{M}(N) \exists w_m . m \xrightarrow{w_m t} m'$ for some m'.

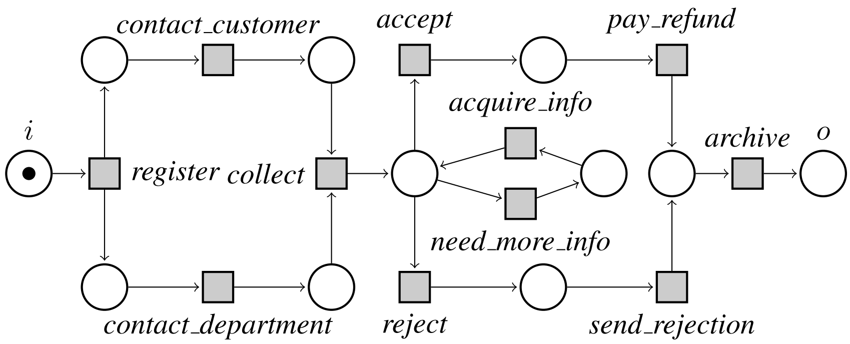


Figure 1. An example for a sound workflow net (drawn without the reset transition $\mathbf{r}$)

The *reachability graph* of a net $N = (P, T, F, W, m_0)$ is the directed graph $G = (V, E)$ with $V = \mathcal{M}(N)$ and $(x, y) \in E$ iff $x \xrightarrow{t} y$ for some $t \in T$. If G is finite, then the five-tuple $A(N) = (Q, \Sigma, \delta, q_0, F)$ given by $Q = \mathcal{M}(N)$, $\Sigma = T$, $q_0 = m_0$, $F = Q$ and $\delta(m, t) := m'$ if m $\xrightarrow{t} m'$ and undefined otherwise, is a DFA. (Note that δ is well-defined, because if $m \xrightarrow{t} m'$ and $m \xrightarrow{t} m''$ then $m' = m''$.) We call it the *marking-DFA* of N. The *language* of N, denoted by $\mathcal{L}(N)$, is defined as the language of $A(N)$.

2.3. Workflow Nets

Loosely speaking, a workflow net is a Petri net with two distinguished input and output places without input and output transitions respectively, and such that the addition of a "reset" transition leading back from the output to the input place makes the net strongly connected (see Figure 1 for an example). Formally, a net $N = (P, T, F, W, m_0)$ is a *workflow net* if there exist places $i, o \in P$ such that ${}^\bullet i = \emptyset = o^\bullet$, $m_0(p) = 1$ for $p = i$ and $m_0(p) = 0$, otherwise, and the net $\tilde{N} = (P, T \cup \{\mathbf{r}\}, F \cup \{(o, \mathbf{r}), (\mathbf{r}, i)\}, W \cup \{(o, \mathbf{r}) \mapsto 1, (\mathbf{r}, i) \mapsto 1\}, m_0)$, where $\mathbf{r} \notin T$, is strongly connected.

A firing sequence w of a workflow net N is a *run* if $m_0 \xrightarrow{w\mathbf{r}} m_0$ in $\tilde{N}$. Note that any proper prefix of a run is *not* a run since the output place o has no output transition besides $\mathbf{r}$. The runs of N are the formalization of the use cases of the business process modelled by the workflow net. A workflow net N is *sound* if $\tilde{N}$ is live and bounded. It is argued in [2] that a well-formed business process can be modelled by a sound workflow net (at a certain level of abstraction). The workflow net in Figure 1 is a very simple model for processing complaints (taken from [1], slightly altered)

The following lemma characterizes soundness. In the rest of the paper we work with this characterization as definition.

Lemma 2.1. A workflow net N is sound iff $\tilde{N}$ is bounded, reversible, and for every transition t there is a reachable marking m such that m enables t.

Proof:
Let $N = (P, T, F, W, m_0)$ be workflow net.
($\Rightarrow$): Assume N is sound. Then $\tilde{N}$ is bounded and live. We show $\tilde{N}$ is reversible. Let m be an arbitrary reachable marking of $\tilde{N}$. Then $m_0 \xrightarrow{w} m$ for some $w \in (T \cup \{\mathbf{r}\})^*$. Since $\tilde{N}$ is live, there is a firing

sequence w such that $m \xrightarrow{w\mathbf{r}} m'$ for some marking m'. We claim $m' = m_0$. Assume $m' \neq m_0$. Then, since $m'(i) > 0$, we have $m'(p) \geq m_0(p)$ for every place p, and $m'(p) > m_0(p)$ for some p. So m' strictly covers m_0, and so N is not bounded.

($\Leftarrow$): Assume $\tilde{N}$ is bounded, reversible and every transition is enabled at some reachable marking. We show that $\tilde{N}$ is live, which implies that N is sound. Let m be an arbitrary reachable marking of $\tilde{N}$, and let $t \in T \cup \{\mathbf{r}\}$. Since $\tilde{N}$ is reversible, $m \xrightarrow{w} m_0$ for some $w \in (T \cup \{\mathbf{r}\})^*$, and since t occurs in some firing sequence $m_0 \xrightarrow{vt} m'$ for some $v \in (T \cup \{\mathbf{r}\})^*$ and some m'. So $\tilde{N}$ is live (and bounded by assumption) and therefore N is sound. □

2.4. Synthesis of Petri Nets from Languages and from Automata

In [8], Darondeau et al. address two synthesis problems of Petri nets from a minimal DFA A over an alphabet T:

(S1) Decide if there is a bounded net N with T as set of transitions such that $\mathcal{L}(N) = \mathcal{L}(A)$, and if so return one. We call this problem *synthesis up to language equivalence*.

(S2) Decide if there is a bounded net N with T as set of transitions such that the reachability graph of N is isomorphic to A, and if so return one. We call this problem *synthesis up to isomorphism*.

The algorithm of [8] for synthesis up to language equivalence works in two phases. Firstly, A is transformed into an equivalent automaton A' in a certain normal form. In the worst case, A' can be exponentially larger than A. The second phase constructs the net N, if it exists, in polynomial time in A'. The algorithm requires *exponential time* in A. The algorithm of [8] for synthesis up to isomorphism, on the contrary, needs only *polynomial time* in A. Notice that, in general, if one knows the language $\mathcal{L}(N)$ of a net, one does not know directly its marking-DFA. In particular, the minimal automaton recognizing $\mathcal{L}(N)$ may not be the marking-DFA of any net. The basic algorithm in [8] can only handle pure nets, but a generalization to non-pure nets can be found in [9].

Hints on how to obtain nets that are more "visually appealing" (i.e. have few arcs, no redundant places, etc.) than those generated by standard synthesis algorithms can be found in [10], where net synthesis was applied to process mining from event logs.

3. A Learning Algorithm for Sound Workflow Nets

Our goal is to develop a learning algorithm for sound workflow nets which is guaranteed to terminate, and in which a teacher only needs to answer membership queries.

The precise learning setting is as follows. We have a *Leaner* and a *Teacher*. The Learner is given a set T of transitions, where each transition corresponds to a dedicated *task* (in the sense of [2]) of the business process. The Learner repeatedly asks the Teacher *workflow membership queries*. A query is a sequence $\sigma \in T^*$, and is answered by the Teacher as follows: if σ can be extended to a use case (i.e., a sequence corresponding to a complete instance of the business process), then the Teacher returns this use case in the form of a transition sequence $\sigma\tau\mathbf{r}$, where $\tau \in T^*$ (actually, we assume that the Teacher just returns $\sigma\tau$, the reset transition is implicitly always there). Otherwise, the Teacher answers "no". In our running example the Learner is given the set of transitions of the net of Figure 1, and the Teacher's answers are compatible with this net, i.e., acts as if it knew the net. Note that in practice, this only means that the Teacher can either extend the query to a use case of the net to learn or can reject the query. Two possible queries are

$$\begin{array}{lll} \textit{register} & \textit{contact\_customer} & \textit{contact\_department} \\ \textit{register} & \textit{contact\_customer} & \textit{collect} \end{array}$$

A possible answer to the first query is the run

$$\textit{register contact\_customer contact\_department collect accept pay\_refund archive}$$

while the answer to the second query is “no”.

Assuming that the Teacher’s answers are compatible with a k-bounded and reversible net N, the goal of the Learner is to produce a net N' such that $\mathcal{L}(N) = \mathcal{L}(N')$. It is easy to see that a (very inefficient) learning algorithm exists for a given k known in advance. Since N is k-bounded, we need only consider nets N' with arc weights smaller than or equal to k, and where no two places p, p' satisfy $W(p,t) = W(p',t)$ and $W(t,p) = W(t,p')$ for every transition t. (We can always remove one of p and p' without changing the language of the net.). Under these constraints we have:

(1) A net with n transitions has at most $c_1 := 2^{(2k+2)n}$ places. (For each transition t we have $k+1$ possible values for each of $W(p,t)$ and $W(t,p)$.)

(2) By (1), N has at most $c_2 := (k+1)^{c_1}$ reachable markings. Therefore, there exists a minimal DFA A with at most c_2 states such that $\mathcal{L}(N) = \mathcal{L}(A)$.

(3) The minimal DFA A can be learned by querying all words over T of length $2c_2$, i.e., after at most $c_3 := n^{2c_2}$ queries.
To see this, observe first that any two prefix-closed minimal DFAs with c_2 states differ in some word of length c_2. We now apply the Myhill-Nerode Theorem to construct A from the answers to the queries as follows. The states of A are the equivalence classes of words of $\mathcal{L}(N)$ of length up to c_2, where two words w, v are equivalent if for every word u of length up to c_2 either wu and vu belong to $\mathcal{L}(N)$, or none of them does [19, 15]. The initial state is the equivalence class of the empty word, and all states are final. There is a transition $[w] \xrightarrow{a} [wa]$ for every word w of length at most c_2.

(4) The net N is obtained from A by means of the algorithm of [8] for synthesis up to language equivalence (see problem (S1) in Section 2.4). The algorithm runs in $2^{\mathcal{O}(p(c_2))}$ time for some polynomial p.

The query complexity of this naive algorithm, i.e. the number of queries it needs to ask, is triple exponential in the number n of transitions. In this section we prove a series of results ending in an improved algorithm with single exponential query and time complexity (notice that single exponential time complexity implies single exponential query complexity, but not vice versa).

3.1. An Upper Bound on the Number of Reachable Markings

We show that the naive bound on the number of states of A obtained in (2) above, which is double exponential in n, can be improved to a single exponential bound.

Given a net $N = (P, T, F, W, m_0)$ with incidence matrix C, we let $C(p)$ denote the p-th row vector $(C(p, t_1), \dots, C(p, t_{|T|}))$. We say that a place p is a *linear combination* of the places $p_1, \dots, p_k$ if there are real numbers $\lambda_1, \dots, \lambda_k$ such that $C(p) = \sum_{i=1}^{k} \lambda_i \cdot C(p_i)$.

The following lemma is well known.

Lemma 3.1. Let $N = (P, T, F, W, m_0)$ be a net with incidence matrix C, and let $C(p) = \sum_{i=1}^{k} \lambda_i C(p_i)$. Then for every reachable marking m: $\forall p \in P.\ m(p) = m_0(p) + \sum_{i=1}^{k} \lambda_i (m(p_i) - m_0(p_i))$.

Proof:
Since m is reachable, there is $w \in T^*$ such that $m_0 \xrightarrow{w} m$. By the marking equation $m = m_0 + C \cdot P(w)$, and so in particular $m(p) = m_0(p) + C(p) \cdot P(w)$, and $m(p_i) = m_0(p_i) + C(p_i) \cdot P(w)$ for every $1 \leq i \leq k$. So $m(p) = m_0(p) + \sum_{i=1}^{k} \lambda_i C(p_i) \cdot P(w) = m_0(p) + \sum_{i=1}^{k} \lambda_i (m(p_i) - m_0(p_i))$ □

Theorem 3.1. Let $N = (P, T, F, W, m_0)$ be a k-bounded net with n transitions. Then N has at most $(k+1)^n$ reachable markings.

Proof:
The incidence matrix C has $|P|$ rows and n columns, and so it has rank at most n. So there are l places $p_1, \dots, p_l$, $l \leq n$, such that $C(p_1), \dots, C(p_l)$ are linearly independent. So every place p is a linear combination of $p_1, \dots, p_l$. It follows from Lemma 3.1 that for every two reachable markings m, m', if $m(p_i) = m'(p_i)$ for every $1 \leq i \leq l$, then $m(p) = m'(p)$ for every place p. In other words, if two markings coincide on all of $p_1, \dots, p_l$, they are equal. Since for every reachable marking m we have $0 \leq m(p_i) \leq k$, the number of projections of the reachable markings onto the places $p_1, \dots, p_l$ is at most $(k+1)^l \leq (k+1)^n$. So N has at most $(k+1)^n$ reachable markings. □

3.2. Minimality of the Marking-DFA

We still have to solve two problems of step (3) in the naive algorithm. First, the algorithm does not learn the marking-DFA of the net, it only learns the minimal DFA for its language. Therefore, in step (4) it needs to apply the exponential algorithm for synthesis up to language equivalence (Problem (S1)), instead of the polynomial algorithm of [8] for synthesis up to isomorphism (Problem (S2)). Second, the number of queries of the algorithm is exponential in the number of reachable states: if n and r are the number of transitions and reachable markings of the net, then the algorithm requires n^r membership queries.

In the next section we provide an algorithm that solves both problems at the same time when the net is k-bounded and reversible: the algorithm computes the marking-DFA, and only asks $\mathcal{O}(n \cdot r^2)$ membership queries. However, before presenting the algorithm, we prove in this section that in fact the first problem *disappears* for bounded and reversible nets: When the net N is bounded and reversible, its marking-DFA *coincides* with the minimal DFA for $\mathcal{L}(N)$, and so any learning algorithm for the minimal DFA actually learns the marking-DFA. This is relevant when considering alternatives to our algorithm (see Section 3.6 for a discussion).

The proof is based on Lemma 3.2 below. Readers familiar with the Myhill-Nerode Theorem (see also Section 2) will probably need no proof, but we include one for completeness. Recall that we identify a DFA A with a single trap state and the DFA A' obtained by removing the trap state of A together with its ingoing and outgoing transitions.

Lemma 3.2. A DFA $A = (Q, \Sigma, \delta, q_0, F)$ is minimal iff the following two conditions hold:

(1) every state lies in a path leading from q_0 to some state of F, and

(2) $\mathcal{L}(q) \neq \mathcal{L}(q')$ for every two distinct states $q, q' \in Q$.

Proof:

($\Rightarrow$): We prove the contrapositive. For (1), if some state q does not lie in any path from q_0 to some final state, then it can be removed without changing the language, and so A is not minimal. For (2), if two distinct states q, q' of A satisfy $\mathcal{L}(q) = \mathcal{L}(q')$, then $[q] = [q']$, and so the quotient automaton has fewer states than A. So A is not minimal.

($\Leftarrow$): Assume (1) and (2) hold. We prove that for every state q the language of the words w such that $\delta(q_0, w) = q$ is an equivalence class of L-equivalence. It follows that the number of states of A is at most as large as the number of equivalence classes of L-equivalence, which implies that A is the minimal DFA for L.

It suffices to show:

- If $\widehat{\delta}(q_0, w) = q = \widehat{\delta}(q_0, v)$, then $w \sim_L v$.
  This follows immediately from the definition of L-equivalence.

- If $\widehat{\delta}(q_0, w) = q$ and $\widehat{\delta}(q_0, v) = q'$ for some $q' \neq q$, then $w \not\sim_L v$.
  Since $\mathcal{L}(q) \neq \mathcal{L}(q')$, w.l.o.g. there is a word $u \in \mathcal{L}(q) \setminus \mathcal{L}(q')$. So $wu \in L$ and $vu \notin L$, which implies $w \not\sim_L v$.

$\square$

Theorem 3.2. Let $N = (P, T, F, W, m_0)$ be a bounded and reversible Petri net. The marking-DFA $A(N)$ of N is a minimal DFA.

Proof:

Since N is bounded, $A(N)$ is a DFA. Assume that $A(N)$ is not minimal. Since every state of $A(N)$ is final, by Lemma 3.2 there are two states of $A(N)$, i.e., two reachable markings $m_1 \neq m_2$ of N, such that $\mathcal{L}(m_1) = \mathcal{L}(m_2)$. As $m_1 \neq m_2$ there exists $p \in P$ with $m_1(p) \neq m_2(p)$. Assume w.l.o.g. $m_1(p) < m_2(p)$. Let m be a reachable marking such that $m(p)$ is minimal, i.e. there is no other reachable marking m' s.t. $m'(p) < m(p)$. Since m is reachable and N is reversible, there is $w \in T^*$ such that $m_2 \xrightarrow{w} m$. Since $\mathcal{L}(m_1) = \mathcal{L}(m_2)$, there is a reachable marking m' such that $m_1 \xrightarrow{w} m'$. It follows

$$m'(p) = m_1(p) + C(p) \cdot P(w) < m_2(p) + C(p) \cdot P(w) = m(p)$$

contradicting the minimality of $m(p)$. $\square$

3.3. Learning the Marking-DFA by Exploration

We still have to solve the second problem of step (3) in the naive algorithm: if n and r are the number of transitions and reachable markings of the net, then the algorithm requires n^r membership queries.

In this section we give an algorithm for the case in which the answers of the Teacher are compatible with a k-bounded and reversible net, which requires only $\mathcal{O}(n \cdot r^2)$ queries.

Recall the standard search approach for constructing the reachability graph of a net *if the net is known*. We maintain a queue of markings, initially containing the initial marking, and two sets of already visited markings and transitions (transitions between markings). While the queue is non-empty, we take the marking m at the top of the queue, and check for each transition a whether a is enabled at m. If so, we compute the marking m' such that $m \xrightarrow{a} m'$, and proceed as follows: if m' has been already visited, we add $m \xrightarrow{a} m'$ to the set of visited transitions; if m' had not been visited yet, we add m' to the set of visited markings and to the queue, and add $m \xrightarrow{a} m'$ to the set of visited transitions.

Our learning algorithm closely mimics this behaviour, but works with firing sequences of N instead of reachable markings (the Learner does not know the markings of the net, it does not even know its places). We maintain a queue of firing sequences, initially containing the empty sequence, and two sets of already visited firing sequences and transitions. While the queue is non-empty, we take the firing sequence $w \in (T \cup \{\mathbf{r}\})^*$ at the top of the queue, and ask the Teacher for each transition a whether wa is also a firing sequence of N. This is equivalent to asking the Teacher to provide the transitions enabled after the execution of w. Then, for each enabled transition we first determine whether each already visited firing sequence u leads to the same marking as wa. Notice that it is not obvious how to do this—this is the key of the learning algorithm. If some firing sequence u leads to the same marking as wa, then we add $w \xrightarrow{a} u$ to the set of visited transitions; otherwise, we add wa to the set of visited firing sequences and to the queue, and add $w \xrightarrow{a} wa$ to the set of visited transitions. The algorithm in pseudo code can be found below (Algorithm 1), where Equiv(u, v) denotes that there is a marking m such that $m_0 \xrightarrow{u} m$ and $m_0 \xrightarrow{v} m$.

Algorithm 1: Learning the reachability graph

Output: graph (V, E) isomorphic to the reachability graph of N

$V \longleftarrow \emptyset$; $E \longleftarrow \emptyset$
$F \longleftarrow \{\epsilon\}$ `// queue of firing sequences`
while $F \neq \emptyset$ **do**
  pick w from F
  forall the $a \in \text{Enabled}(w)$ **do**
    /* This means $wa \in \mathcal{L}(N)$ */
    if there exists $u \in V$ such that Equiv(u, wa) **then** add $w \xrightarrow{a} u$ to E
    else add wa to V and F, and add $w \xrightarrow{a} wa$ to E

The correctness of the algorithm is immediate: we just simulate a search algorithm for the construction of the reachability graph, using a firing sequence u to represent the marking m such that $m_0 \xrightarrow{u} m$. The check Equiv(u, wa) guarantees that each marking gets exactly one representative.

The problem is to implement Equiv(u, wa) using only membership queries. In general this is no easy task, but in the case of reversible nets it can be easily done as follows. When checking Equiv(u, wa) the word u has been already added to V, and so the Learner has established that $u \in \mathcal{L}(N)$. So in particular the Teacher has answered positively a query about u and, due to the structure of workflow membership queries, it *has returned a run* uu_c, where $u_c\mathbf{r}$ is a transition sequence leading back to the initial marking.

We prove that Equiv(x, y) holds if and only if the sequence xy_c is a run of N:

Proposition 3.1. In Algorithm 1, Equiv(u, wa) = **true** if and only if uw_c is a run of N, where waw_c is the run reported by the Teacher when positively answering the query about $wa \in \mathcal{L}(N)$.

Proof:
If Equiv(u, wa) = **true**, then there is a marking m such that $m_0 \xrightarrow{u} m$ and $m_0 \xrightarrow{wa} m$. Because $m_0 \xrightarrow{wa} m \xrightarrow{w_c\mathbf{r}} m_0$, we have $m_0 \xrightarrow{u} m \xrightarrow{w_c\mathbf{r}} m_0$, which implies that uw_c is a run.

If $u \cdot w_c$ is a run, then we have $m_0 \xrightarrow{waw_c\mathbf{r}} m_0$ and $m_0 \xrightarrow{uw_c\mathbf{r}} m_0$. Let m be the marking such that $m_0 \xrightarrow{wa} m$. We then have $m \xrightarrow{w_c\mathbf{r}} m_0$. Moreover, m is the only marking such that $m \xrightarrow{w_c\mathbf{r}} m_0$ (Petri nets are backward deterministic: given a firing sequence and its target marking, the source marking is uniquely determined). Since $m_0 \xrightarrow{uw_c\mathbf{r}} m_0$, we then necessarily have $m_0 \xrightarrow{u} m \xrightarrow{w_c\mathbf{r}} m_0$, and so in particular $m_0 \xrightarrow{u} m$. So both wa and u lead to the same marking m, and we have Equiv(u, wa) = **true**. □

We can now easily show that checking Equiv(u, wa) reduces to one single membership query.

Proposition 3.2. The check Equiv(u, wa) can be performed by querying whether uw_c belongs to $\mathcal{L}(N)$: Equiv(u, wa) holds if and only if the Teacher answers positively and returns the sequence uw_c itself as a run.

Proof:
There are three possible cases:

- The answer is negative. Then $uw_c \notin \mathcal{L}(N)$, and so in particular it is not a run of N. So Equiv(u, wa) = **false**.
- The answer is positive *and* the Teacher returns uw_c as run. Then Equiv(u, wa) = **true** by Proposition 3.1.
- The answer is positive, but the Teacher returns uw_cv for some $v \neq \epsilon$ as a run. Since no proper prefix uw_cv' is a run (see the definition of a run in section 2.3), we have in particular by taking $v' = \epsilon$ that uw_c is not a run. By Proposition 3.1 we have Equiv(u, wa) = **false**.

□

There is a technical issue that should be mentioned at this point. The algorithm delivers a net N' such that the reachability graphs of N and N' are isomorphic. It follows that N' is reversible and bounded. However, we cannot guarantee that N' has the same bound as N. For a trivial example, consider a net consisting of a single circuit in which all arcs have weight $k \geq 1$, with an initial marking that puts k tokens in a place and no tokens in the rest. Obviously, the reachability graph does not depend on k up to isomorphism. We consider this fact a minor problem, since N' and N are equivalent for behavioural purposes.

3.3.1. Complexity

It would seem natural to define the complexity of the algorithm as the total number of queries made to the Teacher. (Since, as we have just shown, an equivalence query reduces to a membership query, there is no need to distinguish between membership and equivalence queries). However, a finer analysis shows that this measure is not adequate. In order to compute the set Enabled(w), the algorithm asks the Teacher for every transition a whether wa is a firing sequence. This would count as $|T|$ membership queries. However, in a real-world workflow, even of moderate size, the number of transitions enabled at a marking is much smaller than the total number of transitions, and so most of the queries are answered negatively. As an extreme case, consider the task of learning the sequence of calendar months. The algorithm starts with the empty sequence, and then ask twelve membership queries of the form "Can January, February, ..., December ?" . Obviously, any human Teacher would prefer to be asked "Which is the first month of the year?" (or "Which could be the first months of the year?" if the Learner does not know that months are totally ordered).

This would suggest to count a call to Enabled() as one single query. However, this is also misleading: because of the way equivalence queries are dealt with, the Teacher must provide for every positively answered query an extension of the query sequence to a run. For this reason, it is more adequate to use the following complexity measure: if a call to Enabled() returns k enabled transitions, then we assign to it a cost of k membership queries, but each call to Equiv() counts as one membership query. (Notice that the membership query for an equivalence query asks if uw_c is fireable, where only u is known to be fireable.)

Let r be the number of reachable markings of N, and let d be the *enabling degree* of N, i.e., the maximum number of transitions of N that can be enabled by a reachable marking. It follows from the description of Algorithm 1 that the number of firing sequences added to the queue is equal to r. Now, let w be the i-th sequence taken from the queue (i.e., w is taken from the queue when $|V| = i - 1$). Processing w requires at most $d \cdot i$ membership queries: d queries for the call to Enabled(), and at most $d \cdot (i - 1)$ equivalence checks of the form Equiv(u, wa) (because $u \in V$ and $a \in E$). So the algorithm performs at most $\sum_{i=1}^{r} d \cdot i = dr(r+1)/2 \in \mathcal{O}(d \cdot r^2)$ queries.

Putting all the pieces of our discussion together, we obtain WNL, an algorithm to learn a bounded and reversible net N. WNL proceeds as follows:

- Using Algorithm 1, learn the marking-DFA A of N.
- Apply the polynomial algorithm of [8] for synthesis up to isomorphism (S2).

The following theorem sums up the results of the section.

Theorem 3.3. (Learning by Exploration)
Let N be a k-bounded and reversible net N with n transitions and enabling degree d. WNL returns a bounded net N' such that $\mathcal{L}(N) = \mathcal{L}(N')$. Moreover, WNL requires at most $\mathcal{O}(d \cdot (k+1)^{2n})$ queries, and the total runtime of the Learner is polynomial in k and d, and single exponential in n.

Proof:
The correctness follows immediately from the fact that Algorithm 1 returns the marking-DFA of N (up to isomorphism).

For the bound on the number of queries, we observe that, by the discussion above, Algorithm 1 asks $\mathcal{O}(d \cdot r^2)$ queries, where r is the number of reachable states, and that $r \leq (k+1)^n$ holds by Theorem 3.1.

For the exponential bound on the runtime, observe that Algorithm 1 explores firing sequences by increasing length. Therefore, if a sequence w leading to a marking m is added to the queue, then w is a shortest sequence leading to w. Since the net has at most $(k+1)^n$ reachable markings, the length of w is at most $(k+1)^n$. Therefore, the Learner needs $\mathcal{O}((k+1)^n)$ time to perform an operation on a sequence. Since the number of operations is polynomial in the number of queries, the runtime is in $\mathcal{O}(d \cdot r^2 \cdot (k+1)^n)$, and thus polynomial in k and d, and single exponential in n. $\square$

Notice that we only give a bound for the runtime of the Learner. The runtime of the Teacher depends on the computational model associated to her. For the case in which the Teacher consults an event log (i.e., a database of runs), see Section 3.4.

A final question is what happens if the Teacher's answers are not compatible with any k-bounded and reversible net N. In this case there are two possibilities: they are not compatible with any DFA having at most $(k+1)^n$ states, or they are compatible with some such DFA, but this DFA is not the marking-DFA of any net. In the first case the algorithm can stop when the number of generated states exceeds $(k+1)^n$. In the second case, the algorithm terminates and produces a DFA, but the synthesis algorithm of [8] does not return a net.

3.4. Mixing Process Mining and Learning

The algorithm we have just presented does not assume the existence of an event log: the Learner only gets information from membership queries. However, as explained in the introduction, we consider our learning approach as a way of complementing log-based process mining. In this section we explain how to modify the algorithm accordingly.

We assume the existence of an event log consisting of use cases. Given the set of tasks T of the business process, we can think of each use case as a word $w \in T^*$, such that w corresponds to a run of the reversible net to be learnt. The event log then corresponds to a language $L \subseteq T^*$.

In a first step we construct a minimal DFA for the language L. This can be done space-efficiently in a number of ways. For instance, we can divide the set of runs in two halves L_1, L_2, recursively compute minimal DFAs A_1, A_2 recognizing L_1 and L_2, and then compute the minimal DFA for L from A_1, A_2 using an algorithm very similar to the one that computes the union of two binary decision diagrams [6]. Once this is done, we easily get the minimal DFA A for the language of prefixes of $(L\mathbf{r})^*$ (this requires to add one extra state and make all states final).

Once A is computed, we assign to each state q of A a word w_q such that $q_0 \xrightarrow{w_q} q$. For every two states q_1, q_2, we check whether the states correspond to the same reachable marking by calling Equiv(w_{q_1}, w_{q_2}). After this step we are in the same situation we would have reached if the algorithm would have queried all the words w_q.

In practice we would first apply the reduction techniques described in Section 4 before actually calling Equiv() for each pair of states, which would be quite inefficient.

3.5. A Lower Bound for Petri Net Learning

We now show that we cannot in general solve the learning problem in subexponential time, by providing a hard-to-learn instance. We will show that any learning algorithm has to ask $\Omega(2^n)$ membership queries in the worst case to derive the correct net, where n is the number of transitions.

Consider the following set $\mathcal{N}$ of workflow nets. All the nets in $\mathcal{N}$ have the same number $n+3$ of transitions: two transitions *init* and *final*, transitions called $a_1, \ldots, a_n$, and a transition t (see Figure 2). The pre- and postsets of all transitions but t, which are identical for all nets of $\mathcal{N}$, are shown in the figure. The postset of t is always the place o. The preset of t always contains for each i exactly one of the places

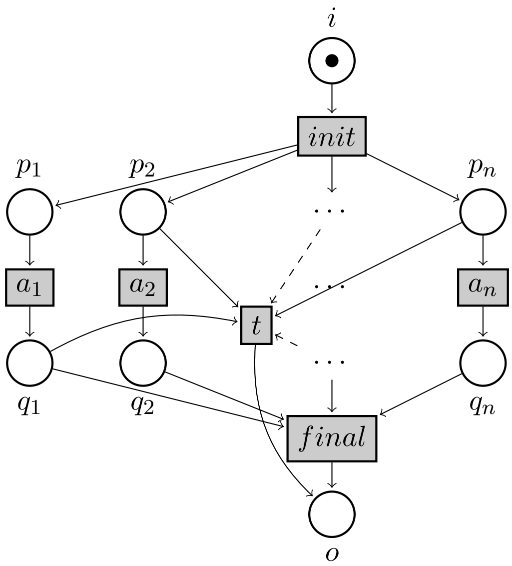


Figure 2. Hard-to-learn instance for Petri net learning

p_i or q_i, and the only difference between two nets in $\mathcal{N}$ is their choice of p_i or q_i. Clearly, the set $\mathcal{N}$ contains 2^n workflow nets, all of them sound.

For each net $N \in \mathcal{N}$ there is exactly one subset of $\{a_1, \ldots, a_n\}$ such that t can fire after the transitions of the set have fired. We call this subset S_N. Notice that if we know S_N then we can infer ${}^\bullet t$.

We ease the task for the Learner by assuming she knows that the net to be learned belongs to $\mathcal{N}$. Her task consists only of finding out ${}^\bullet t$, or, equivalently, the set S_N.

A query of an optimal Learner has always the form $a_{i_1} a_{i_2} \cdots a_{i_k} t$, because querying any a_i after t does not provide the Learner with any information. Furthermore the order of the a_i is not important—all these transitions are independent and the Learner already knows this. So we can view a query just as a subset S of the set of all transitions. A negative answer to a query S always rules out *exactly one* of the nets of $\mathcal{N}$, namely the one in which ${}^\bullet t = S$. The worst case appears when the Learner ask queries "in the worst possible order", eliminating all nets of $\mathcal{N}$ but the right one. This requires $2^n - 1$ queries.

3.6. Alternatives to Algorithm 1

The literature on machine learning contains many different algorithms to learn the minimal DFA A for a regular language L under different learning models. By Theorem 3.2 shows that, when these algorithms are applied to the problem of learning the language of a bounded and reversible net N, they in fact learn the marking-DFA of N.

In this section we briefly discuss Angluin's algorithm, which learns A under the assumption that the Learner can ask membership and *equivalence* queries [7]. In a membership query, the Learner sends the Teacher a word w, and asks her if the word is accepted by A. In an equivalence query, the Learner sends an automaton H, usually called the *hypothesis*, and asks the Teacher whether $\mathcal{L}(A) = \mathcal{L}(H)$ holds[1].

[1]Notice that "equivalence" refers here to equivalence of automata, and not to equivalence of firing sequences, as in Algorithm 1

If $\mathcal{L}(A) = \mathcal{L}(H)$ the Teacher answers an equivalence query with "yes" or provides the Learner with a counterexample w otherwise, i.e. w is a word in the symmetric difference of $\mathcal{L}(A)$ and $\mathcal{L}(H)$. If m is the maximum length of a counterexample provided by the Teacher, Angluin's algorithm learns a minimal DFA with r states and alphabet size n using at most $\mathcal{O}(n \cdot m \cdot r^2)$ membership queries and r equivalence queries. Observe that the answer to a membership query is only "yes" or "no", the Teacher is not required to extend a firing sequence of the net to a run.

There is a trade-off between our learning setting and Angluin's. Angluin's setting requires an equivalence oracle, but membership queries can be answered with a simple "yes/no". On the other hand, our setting does not need equivalence queries — at the price of requiring the Teacher to provide full runs. Depending on the application (workflow design, requirements elicitation, reverse engineering of "black-box" systems, etc.) both approaches can be useful, and Theorem 3.2 shows that both can be applied if the workflow models are bounded and reversible.

4. Optimizations

In this section we discuss some simple heuristics and optimizations that can greatly reduce the number of membership queries that the Teacher needs to answer.

There are some easy properties of Petri nets that can be used to reduce the number of queries in many situations. Additionally, they are helpful to trim an initial DFA constructed from event logs.

4.1. Memorizing and Deducing Answers from a Partial DFA

A first obvious improvement on Algorithm 1 consists of storing the queries that have been already been asked, together with the Teacher's answer, and making sure that the Teacher is never asked the same query twice. This also means not asking the Teacher to produce a run when one is already available. Imagine that when asked Enabled(w) the Teacher answers that a is enabled by w, and $wabc$ is a run. If the algorithm asks now after Enabled(wa), it already knows that b belongs to it, and $wabc$ is also a witness run. So the Teacher is told not to provide this run.

A second easy optimization looks as follows. During its execution, Algorithm 1 learns that some transition sequences are fireable, and others are not: more precisely, for each dequeued sequence w the algorithm learns that wa is fireable for every $a \in \text{Enabled}(w)$, and non-fireable for every $a \notin \text{Enabled}(w)$. Moreover, the algorithm also learns that every prefix of a run returned by the Teacher is fireable. An obvious optimization is to use this information to reduce the number of equivalence queries passed to the Teacher. Recall that, when calling Equiv(u,wa), Algorithm 1 asks the Teacher if $uw_c \in \mathcal{L}(N)$, where waw_c is the run reported by the Teacher when positively answering the query about $wa \in \mathcal{L}(N)$ (see Proposition 3.2). Before asking the Teacher whether $uw_c \in \mathcal{L}(N)$, the improved algorithm first checks if it already knows whether uw_c is a prefix of some fireable sequence, or whether some prefix of uw_c is not fireable. Our implementation (cf. Section 5) incorporates this improvement.

Notice that the equivalence query Equiv(u,wa) can also be answered by deciding if $wau_c \in \mathcal{L}(N)$ holds, and is a run. However, in this case the algorithm would never be able to answer the query without passing it to the Teacher, because wa is fireable, and the algorithm has not yet explored any sequence having wa as a proper prefix.

4.2. Bounded Lookahead

In Algorithm 1, the answer to a query Equiv(u,wa) is positive iff the sequences u and wa are both fireable, and lead to the same marking. So, in particular, if Equiv(u,wa) holds then for every transition sequence v we have that uv is fireable iff wa is fireable. We can use this property to simplify the task of the Teacher. Instead of directly asking whether Equiv(u,wa) holds, the algorithm asks the Teacher to first provide the sets Enabled(u) and Enabled(wa). If the answers are different, then there is a transition b such that exactly one of ub and wab is fireable, and so Equiv(u,wa) does not hold. (Note that the Teacher does not have to provide full runs for any of these possible extensions so this is quite a simple task.) We call this heuristic *bounded lookahead of depth 1*. The heuristic can be further extended by asking the Teacher to provide the sequences of length d enabled by u and wa (lookahead of depth d). If we explore possible extensions up to depth d we may ask up to $|T|^d$ unnecessary queries in case of an already discovered state, but we can save up to n calls to Equiv(), if n is the number of states currently discovered.

4.3. Parikh Vectors

It is well known that if $w_1, w_2 \in \mathcal{L}(N)$ and $P(w_1) = P(w_2)$ (i.e., w_1 and w_2 have the same Parikh vector) then firing w_1 and w_2 leads to the same marking. This fact can be used to reduce the number of calls to Equiv() in our algorithm. For each state q in our currently explored automaton, we save the Parikh vectors P_q of the words leading to it. Now, if the algorithm is currently processing w, and the Teacher reported that w enables a transition a, then we check if $P(wa) \in P_q$ for some state q. If that is the case, then we simply add the transition $p \xrightarrow{a} q$.

4.4. Diamond Property for Pure Nets

This heuristic saves queries when the net is required to be pure (i.e., when one can assume that if there is a workflow net modelling the system, then there is also a pure workflow net). A *diamond* is a subgraph in the reachability graph of a net with four states that are connected in the following way: $m_1 \xrightarrow{a} m_2$, $m_1 \xrightarrow{b} m_3$, $m_2 \xrightarrow{b} m_4$, $m_3 \xrightarrow{a} m_4$. A diamond is *incomplete* if it is missing exactly one transition (see Figure 3). It is well known that the reachability graphs of pure nets do not have incomplete diamonds (the *diamond property*). Therefore, if the algorithm discovers that the part of the reachability graph constructed so far has an incomplete diamond, it can directly add the missing transition, saving one membership query and n equivalence queries, where n is the number of states constructed so far.

The diamond property can also be used to merge states of a DFA generated from an event log (see Section 3.4) as indicated in Figure 3.

4.5. An Example

We show how Algorithm 1 learns the reachability graph of the net of Figure 4, the first part of the net in Figure 1. Table 1 shows the list queries submitted to the Teacher when none of the optimizations we have just discussed are applied. To simplify the presentation some queries are grouped together. The "Query" column contains the queries being asked; Equiv($\{w_1, \ldots, w_n\}$, w) is an abbreviation for Equiv(w_1, w), ..., Equiv(w_n, w). The notation $\rightsquigarrow \{w_1, \ldots, w_n\} \cdot v$ indicates that the equivalence queries are transformed into the membership queries $w_1 v, \ldots w_n v$. The "Answer" column contains the run or

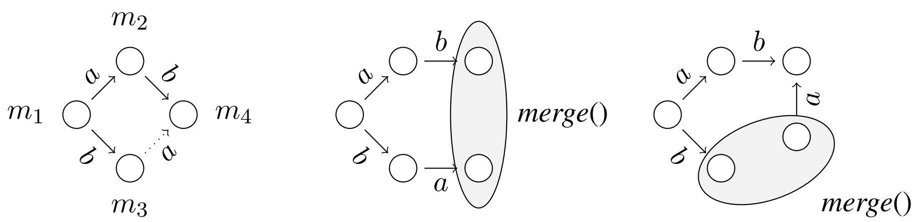


Figure 3. Incomplete diamond (left), states merged because of equal Parikh vectors (middle) or by using the diamond property (middle and right)

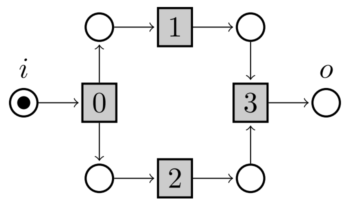


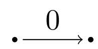
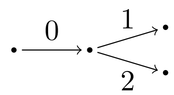
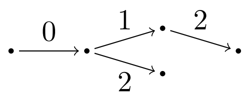
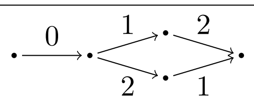
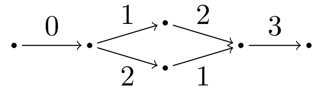
Figure 4. Initial part of the net of Figure 1

runs ww_c returned by the Teacher as part of the answer to a query of the form Enabled(w). The notation is $w(w_c)$, i.e., the part between parenthesis is the continuation of w to a run. Notice that if the Teacher provides several runs, then we count this as several queries. Finally, the column "Current Automaton" gives the current knowledge of the algorithm about the reachability graph.

The algorithm needs 27 queries, a high number for such a small net. But let us know consider the effect of some of the optimizations.

- Query 3 is redundant, because after query 1 the algorithm already knows that 0123 is a run.
- Queries 5 and 7 ($23, 13 \in \mathcal{L}(N)?$) are deducible: after query 1 the algorithm knows that the sequences 2 and 1 (both of length one) are not fireable, and so $23 \notin \mathcal{L}(N)$ and $13 \notin \mathcal{L}(N)$.
- Query 10 is redundant, after query 1 the algorithm already knows that 0123 is a run.
- Queries 11 and 12 ($3, 03 \in \mathcal{L}(N)?$) are deducible: after queries 1 and 3 the algorithm knows that 3 and 03 are not fireable.
- Queries 13 and 14 ($013, 023 \in \mathcal{L}(N)?$) have been asked before (queries 8 and 6, respectively).
- Query 15 is redundant, after query 4 the algorithm already knows that 0213 is a run.
- Queries 16-19 ($3, 03, 013, 023 \in \mathcal{L}(N)?$) have been asked before (queries 11,12,8, and 14, respectively).
- Query 20 (Equiv(012,021)?) is deducible: since both sequences have the same Parikh mapping the answer is positive.
- Query 21 is redundant, after query 1 the algorithm already knows that 0123 is a run.

Table 1. Execution of Algorithm 1 on the net of Figure 4

#	Query	Answer	Current Automaton
1	Enabled(ε) ?	$0(123)$	
2	Equiv($\varepsilon, 0$) ? $\rightsquigarrow\ \varepsilon \cdot 123$	no	
3-4	Enabled(0) ?	$01(23)$ $02(13)$	
5-6	Equiv($\{\varepsilon, 0\}$, 01) ? $\rightsquigarrow\ \{\varepsilon, 0\} \cdot 23$	no	
7-9	Equiv($\{\varepsilon, 0, 01\}$, 02) ? $\rightsquigarrow\ \{\varepsilon, 0, 01\} \cdot 13$	no	
10	Enabled(01) ?	$012(3)$	
11-14	Equiv($\{\varepsilon, 0, 01, 02\}$, 012) ? $\rightsquigarrow \{\epsilon, 0, 01, 02\} \cdot 3$	no	
15	Enabled(02) ?	$021(3)$	
16-20	Equiv($\{\varepsilon, 0, 01, 02, 012\}, 021$) ? $\rightsquigarrow \{\varepsilon, 0, 01, 02, 012\} \cdot 3$	yes (012) no (rest)	
21	Enabled(012) ?	$0123(\varepsilon)$	
22-27	Equiv($\{\varepsilon, 0, 01, 02, 012, 021\}$, 0123) ? $\rightsquigarrow \{\varepsilon, 0, 01, 02, 012, 021\} \cdot \varepsilon$	no	

- Queries 22-27 ($\varepsilon, 0, 01, 02, 012, 021 \in \mathcal{L}(N)$?) are all deducible, because the algorithm knows that 0123 and 021 are fireable (queries 21 and 15, respectively).

So out of the 27 queries asked by the naive algorithm, only 6 need to be answered by the Teacher: queries 1, 2, 4, 6, 8, and 9.

5. Experiments

To get insights into the practical feasibility of the derived algorithm WNL, we have developed a prototype learning and synthesis tool for workflow nets and examined its practical performance on a number of examples.

5.1. Implementation

Our prototype, which we call WNL for Workflow Net Learner, is written in C++. It has approximately 3,000 lines of code and uses LIBALF for dealing with automata. LIBALF is part of the automata learning factory currently developed jointly at RWTH Aachen and TU MĂĽnchen[2] [14].

The synthesis algorithm (S2) of [8] is implemented using the LP_SOLVE[3] framework to efficiently solve the linear programs needed for computing the places of the net. Furthermore LP_SOLVE is used for eliminating redundant places after the net has been synthesized to reduce its size and to make it look more appealing. The implementation is currently not tailored to user interaction but consults pre-existing workflow nets for queries. Outputs are given in form of dot-files that can be visualized using the GRAPHVIZ toolkit.

5.2. Experimental Results

We have tested the performance of WNL on several pure, safe and reversible net models. The list of models can be found in Table 2. The `buf_n` net models a n-buffer with 2 reachable markings, and is especially suitable to illustrate scalability issues of the algorithms. The `mutex_n` models a mutual exclusion algorithm for n processes. The `absence` workflow is loosely modelled after an example from [18], and the `complaint` workflow is the example presented in our background section (Figure 1). The `order_simp` and `transit1` models are case studies taken from [20].

We have conducted two series of experiments. First, we apply WNL without any event logs as initial knowledge, and then again with randomly generated logs as input. In both cases we count the number of queries needed to learn the model. Recall that, besides counting the queries needed for Equiv(), we only count queries answered positively by the Teacher, as these correspond to runs supplied by him, and thus reflect the actual work to be done by an expert in an adequate manner.

5.2.1. Learning from Scratch

Table 2 collects the performance of WNL when learning a model "from scratch" with respect to the size of the alphabet and the reachability graph, see Figure 2. The first two columns of the table give the alphabet size and the number of reachable states of the model. The third column gives the number of queries asked by the "non-optimized" algorithm. This is the algorithm that memorizes and deduces answers to queries (see Section 4.1), but uses none of the further optimizations (bounded lookahead, diamond property, Parikh-vectors).

The series of the n-cell buffer examples from $n = 2$ to $n = 8$ suggests that the practical performance of WNL is even better than quadratic in the number of reachable markings. The time needed for learning the nets in an applied setting is of course dominated by the number of queries a user has to answer. Synthesizing the resulting Petri net using the method proposed by Darondeau et al. (see Section 2.4) together with some post-processing to remove redundant places needs just a few seconds in the worst case and is therefore negligible.

[2] `http://libalf.informatik.rwth-aachen.de/`
[3] `http://lpsolve.sourceforge.net/`

Table 2. Learning without logs: impact of optimizations on the number of queries. B=bounded lookahead of depth 1, D=diamond heuristic, P = Parikh heuristic

Model	$\|T\|$	$\|RG\|$	non-opt.	+B	+D	+P	+B+D	+B+P	+B+P+D
`buf_2`	3	4	6	12	5	6	10	12	10
`buf_3`	4	8	16	32	11	15	21	29	21
`buf_4`	5	16	50	85	31	43	46	69	46
`buf_5`	6	32	171	216	97	128	99	158	99
`buf_6`	7	64	599	538	312	390	216	359	216
`buf_7`	8	128	2115	1304	1013	1197	461	798	461
`buf_8`	9	256	7622	3107	3357	3778	994	1765	994
`mutex_2`	6	8	23	40	19	22	30	37	27
`mutex_3`	9	20	119	168	98	106	102	148	82
`mutex_4`	12	48	578	594	502	478	310	509	225
`absence`	11	8	21	32	21	21	32	32	32
`complaint`	12	11	26	37	24	25	34	35	34
`order_simp`	9	7	17	23	17	17	23	23	23
`transit1`	25	77	883	474	439	504	221	287	221

Impact of Optimizations. The rest of Table 2 describes the impact of the different optimizations and combinations thereof on the total number of queries. Observe that the bounded lookahead optimization sometimes results in an increase in the total number of queries. these are cases in which the number additional lookahead queries was larger than the number of saved queries. The overall results show that the heuristics bring large savings. The combination BD already accounts for most of the savings, while the contribution of the Parikh heuristic is smaller.

Conceptually[4] there are two kinds of membership queries, depending on the stage in the algorithm at which they are asked. *Discovery* queries are used to discover potentially new states, while *equivalence* queries try to tell states apart. From our discussion of the different optimization techniques we cannot hope for a dramatic reduction in discovery queries (only the diamond-heuristic can reduce their number slightly) but we expect quite an impact on the number of equivalence queries. As the queries arising from Equiv() are the most complex (i.e. longest) membership queries from a user's point of view it is highly desirable to minimize their number.

Table 3 analyses the impact of the heuristics on the number of equivalence queries. It shows that bounded lookahead leads to very large saves in the number of equivalence queries, especially for more complex models. Only for very small or simple models is the overhead larger than the gain. The diamond heuristic alone also significantly cuts down the number of equivalence queries, although it is not as

[4]Syntactically they are of course indistinguishable.

Table 3. Learning without logs: number of calls to Equiv() with and without the optimizations, B=bounded lookahead of depth 1, D=diamond heuristic, P=Parikh heuristic

Model	non-opt.	+B	+D	+P	+B+D	+B+P	+B+P+D
`buf_2`	0	0	0	0	0	0	0
`buf_3`	3	1	2	2	0	0	0
`buf_4`	21	6	14	14	1	1	1
`buf_5`	106	19	64	63	2	2	2
`buf_6`	454	57	247	245	7	7	7
`buf_7`	1794	151	884	876	12	14	12
`buf_8`	6917	386	3100	3073	33	37	33
`mutex_2`	8	1	8	7	1	0	0
`mutex_3`	70	5	70	57	5	0	0
`mutex_4`	433	17	433	333	17	0	0
`absence`	9	3	9	9	3	3	3
`complaint`	11	3	10	10	2	2	2
`order_simp`	7	0	7	7	0	0	0
`transit1`	739	66	361	360	0	0	0

effective as the bounded lookahead. The Parikh and diamond heuristics have nearly the same effect on the number of equivalence queries. Since moreover they both optimize in the same "direction", their combination is not too fruitful. Our main finding is that the combination of bounded lookahead and the Parikh/diamond heuristic eliminates most of the calls to Equiv().

5.2.2. Learning with Initial Knowledge.

Our learning approach is not designed to replace workflow mining, but to complement the information of incomplete logs. To analyse the effect of an existing log, we have generated random event logs for our models. For this, we have randomly generated firing sequences of the models as follows: starting at the initial marking, we compute the set of enabled transitions, and choose one of them randomly with equal probability. We iterate this process until either the initial marking is reached again, or a certain bound is reached. When the bound is reached, the generated sequence is extended to a run by means of a search (see Figure 5 for an example log). Notice that the runs in the generated log-files are not unique—runs that are more likely will probably appear multiple times, which is also the case for real-world event logs. For each log size we generated 100 logs of that size, and computed the average number of queries needed by WNL to learn the model starting from each of the 100 logs.

We found out that for small models like the 2-cell buffer or the `complaint` workflow very small logs (< 10 runs) are complete, and the Teacher does not have to supply any additional run. Clearly, for larger models, we can only expect that the Teacher's work is reduced but not completely eliminated. Table 6)

```
.a.b.a.c.d.b.c.a.d.b.c.d.
.a.b.c.d.
.a.b.c.a.b.d.c.a.d.b.c.d.
.a.b.a.c.d.b.c.d.
.a.b.a.c.b.a.d.c.d.b.c.d.
.a.b.c.a.d.b.a.c.d.b.c.d.
.a.b.c.a.b.d.c.d.
.a.b.a.c.b.d.a.c.d.b.c.d.
.a.b.c.d.
.a.b.c.d.
```

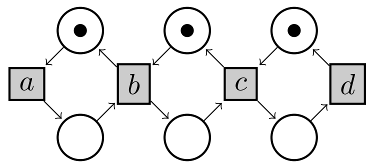


Figure 5. Example event log for 3-cell buffer

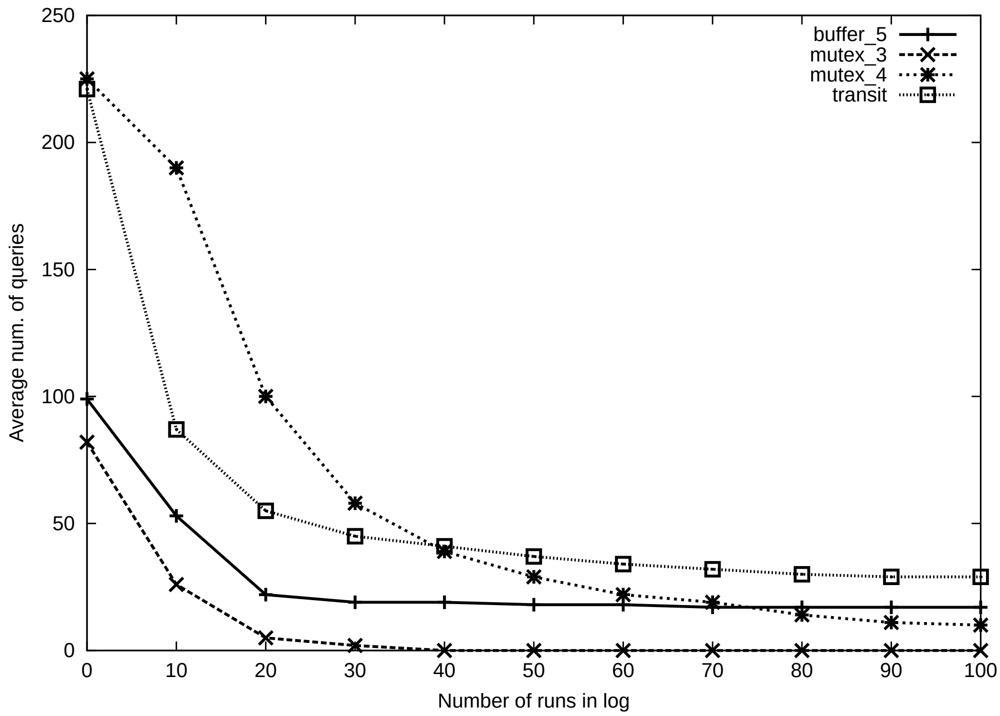


Figure 6. Average number of queries needed by WNL applied to event logs of different sizes

shows the reduction for the `buf_5`, `mutex_3`, `mutex_4`, and `transit` models as a function of the log size. All optimizations described in Section 4 were employed for these tests.

We observe that small logs already reduce the number of queries to be answered quite drastically. At the same time, because our logs may contain many identical entries, larger logs contribute less and less new knowledge. This reflects the situation of real-life logs, which mostly contain common executions of a workflow but lack less common runs.

The results depicted in Figures 2, 3 and 6 suggest that, despite the seemingly intimidating result in Section 3.5, learning of workflow models is quite feasible for practical applications.

6. Conclusion

We have presented a new approach for mining workflow nets based on learning techniques. The approach palliates the problem of incompleteness of event logs: if a log is incomplete, our algorithm derives membership queries identifying the missing knowledge. The queries can be passed to an expert, whose answers allow to produce a model.

We have shown the correctness and completeness of our approach given a teacher answering workflow membership queries. Starting with general combinatorial arguments showing that workflow models can in principle be learned, we have derived a learning algorithm requiring a single exponential number of queries in the worst case, and we have given a matching lower bound. We have also shown experimental evidence indicating that the combination of an event log, even of small size, and a Teacher responsible for providing information on "corner cases" allows to efficiently produce models in practically relevant cases.

There are several promising paths for further research. One aspect is the application of learning to the *design* of workflows. In this approach an expert on business processes (the Teacher) and a modelling expert (the Learner) cooperate. The Learner asks the Teacher queries about the expected behaviour of the workflow, and proposes models. The models are criticized by the Teacher, who continues to ask queries and refine the model until the Teacher is satisfied. The proposal of a model by the Learner and its acceptance or rejection by the Teacher can be naturally modelled as an equivalence query (see the discussion in Section 3.6). We expect to transfer ideas from the field of learning models of software systems [13] to workflow systems, and develop "teaching assistants" that filter the queries, automatically answering those for which the answer can be deduced from current information (for instance because it is known that two tasks must be concurrent), and only passing to the expert the remaining ones. Here we expect to profit from related work by Desel, Lorenz and others [12]. An important point for process mining and even more for process design is designing fault tolerance techniques allowing to cope with false answers by the Teacher. Finally, learning more general classes of Petri nets, and applications to modelling/reconstruction of distributed systems, or biological/chemical processes, are also promising paths for future work.

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