1 CTL and LTL Specifications

(a) * Show that $AfUg$ is equivalent to $\neg[(E(\neg gU(\neg g \land \neg f)) \lor EG\neg g]$.

(b) Given a formula $EaUb$, and a Kripke structure $K = (S, R, L)$, describe an algorithm which labels all states $s \in S$ where $K, s \models EaUb$, in linear time, i.e., in time $O(|S| + |R|)$.

Note: the algorithm has to label all states where $K, s \models EaUb$, not just find some such states. For linear time, operations on lists, sets, etc. have to be counted.

(c) ** Same as above, for $EGb$.

(d) * Let $K_1 = (S_1, R_1, L_1)$ and $K_2 = (S_2, R_2, L_2)$. We define $K_1 \leq K_2$ if $S_1 = S_2$, $R_1 \subseteq R_2$, and $L_1 = L_2$.

i) Show that $\leq$ is a partial order.

ii) Show the following lemma:

Let $\phi$ be an LTL specification, and $K_1 \leq K_2$ Kripke structures. If $K_2, s \models \phi$ then $K_1, s \models \phi$.

iii) Show that there exists a CTL specification which cannot be expressed in LTL.

Hint: Use the previous Lemma on the formula $EFp$.

(e) ** Find a Kripke structure $K, s$ such that $K, s \models AFGp$ but $K, s \not\models AFAGp$.

(f) *** Show that there exists an LTL specification which cannot be expressed in CTL.

2 Fixpoint Characterizations

Reconsider the Kripke structure in Figure 1. Using the fixpoint characterizations of CTL, show which states satisfy the following specifications:

(a) $EFa$.

(b) $AGa$.

(c) $EaUb$.

(d) $(a \lor q) \rightarrow EXb$.

Figure 1: Kripke structure.