Space-Efficient Scheduling of Stochastically Generated Tasks

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Task Systems

We study tasks that stochastically generate new tasks.
The execution of a task can generate subtasks.

Examples:

- Divide-and-conquer Algorithms:
  Either solve the problem directly or solve sub-instances.
- Multi-Threaded Programs:
  Either terminate, or spawn new thread, or none of that.

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- Divide-and-conquer Algorithms:
  Either solve the problem directly or solve sub-instances.
- Multi-Threaded Programs:
  Either terminate, or spawn new thread, or none of that.

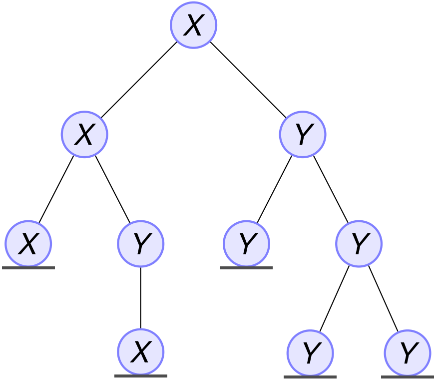
How much memory is needed?

Task Systems

Formally, task systems are like context-free grammars.

$$X \hookrightarrow XY$$
$$X \hookrightarrow \varepsilon$$

$$Y \hookrightarrow YY$$
$$Y \hookrightarrow X$$
$$Y \hookrightarrow \varepsilon$$



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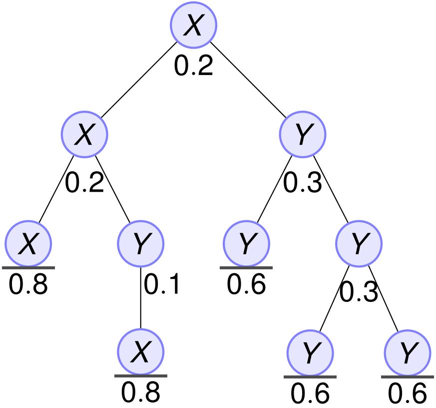

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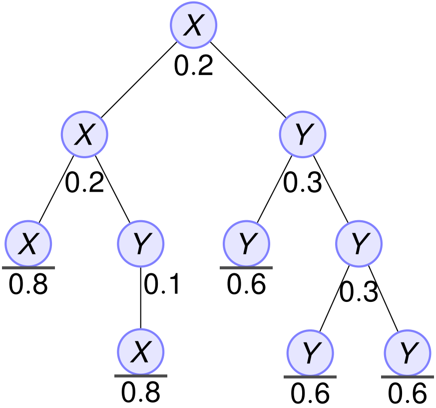

Probability of this tree:
$0.2 \cdot 0.2 \cdot 0.3 \cdot 0.8 \cdot 0.1 \cdot 0.6 \cdot 0.3 \cdot 0.8 \cdot 0.6 \cdot 0.6$

A Derivation

$$X \hookrightarrow XY$$

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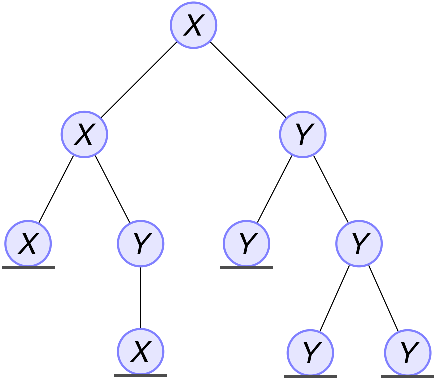

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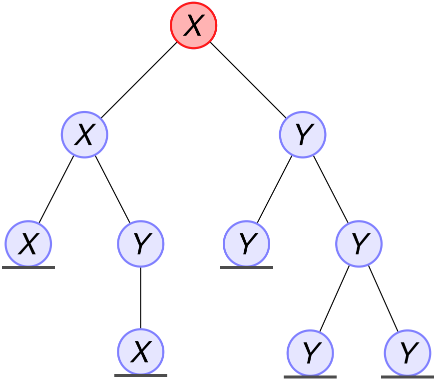

X

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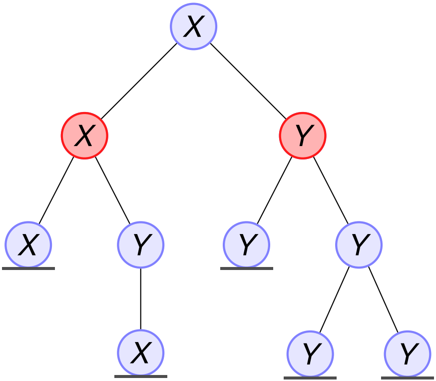

$X \Rightarrow XY$

A Derivation

$X \hookrightarrow XY$

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$Y \hookrightarrow YY$

$Y \hookrightarrow X$

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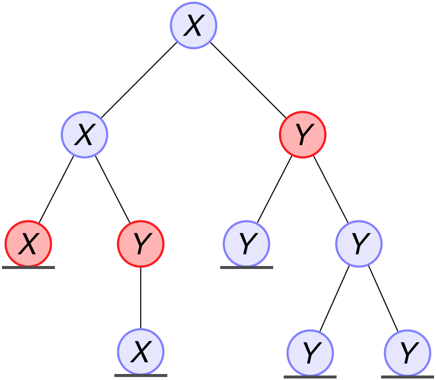

$X \Rightarrow XY \Rightarrow XXY$

A Derivation

$$X \hookrightarrow XY$$

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$$Y \hookrightarrow YY$$

$$Y \hookrightarrow X$$

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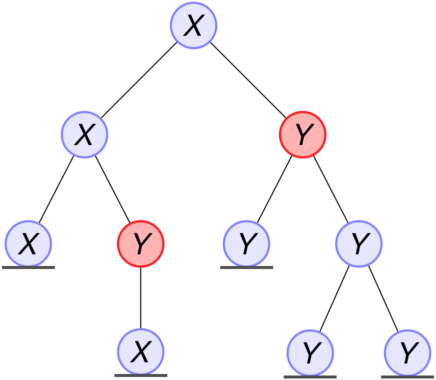

$$X \Rightarrow XY \Rightarrow XXY \Rightarrow \color{red}{YY}$$

A Derivation

$$X \hookrightarrow XY$$
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$$Y \hookrightarrow YY$$
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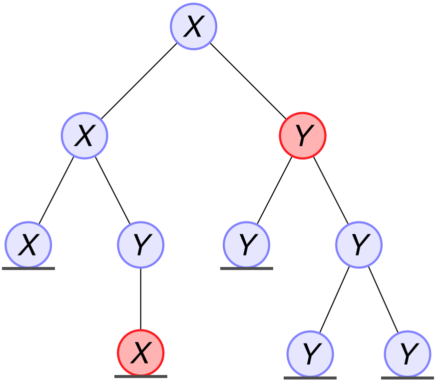

$$X \Rightarrow XY \Rightarrow XXY \Rightarrow YY \Rightarrow {\color{red}XY}$$

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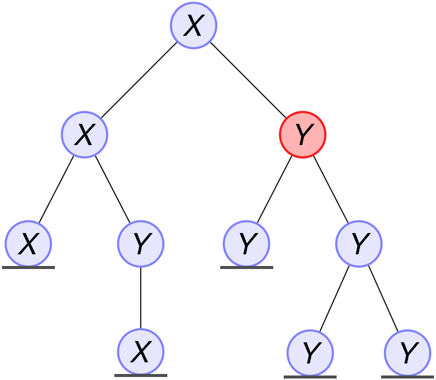

$$X \Rightarrow XY \Rightarrow XXY \Rightarrow YY \Rightarrow XY \Rightarrow Y$$

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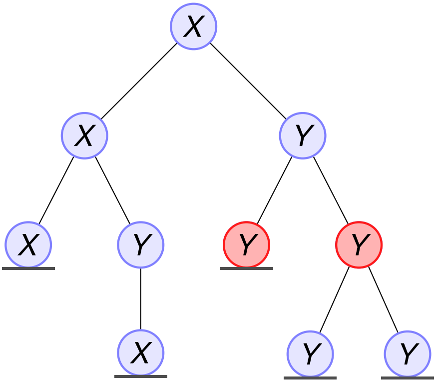

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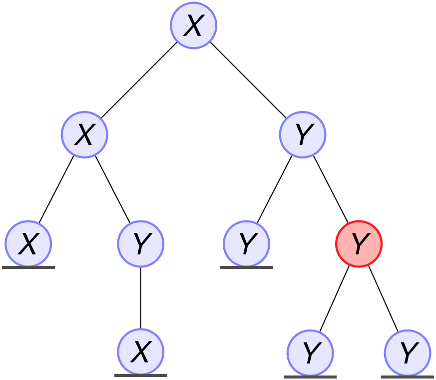

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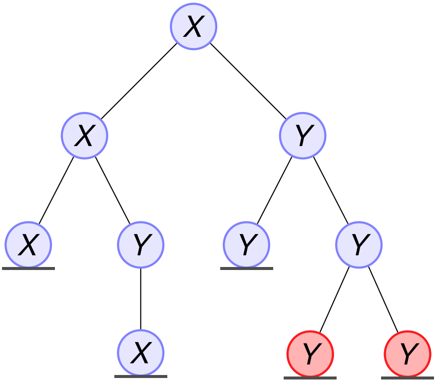

$$X \Rightarrow XY \Rightarrow XXY \Rightarrow YY \Rightarrow XY \Rightarrow Y \Rightarrow YY \Rightarrow Y \Rightarrow \color{red}{YY}$$

A Derivation

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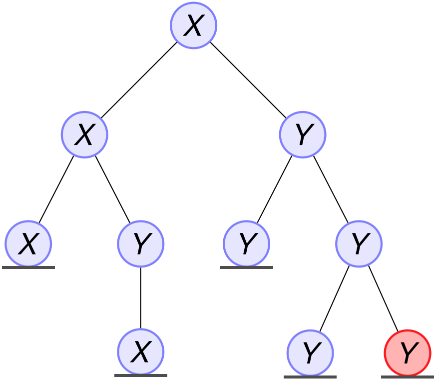

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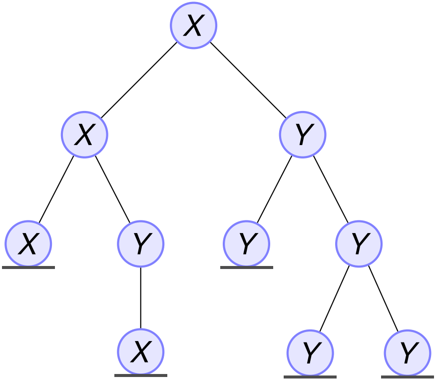

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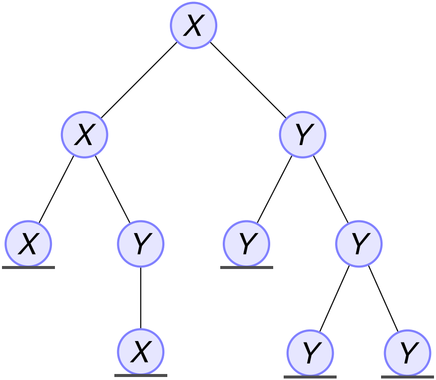

$$X \Rightarrow XY \Rightarrow XXY \Rightarrow YY \Rightarrow XY \Rightarrow Y \Rightarrow YY \Rightarrow Y \Rightarrow YY \Rightarrow Y \Rightarrow \varepsilon$$

Time: 10 (number of nodes)
Space: 3

A Derivation

$$X \hookrightarrow XY$$

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$$Y \hookrightarrow YY$$

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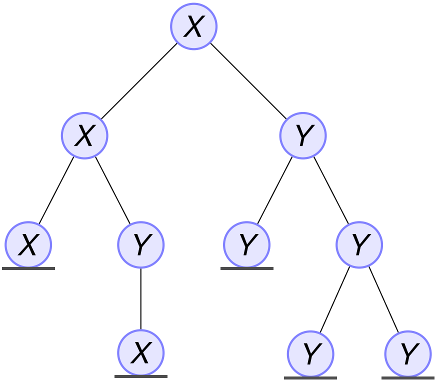

$$X \Rightarrow XY \Rightarrow {\color{red}XXY} \Rightarrow YY \Rightarrow XY \Rightarrow Y \Rightarrow YY \Rightarrow Y \Rightarrow YY \Rightarrow Y \Rightarrow \varepsilon$$

Time: 10 (number of nodes)

Space: 3

Another Derivation

$$X \hookrightarrow XY$$
$$X \hookrightarrow \varepsilon$$
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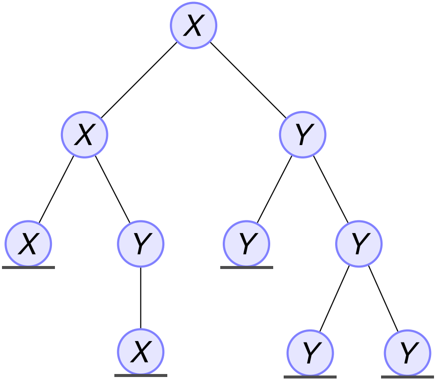

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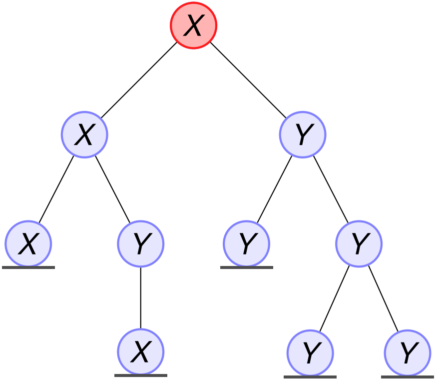

X

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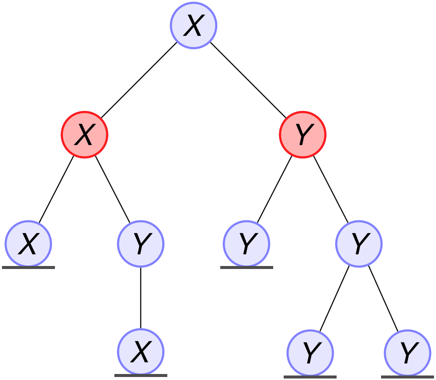

$X \Rightarrow \color{red}{XY}$

Another Derivation

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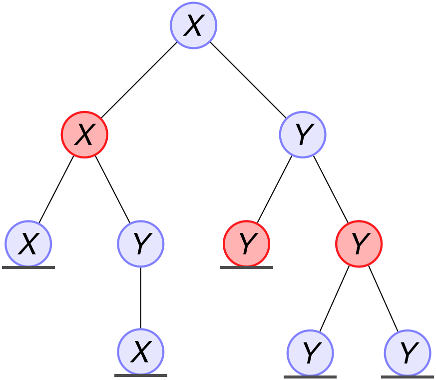

$$X \Rightarrow XY \Rightarrow XYY$$

Another Derivation

$$X \hookrightarrow XY$$

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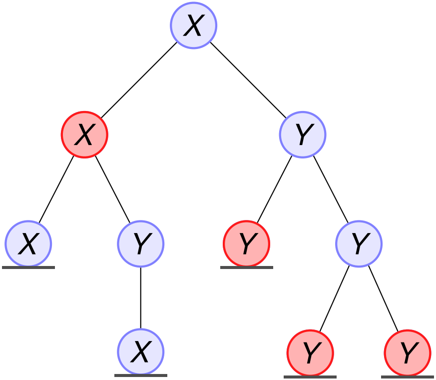

$$X \Rightarrow XY \Rightarrow XYY \Rightarrow XYYY$$

Another Derivation

$X \hookrightarrow XY$

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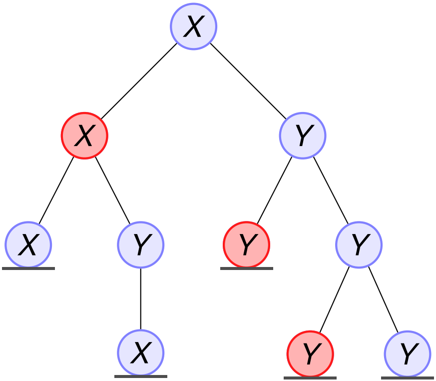

$X \Rightarrow XY \Rightarrow XYY \Rightarrow XYYY \Rightarrow$ XYY

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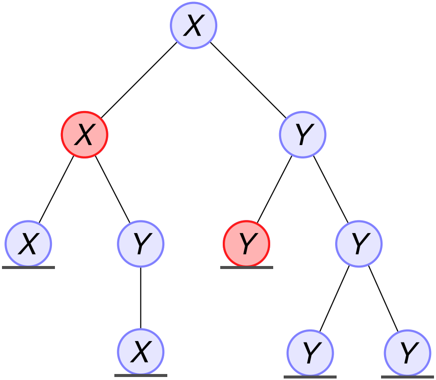

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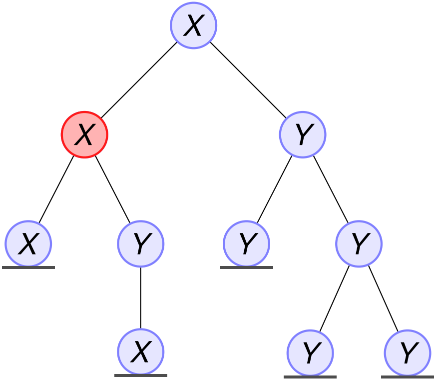

$$X \Rightarrow XY \Rightarrow XYY \Rightarrow XYYY \Rightarrow XYY \Rightarrow XY \Rightarrow {\color{red}X}$$

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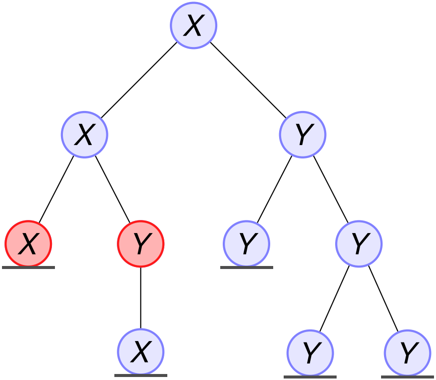

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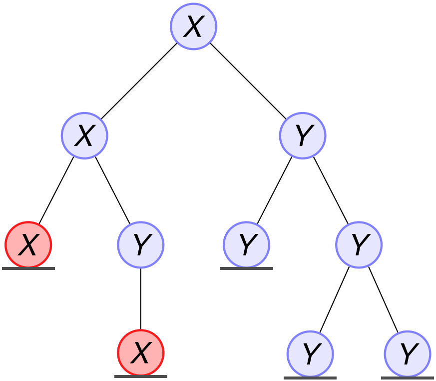

$$X \Rightarrow XY \Rightarrow XYY \Rightarrow XYYY \Rightarrow XYY \Rightarrow XY \Rightarrow X \Rightarrow XY \Rightarrow \color{red}{XX}$$

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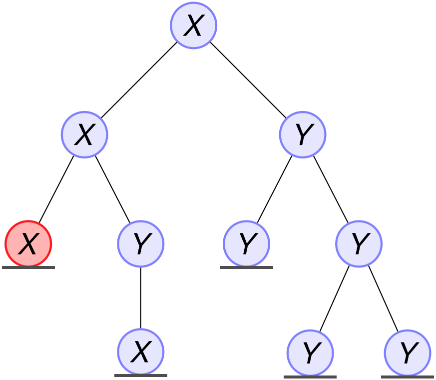

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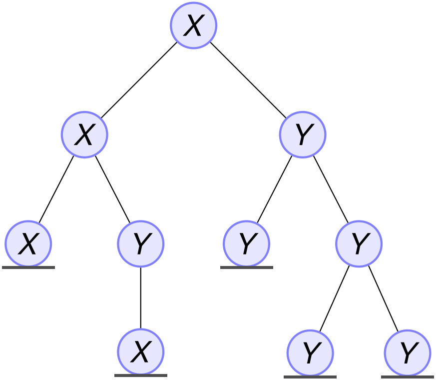

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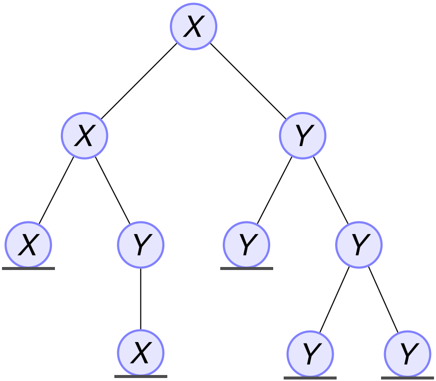

$$X \Rightarrow XY \Rightarrow XYY \Rightarrow XYYY \Rightarrow XYY \Rightarrow XY \Rightarrow X \Rightarrow XY \Rightarrow XX \Rightarrow X \Rightarrow \varepsilon$$

Time: 10 (number of nodes)

Space: 4

Another Derivation

$$X \hookrightarrow XY$$

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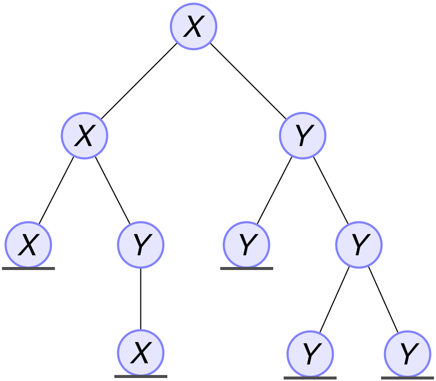

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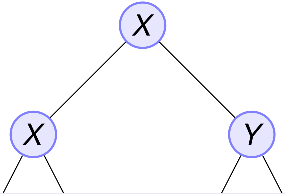

Each finite tree has its

- Probability (does not depend on scheduler)
- Time (does not depend on scheduler)
- Space (depends on scheduler)

Another Derivation

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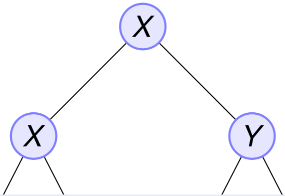
$X \hookrightarrow \varepsilon$



Each finite tree has its

- Probability (does not depend on scheduler)
- Time (does not depend on scheduler)
- Space (depends on scheduler)

Raises questions like:

- What is the expected time and the expected space?
- How are these random variables distributed?

Time has been studied before. We focus on space.

Related Work: Branching Processes

$$X \overset{0.2}{\hookrightarrow} XY$$
$$X \overset{0.8}{\hookrightarrow} \varepsilon$$

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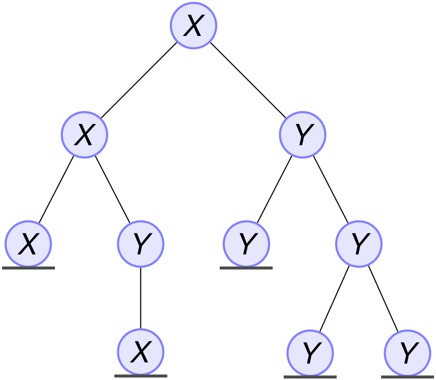

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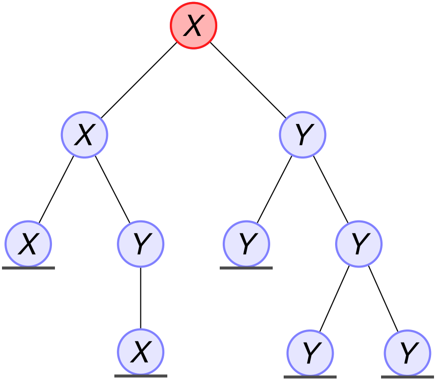

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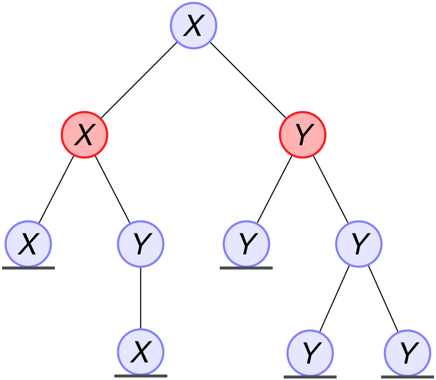

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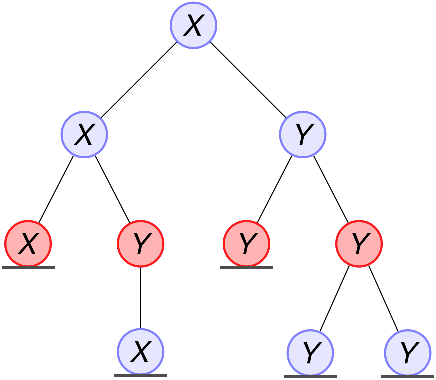

Related Work: Branching Processes

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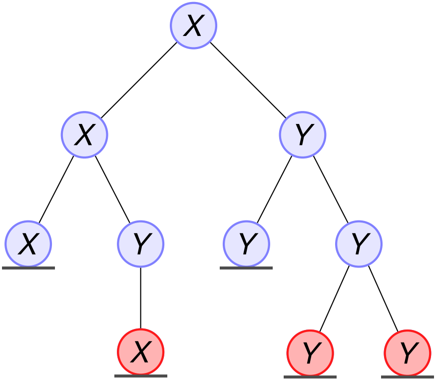

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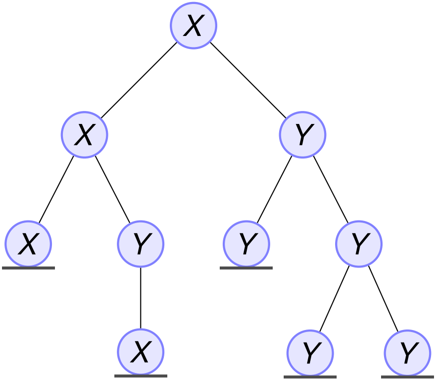

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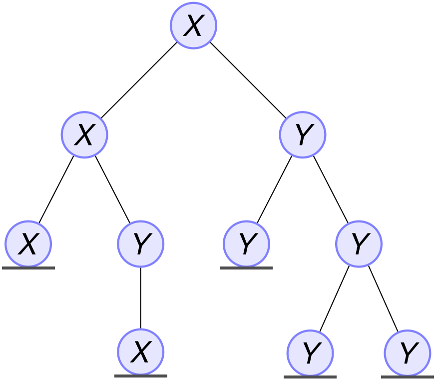

Branching Processes have been extensively studied.
They are models for biological or physical systems.
But they assume an unbounded number of “processors”.

Termination Probability

$X \xhookrightarrow{0.2} XY$

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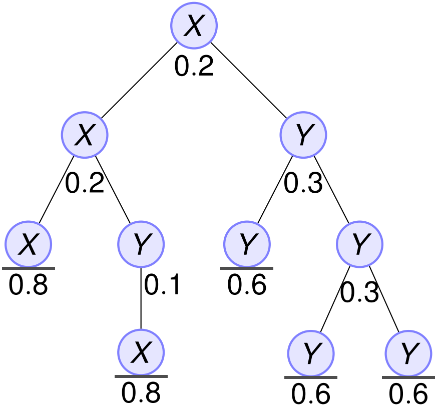


Each tree has its probability.
The sum of these probabilities is the "termination probability".
Is it always 1?

Termination Probability and the function $\boldsymbol{f}$

A task system induces a vector $\boldsymbol{f}(\boldsymbol{x})$.

For our example: $\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix}$ with

$$\left.\begin{array}{l} X \xhookrightarrow{0.2} XY \\ X \xhookrightarrow{0.8} \varepsilon \end{array}\right\} \quad f_x(x,y) = 0.2xy + 0.8$$

$$\left.\begin{array}{l} Y \xhookrightarrow{0.3} YY \\ Y \xhookrightarrow{0.1} X \\ Y \xhookrightarrow{0.6} \varepsilon \end{array}\right\} \quad f_y(x,y) = 0.3y^2 + 0.1x + 0.6$$

Termination Probability and the function $\boldsymbol{f}$

A task system induces a vector $\boldsymbol{f}(\boldsymbol{x})$.

For our example: $\boldsymbol{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ and $\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix}$ with

$$\left.\begin{array}{l} X \xhookrightarrow{0.2} XY \\ X \xhookrightarrow{0.8} \varepsilon \end{array}\right\} \quad f_x(x,y) = 0.2xy + 0.8$$

$$\left.\begin{array}{l} Y \xhookrightarrow{0.3} YY \\ Y \xhookrightarrow{0.1} X \\ Y \xhookrightarrow{0.6} \varepsilon \end{array}\right\} \quad f_y(x,y) = 0.3y^2 + 0.1x + 0.6$$

Proposition (well-known, see [Harris])

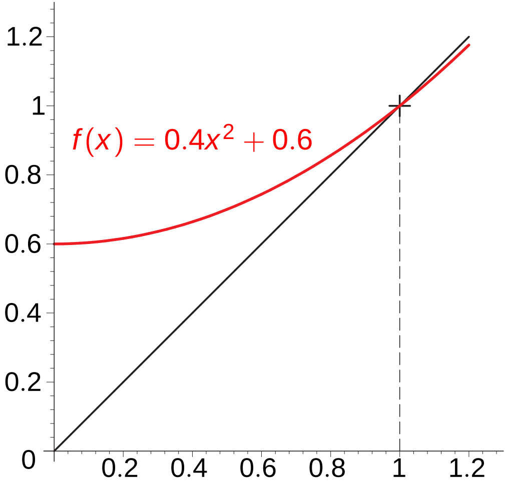
The termination probability is
the (first component of the) least fixed point of $\boldsymbol{f}$.

The function *f*

The subcritical case: $\begin{cases} \text{termination probability} & = 1 \\ \text{expected time} & = \text{finite} \end{cases}$

$X \xhookrightarrow{0.4} XX$

$X \xhookrightarrow{0.6} \varepsilon$

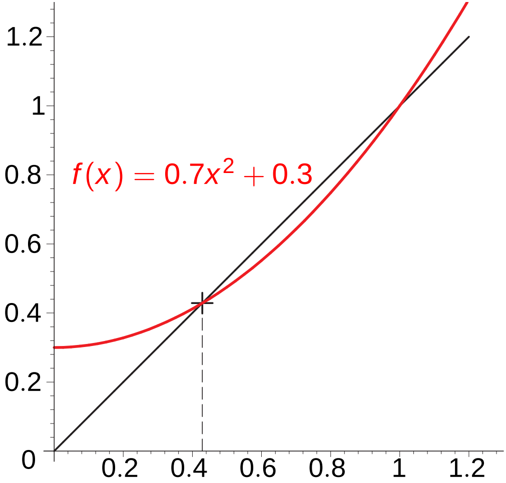


The function *f*

The supercritical case: $\begin{cases} \text{termination probability} & < 1 \\ \text{expected time} & = \infty \end{cases}$

$X \xhookrightarrow{0.7} XX$

$X \xhookrightarrow{0.3} \varepsilon$



The function f

The critical case: $\begin{cases} \text{termination probability} & = 1 \\ \text{expected time} & = \infty \end{cases}$

$$X \xhookrightarrow{0.5} XX$$
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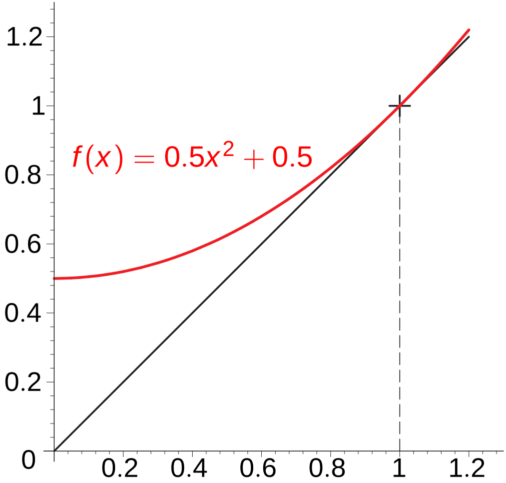

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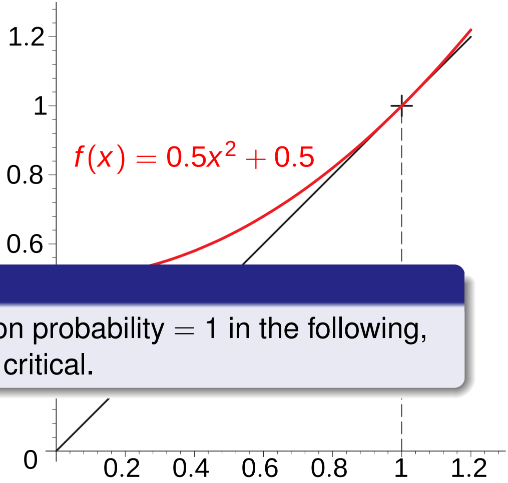
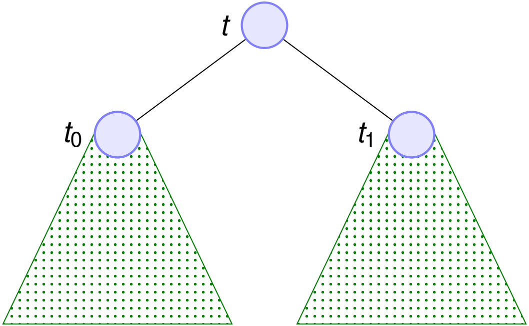


Assumption

We assume termination probability $= 1$ in the following, i.e., subcritical or critical.

Optimal Scheduling

Given a tree t with two children t_0, t_1.
What is the optimal scheduling?



$S^{op}(t) =$

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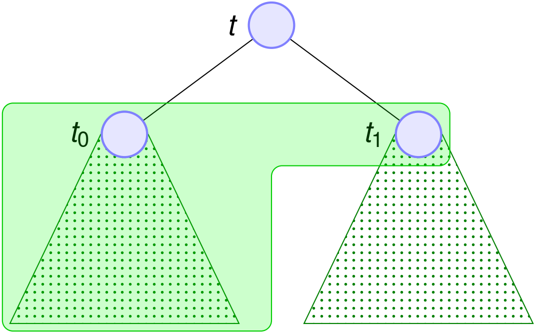

$$S^{op}(t) = \left\{ \begin{array}{l} \max\{\, S^{op}(t_0)+1 \;,\; S^{op}(t_1) \,\} \\ \\ \end{array} \right\}$$

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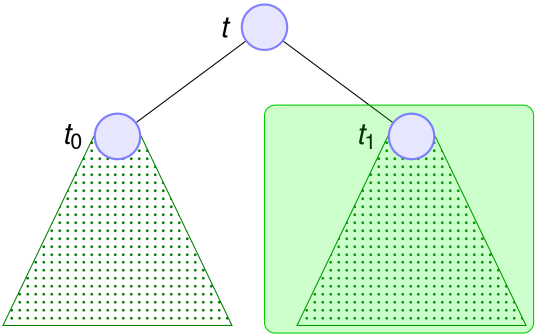

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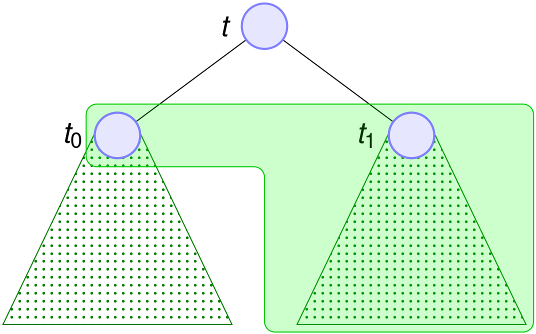

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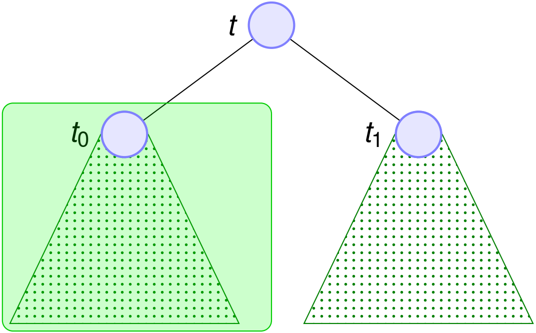

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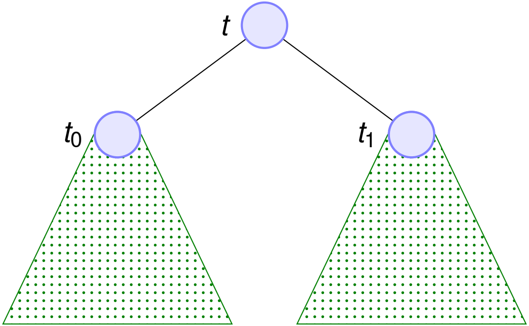
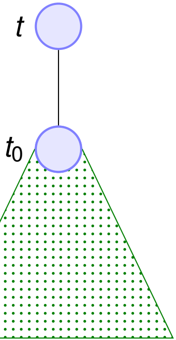


$$S^{op}(t) = \min \left\{ \begin{array}{l} \max\{\ S^{op}(t_0)+1\ ,\ S^{op}(t_1)\ \}, \\ \max\{\ S^{op}(t_1)+1\ ,\ S^{op}(t_0)\ \} \end{array} \right\}$$

Optimal Scheduling

Given a tree t with just one child t_0.
What is the optimal scheduling?



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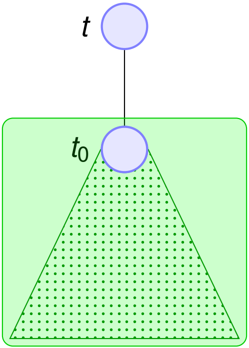

$$S^{op}(t) = S^{op}(t_0)$$

Optimal Scheduling

So we can determine S^{op} for any tree t:

$$S^{op}(t) = \begin{cases} \min\left\{ \begin{array}{l} \max\{S^{op}(t_0)+1, S^{op}(t_1)\}, \\ \max\{S^{op}(t_1)+1, S^{op}(t_0)\} \end{array} \right\} & \text{if } t \text{ has two children } t_0, t_1 \\ S^{op}(t_0) & \text{if } t \text{ has one child } t_0 \\ 1 & \text{if } t \text{ has no children.} \end{cases}$$

Optimal Scheduling

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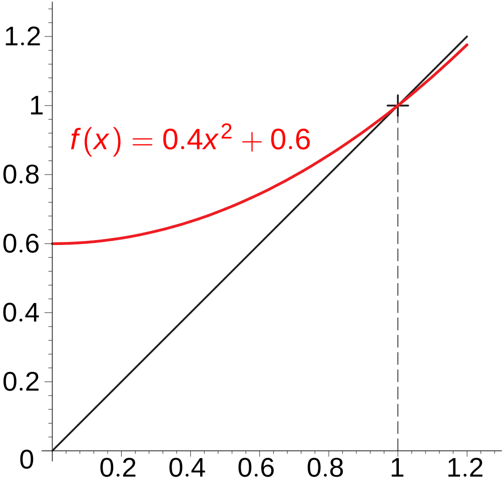
What is the distribution of S^{op}, if trees are randomly generated?

Newton's Method

Let $g(x) := f(x) - x$ and apply Newton's method to $g(x) = 0$:



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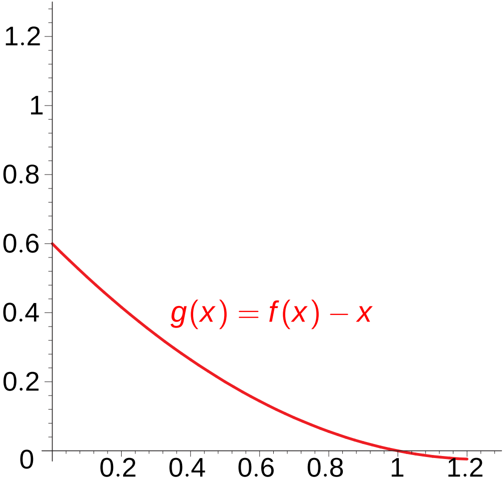

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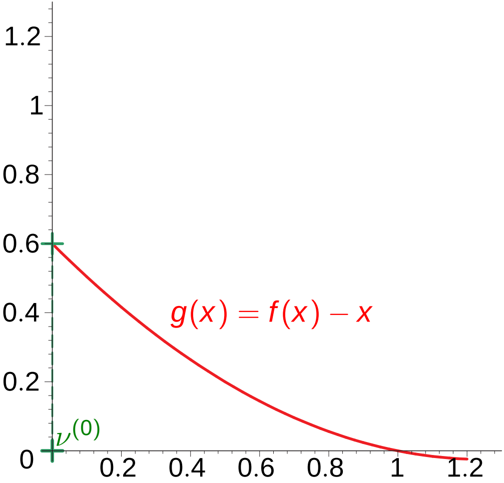

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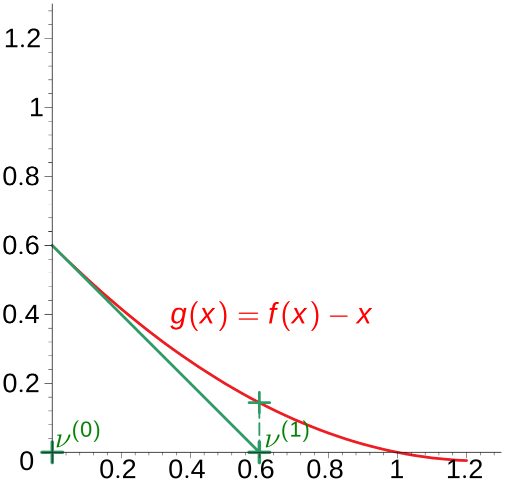

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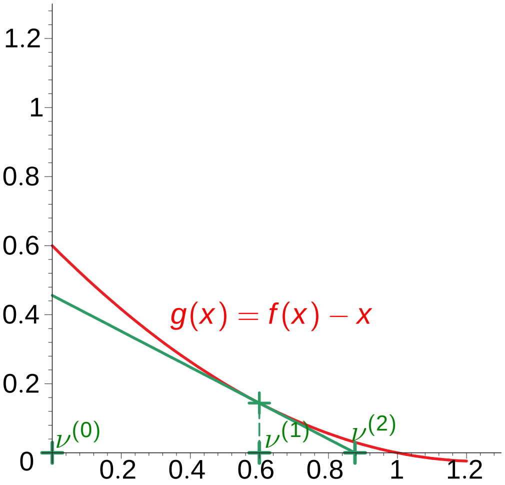

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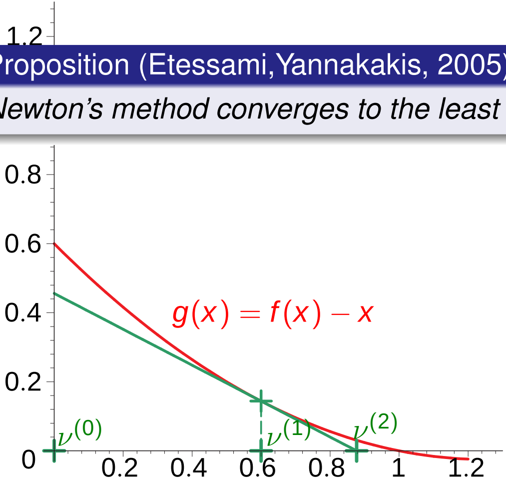

Proposition (Etessami, Yannakakis, 2005)

Newton's method converges to the least solution.

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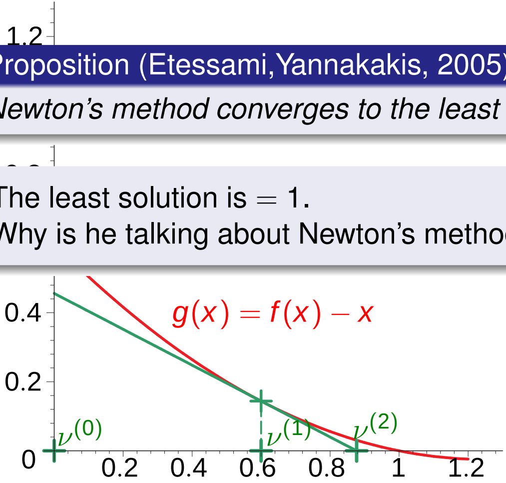
The least solution is $= 1$.
Why is he talking about Newton's method??



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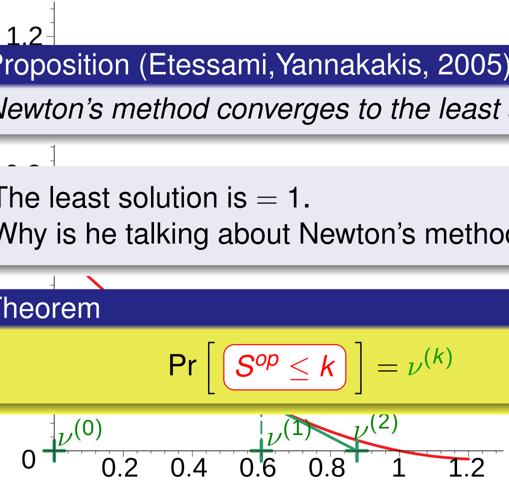

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Theorem

$$\Pr\left[\, S^{op} \le k \,\right] = \nu^{(k)}$$

Tail Bounds for the Optimal Scheduler

Theorem

$$\Pr\left[\, \boxed{S^{op} \leq k} \,\right] = \nu^{(k)}$$

It follows:

$$\Pr\left[\, \boxed{S^{op} \geq k} \,\right] = 1 - \nu^{(k-1)}$$

The $\nu^{(k)}$ converge to 1, so $\Pr\left[\, \boxed{S^{op} \geq k} \,\right]$ goes to 0.
But how fast??

Tail Bounds for the Optimal Scheduler

Theorem

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It follows:

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The $\nu^{(k)}$ converge to 1, so $\Pr\left[\boxed{S^{op} \ge k}\right]$ goes to 0.

But how fast??

Corollary (follows from KLE'07, EKL'08)

general task systems: $\Pr\left[\boxed{S^{op} \ge k}\right] \in \mathcal{O}(d^k) \quad (d < 1)$

subcritical task systems: $\Pr\left[\boxed{S^{op} \ge k}\right] \in \mathcal{O}(d^{2^k}) \quad (d < 1)$

Online Scheduling

$$X \xhookrightarrow{0.2} XY$$

$$X \xhookrightarrow{0.8} \varepsilon$$

$$Y \xhookrightarrow{0.3} YY$$

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X

Online Scheduling

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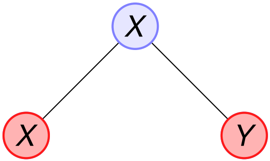

$$X \Rightarrow XY$$

Online Scheduling

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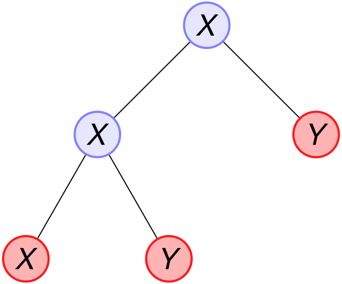

$$X \Rightarrow XY \Rightarrow XYY$$

Online Scheduling

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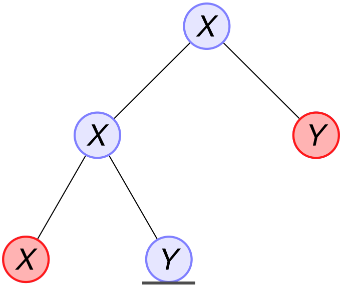

$$X \Rightarrow XY \Rightarrow XYY \Rightarrow {\color{red}XY}$$

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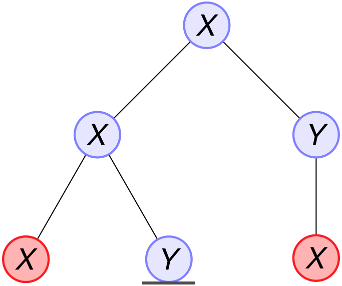

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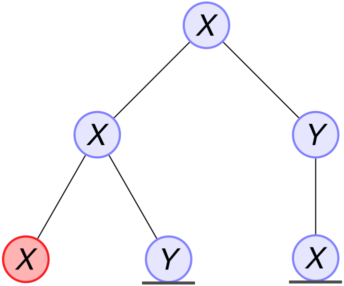

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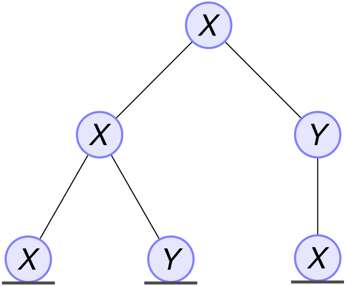

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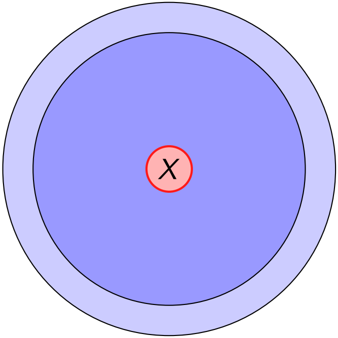

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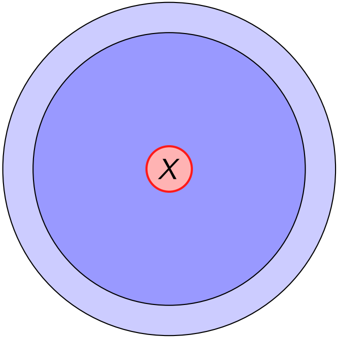

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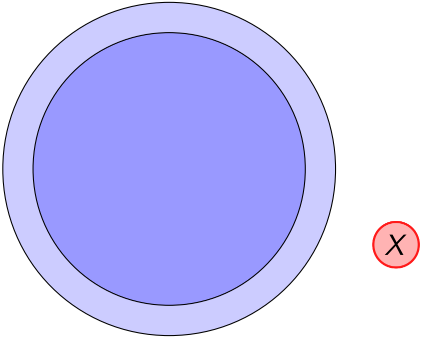

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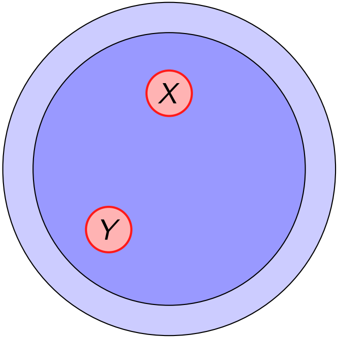

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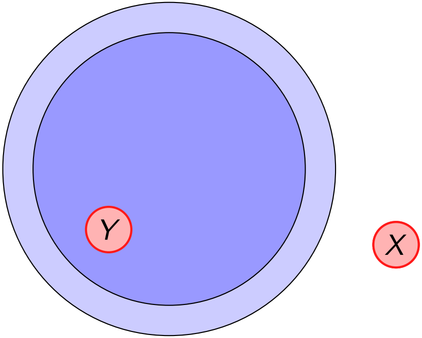

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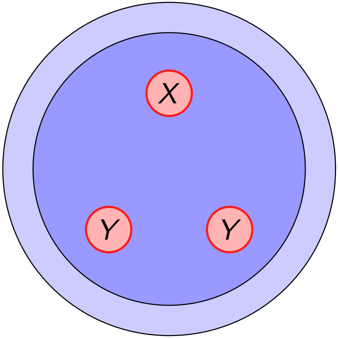

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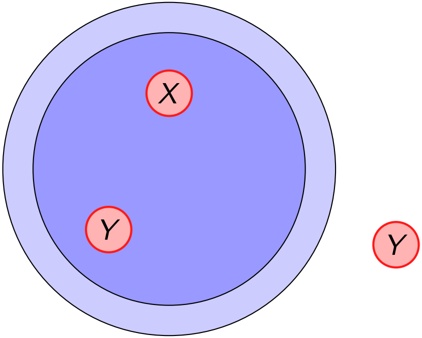

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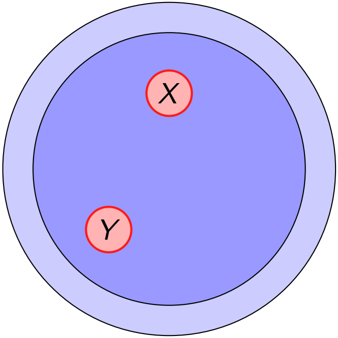

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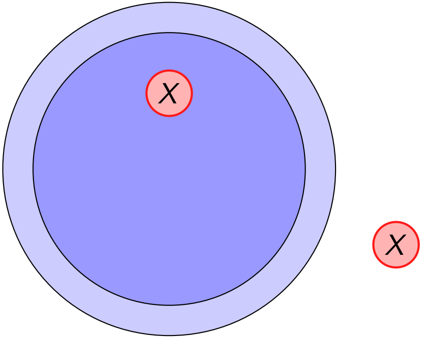

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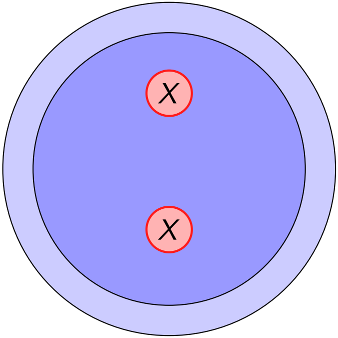

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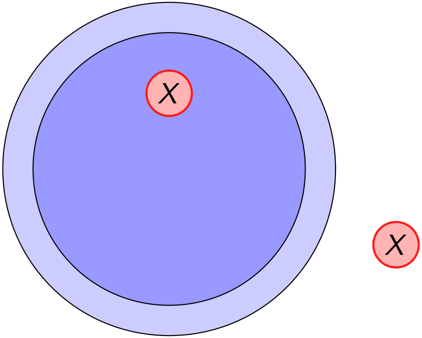

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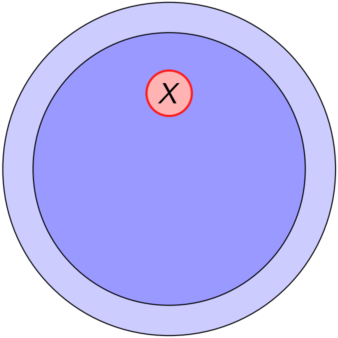

$$X \Rightarrow XY \Rightarrow XYY \Rightarrow XY \Rightarrow XX \Rightarrow X \Rightarrow \varepsilon$$

Online Scheduling

The urn model is more appropriate:

$$X \overset{0.2}{\hookrightarrow} XY$$

$$X \overset{0.8}{\hookrightarrow} \varepsilon$$

$$Y \overset{0.3}{\hookrightarrow} YY$$

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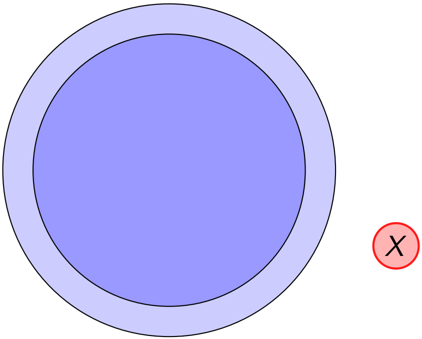

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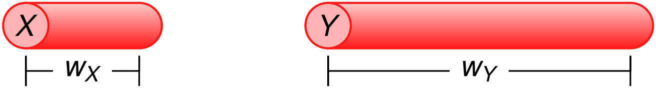
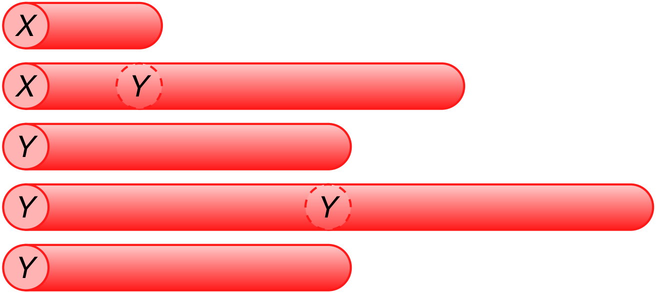
Weights: An Auxiliary Notion

Let $\boldsymbol{v} > \mathbf{1}$ be a vector with $\boldsymbol{v} \geq \boldsymbol{f}(\boldsymbol{v})$.
Choose $h > 1$ and for all types X a weight w_X with

$$h^{w_X} = \boldsymbol{v}_X \quad \text{for all types } X.$$



Denote by W the maximum weight of a derivation.
For instance: $X \Rightarrow XY \Rightarrow Y \Rightarrow YY \Rightarrow Y \Rightarrow \varepsilon$ yields



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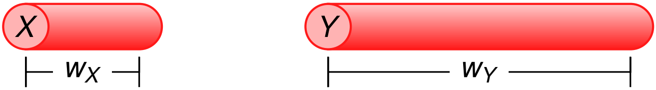

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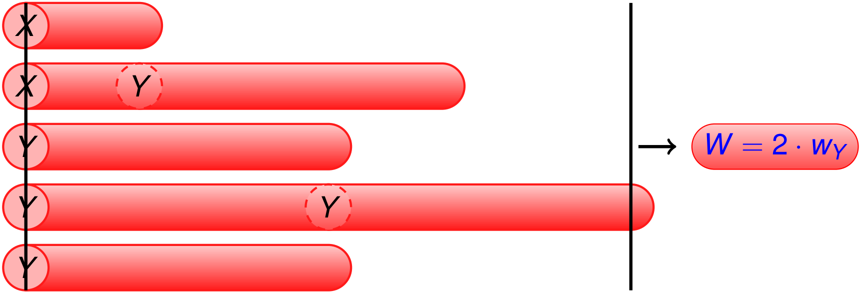

An Upper Bound for All Online Schedulers

(Recall: $\boldsymbol{v} > \mathbf{1}$ with $\boldsymbol{v} \geq \boldsymbol{f}(\boldsymbol{v})$ and $h^{w_X} = \boldsymbol{v}_X$.)

One can show by a martingale argument:

$$\Pr\left[W \geq k \right] \leq \frac{\boldsymbol{v}_{X_0}}{h^k}$$

Note: Whenever $S \geq k$ then $W \geq k \cdot w_{min}$.
So we obtain:

$$\Pr\left[S \geq k \right] \leq \Pr\left[W \geq k \cdot w_{min} \right] \leq \frac{\boldsymbol{v}_{X_0}}{h^{k \cdot w_{min}}} = \frac{\boldsymbol{v}_{X_0}}{\boldsymbol{v}_{min}{}^k}$$

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for all online schedulers σ and all $k \in \mathbb{N}$.

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Example

Consider the task system from the beginning:

$$f_x(x,y) = 0.2xy + 0.8$$
$$f_y(x,y) = 0.3y^2 + 0.1x + 0.6$$

One can show: $\boldsymbol{f}$ has two fixed points: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1.4 \\ 2.2 \end{pmatrix}$.

So: $\frac{c}{2.2^k} \leq \Pr\left[S^\sigma \geq k \right] \leq \frac{1.4}{1.4^k}$ holds for all σ.

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A Light-First Scheduler For Our Example

For the upper bound we used:

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We say, X is the lightest type.

Light-First Scheduler:

Process the lightest type (here: X) whenever it is in the pool.

In our example, the light-first scheduler guarantees: at any time at most one X-task in the pool.

Hence, with the light-first scheduler:

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Now, for the light-first scheduler *lf*:

$$\Pr\left[\, \boxed{S^{lf} \geq k} \,\right] \leq \frac{1.4}{1.4 \cdot 2.2^{k-1}}$$

So, *lf* achieves the optimal tail bound:

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Q: Well, what is a light type, intuitively?
A: Depends on $\boldsymbol{v}$, but if we compute $\boldsymbol{v}$ in a simple way, then the lightest type is the one with the smallest expected **time** (!)

Comparison

Notation:

op = the optimal offline scheduler

σ = any online scheduler

$0 < d < 1$

In the subcritical case:

	tail bound	Expectation
S^{op}	$\Pr\left[S^{op} \geq k \right] \in \mathcal{O}\left(d^{2^k}\right)$	finite
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S^{op}	$\Pr\left[S^{op} \geq k\right] \in \mathcal{O}\left(d^{k}\right)$	finite
S^{σ}	$\Pr\left[S^{\sigma} \geq k\right] \in \Omega\left(\frac{1}{k}\right)$	infinite

Conclusions

- We have studied the space consumption of stochastically generated tasks.
- Such task systems require scheduling.
- The performance of the optimal offline scheduler is closely linked with the convergence speed of Newton's method.
- The performance of the online schedulers is closely linked with fixed points of the function f.
- Light-First schedulers are good online schedulers.
- For critical systems, finite expectation is only achieved by offline schedulers.
- One can efficiently approximate expectations.

End of Talk

Thank you!