Reachability in continuous vector addition systems: from theory to practice

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May 13, 2015

Reachability in continuous vector addition systems: from theory to practice

Vincent Antaki[1], Michael Blondin[1,2] & Pierre McKenzie[1,2]

[1]DIRO, Université de Montréal, Canada

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Project

- Tool for reachability in VASS

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- Relaxations to decide non reachability

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  - Coverability: EXPSPACE/PSPACE-complete

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  - 1-VASS, $\mathbb{Z}$-VASS: NP-complete

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  - Continuous Petri nets: P-complete

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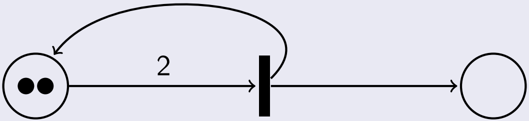
  - 1-VASS, $\mathbb{Z}$-VASS: NP-complete
  - **Continuous Petri nets**: **P-complete**

Continous Petri nets (CPN)

Transitions fired by an amount $\alpha \in \mathbb{R}_{\geq 0}$



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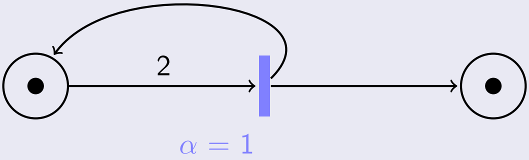

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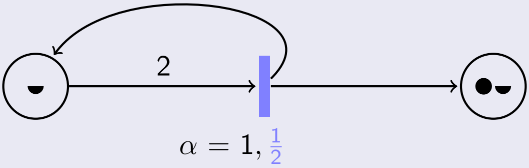

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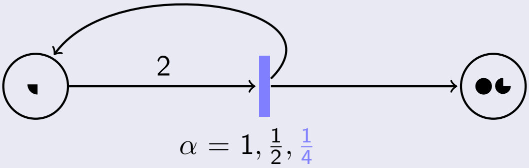

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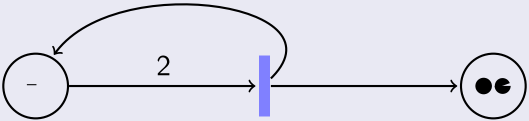

$\alpha = 1, \frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{2^n}$

Continuous vector addition systems with states (CVASS)

- What is a continuous VASS?

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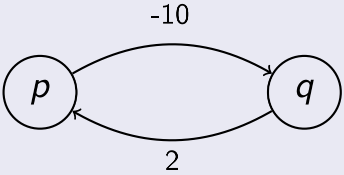
- Not defined in the literature

Continuous vector addition systems with states (CVASS)

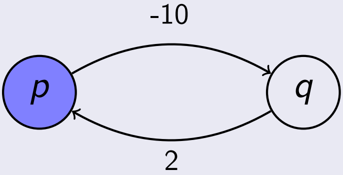
- What is a continuous VASS?
- Not defined in the literature
- Two possible definitions

CVASS with “unique states”



CVASS with “unique states”

1



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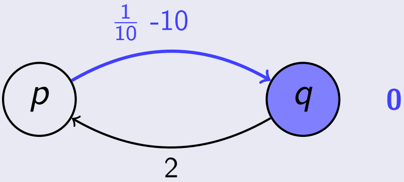

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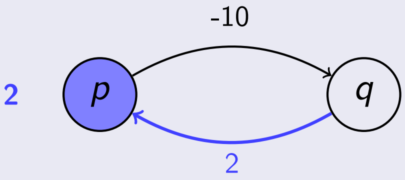

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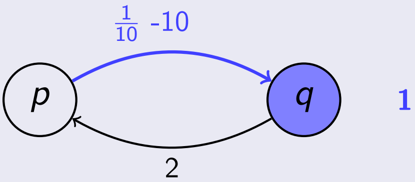

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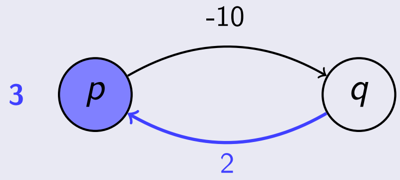

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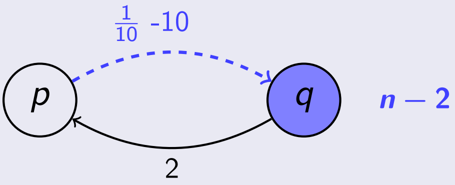

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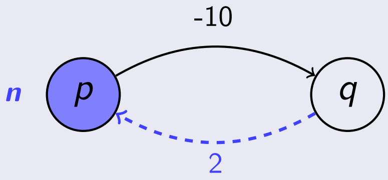
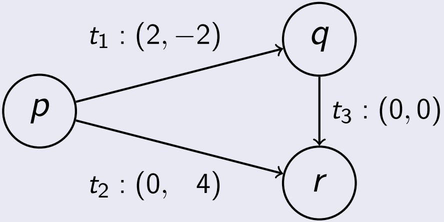


CVASS with “multiple states”



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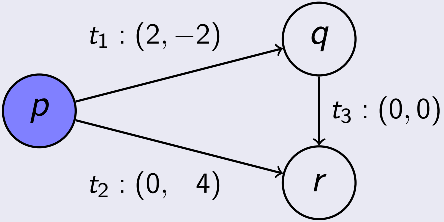
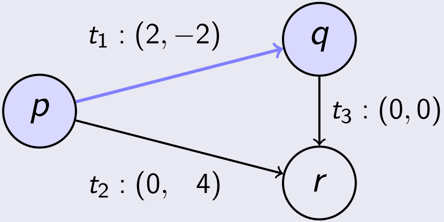


$$\begin{array}{ccc|cc} \boldsymbol{p} & \boldsymbol{q} & \boldsymbol{r} & & \\ (1, & 0, & 0, & 0, & 1) \end{array}$$

CVASS with “multiple states”



$$\begin{array}{l}
\quad \boldsymbol{p} \;\; \boldsymbol{q} \;\; \boldsymbol{r} \\
(1, 0, 0, \;\vdots\; 0, 1) \xrightarrow{\frac{1}{2} t_1} \\
(\frac{1}{2}, \frac{1}{2}, 0, \;\vdots\; 1, 0)
\end{array}$$

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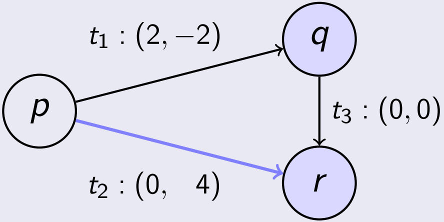

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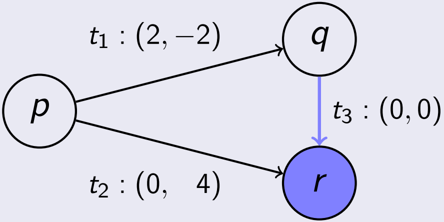
\begin{array}{l}
\quad \boldsymbol{p} \;\; \boldsymbol{q} \;\; \boldsymbol{r} \\
(1, 0, 0, \;\vdots\; 0, 1) \xrightarrow{\frac{1}{2} t_1} \\
(\frac{1}{2}, \frac{1}{2}, 0, \;\vdots\; 1, 0) \xrightarrow{\frac{1}{2} t_2} \\
(0, \frac{1}{2}, \frac{1}{2}, \;\vdots\; 1, 2)
\end{array}
$$

CVASS with “multiple states”



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\quad\;\; \boldsymbol{p} \;\; \boldsymbol{q} \;\; \boldsymbol{r} \\
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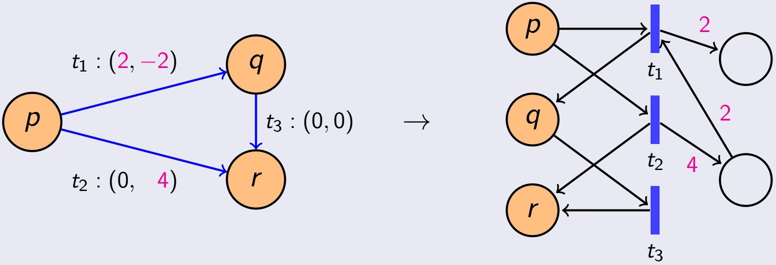
(0, \tfrac{1}{2}, \tfrac{1}{2}, \mid 1, 2) \xrightarrow{\frac{1}{2} t_3} \\
(0, 0, 1, \mid 1, 2)
\end{array}
$$

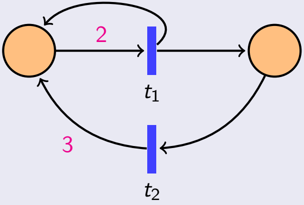
CVASS with "multiple states" $\leq$ CPN

Usual transformation, straightforward proof

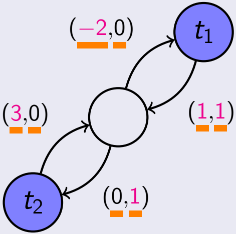


CPN $\leq$ CVASS with "multiple states"

Usual transformation, less straightforward proof



$\longrightarrow$



Our implementation

$\vdots$

$T' \leftarrow T$
while $T' \neq \emptyset$ **do**
  $nbsol \leftarrow 0$; **sol** $\leftarrow \mathbf{0}$
  for $t \in T'$ **do**
    solve $\exists ? \mathbf{v}\ \mathbf{v} \geq \mathbf{0} \wedge \mathbf{v}[t] > 0 \wedge \boldsymbol{C}_{P \times T'} \mathbf{v} = \boldsymbol{m} - \boldsymbol{m}_0$
    if $\exists \mathbf{v}$ **then** $nbsol \leftarrow nbsol + 1$; **sol** $\leftarrow$ **sol** $+ \mathbf{v}$
  end
  if $nbsol = 0$ **then return false else sol** $\leftarrow \frac{1}{nbsol}$**sol**

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- Fraca & Haddad PN'13

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```
t1   = np.array(range(0, n2))
b_eq = np.array(m - m0)

while t1.size != 0:
  l           = t1.size
  nbsol, sol = 0, np.zeros(l, dtype=Fraction)
  A_eq        = incident(subnet(net, t1))

  for t in t1:
    objective_vector = [objective(t, x) for x in range(0, l)]
    result           = solve_qsopt(objective_vector, A_eq, b_eq, t)

    if result is not None:
      nbsol += 1
      sol   += result
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- Fraca & Haddad PN'13
- Reachability in CPN $\in$ P
- python™ with NumPy

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- Reachability in CPN $\in$ P
- python™ with NumPy
- 299 lines of code

  (215 code + 84 docstring)

Polynomial time algorithm (Fraca & Haddad PN'13)

Algorithm 2: Decision algorithm for reachability

`Reachable`$(\langle \mathcal{N}, \boldsymbol{m}_0\rangle, \boldsymbol{m})$: status
Input: a CPN system $\langle \mathcal{N}, \boldsymbol{m}_0\rangle$, a marking $\boldsymbol{m}$
Output: the reachability status of $\boldsymbol{m}$
Output: the Parikh image of a witness in the positive case
Data: *nbsol*: integer; $\mathbf{v}, \mathbf{sol}$: vectors; T': subset of transitions

if $\boldsymbol{m} = \boldsymbol{m}_0$ **then return** (**true**,**0**)
$T' \leftarrow T$
while $T' \neq \emptyset$ **do**
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    end
    if $nbsol = 0$ **then return false else** $\mathbf{sol} \leftarrow \frac{1}{nbsol}\mathbf{sol}$
    $T' \leftarrow [\![\mathbf{sol}]\!]$
    $T' \leftarrow T' \cap$ `maxFS`$(\mathcal{N}_{T'}, \boldsymbol{m}_0[{}^\bullet T'^\bullet])$
    $T' \leftarrow T' \cap$ `maxFS`$(\mathcal{N}^{-1}_{T'}, \boldsymbol{m}[{}^\bullet T'^\bullet])$ `/* deleted for lim-reachability */`
    if $T' = [\![\mathbf{sol}]\!]$ **then return** (**true**,**sol**)
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Relatively easy with `NumPy`

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    $\boxed{\textbf{solve } \exists ? \mathbf{v}\ \mathbf{v} \geq \mathbf{0} \wedge \mathbf{v}[t] > 0 \wedge \boldsymbol{C}_{P \times T'} \mathbf{v} = \boldsymbol{m} - \boldsymbol{m}_0}$ A bit trickier
    if $\exists \mathbf{v}$ **then** $nbsol \leftarrow nbsol + 1$; $\mathbf{sol} \leftarrow \mathbf{sol} + \mathbf{v}$
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System of linear inequalities

$\exists \mathbf{x} \in \mathbb{R}^k$ such that $\mathbf{x} \geq \mathbf{0}$, $\mathbf{x}_t > 0$ and $A\mathbf{x} = \mathbf{b}$?

System of linear inequalities

$\vdots$

$\exists \mathbf{x} \in \mathbb{R}^k$ such that $\mathbf{x} \geq \mathbf{0}$, $\mathbf{x}_t > 0$ and $A\mathbf{x} = \mathbf{b}$?

$\vdots$

Without this condition, could simply use simplex

Handling the strict inequality

1. Solve

$$\begin{array}{ll} \text{Maximize} & \mathbf{x}_t \\ \text{Subject to} & A\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \end{array}$$

Handling the strict inequality

1. Solve

$$\begin{array}{ll} \text{Maximize} & \mathbf{x}_t \\ \text{Subject to} & A\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0} \end{array}$$

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2.
   - If $\mathbf{x}_t > 0$, return $\mathbf{x}$
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2.
   - If $\mathbf{x}_t > 0$, return $\mathbf{x}$
   - If $\mathbf{x}_t = 0$, return `"no solution"`
   - If no solution, return `"no solution"`
   - If unbounded, continue

Handling the strict inequality

3. Solve

$$\begin{array}{ll} \text{Minimize} & \mathbf{x}_t \\ \text{Subject to} & A\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0},\ \mathbf{x}_t \geq 1 \end{array}$$

Handling the strict inequality

3. Solve

$$\begin{array}{ll} \text{Minimize} & \mathbf{x}_t \\ \text{Subject to} & A\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0},\ \mathbf{x}_t \geq 1 \end{array}$$

return $\mathbf{x}$

Simplex implementations

- Usually in floating-point arithmetic

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- Error-prone, even worse with $2|T|^2$ resolutions

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Current solution

`QSopt-Exact`: exact solver from

Exact solutions to linear programming problems

David L. Applegate [a] William Cook [b] Sanjeeb Dash [c] Daniel G. Espinoza [d,*]

Open questions

- Floating-point solver + testing certificates (Farkas' lemma, reconstruct simplex tableaux in $\mathbb{Q}$)

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- Reachability in CVASS with “unique states”?

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- Floating-point solver + testing certificates (Farkas' lemma, reconstruct simplex tableaux in $\mathbb{Q}$)
- Reachability in CVASS with "unique states"?
- Any use for CVASS with "unique states"?

Further work

- Test other solvers

Further work

- Test other solvers
- Benchmarks

Further work

- Test other solvers
- Benchmarks
- Next modules

Thank you! Merci! Danke!